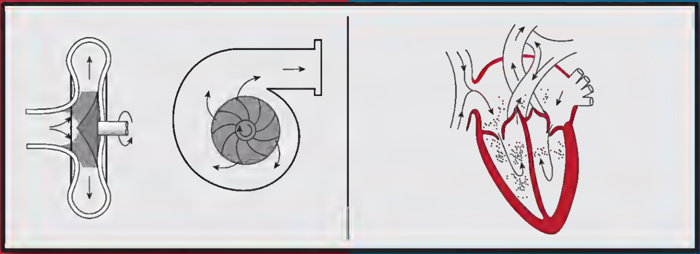
Fundamentals of Momentum, Heat, and Mass Transfer



Welty | Wicks | Wilson | Rorrer

SOLUTION MANUAL

Fifth Edition

CHAPTER 1

1.1 $n = 4\times10^{20}$ MOLECULES$/\text{IN}^3$

$$\bar{V} = \sqrt{kgRT} = 1.32\times10^{4}\ \text{in/s}$$

$$A = \pi/4\,(10^{-3}\text{in})^2$$

$$NA = \tfrac{1}{4}\, n\bar{V}A = 1.04\times10^{18}\ \text{m/s}$$

1.2 FLOW PROPERTIES:

VELOCITY
PRESSURE GRADIENT
STRESS

FLUID PROPERTIES:

PRESSURE
TEMPERATURE
DENSITY
SPEED OF SOUND
SPECIFIC HEAT

1.3 MASS OF SOLID $= \rho_S V_S$

" " FLUID $= \rho_F V_F$

$$x = \frac{\rho_S V_S}{\rho_S V_S + \rho_F V_F}$$

$$\Rightarrow \frac{V_F}{V_S} = \frac{1-x}{x}\,\frac{\rho_S}{\rho_F}$$

$$\rho_{mix} = \frac{\rho_S V_S + \rho_F V_F}{V_S + V_F} = \frac{\rho_S + \rho_F (V_F/V_S)}{1 + V_F/V_S}$$

$$= \frac{\rho_S \rho_F}{x\rho_F + (1-x)\rho_F}$$

1.4 GIVEN $\dfrac{P+B}{P_1+B} = \left(\dfrac{\rho}{\rho_1}\right)^7$

FOR $P_1 = 1$ ATM $\dfrac{\rho}{\rho_1} = 1.01$

$$P = 3001(1.01)^7 - 3000$$

$$= 217\ \text{ATM}$$

1.5 AT CONSTANT TEMPERATURE

$$P/\rho T = \text{CONST.} \Rightarrow P/\rho = \text{CONST.}$$

FOR 10% INCREASE IN ρ

P MUST ALSO INCREASE BY 10%

1.6 SINCE DENSITY VARIES AS

$$\rho = KP$$

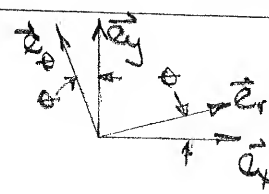
$$\rho_{250,000\ \text{FT}} = \rho_{S.L.}\,\frac{P_{250,000\ \text{FT}}}{P_{S.L.}}$$

& $\rho = nM$ (M = MOLECULAR WT)

$$\therefore\ n_{250,000} = n_{S.L.}\left[\frac{1.5\times10^{-7}}{2.378\times10^{-3}}\right]$$

$$= 4\times10^{20}\,[\ \ ] = 2.5\times10^{16}$$

1.7



$$\vec{e}_r = |\vec{e}_r|_x\,\vec{e}_x + |\vec{e}_r|_y\,\vec{e}_y$$

$$= \cos\theta\,\vec{e}_x + \sin\theta\,\vec{e}_r$$

$$\vec{e}_\theta = |\vec{e}_\theta|_x\,\vec{e}_x + |\vec{e}_\theta|_y\,\vec{e}_y$$

$$= -\sin\theta\,\vec{e}_x + \cos\theta\,\vec{e}_y$$

Q.E.D.

1.8 $$\frac{d\vec{e}_r}{d\theta} = -\sin\theta\,\vec{e}_x + \cos\theta\,\vec{e}_y = \vec{e}_\theta$$

$$\frac{d\vec{e}_\theta}{d\theta} = -\cos\theta\,\vec{e}_x - \sin\theta\,\vec{e}_y = -\vec{e}_r$$

Q.E.D.

1.9 TRANSFORMATION FROM (x,y) TO (r,θ)

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial x}\frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial y} = \frac{\partial r}{\partial y}\frac{\partial}{\partial r} + \frac{\partial \theta}{\partial y}\frac{\partial}{\partial \theta}$$

$$r^2 = x^2+y^2 \qquad \theta = \tan^{-1} y/x$$

SO: $$\frac{\partial r}{\partial x} = \frac{x}{(x^2+y^2)^{1/2}} = \frac{r\cos\theta}{r} = \cos\theta$$

$$\frac{\partial \theta}{\partial x} = -\frac{y}{x^2+y^2} = -\frac{r\sin\theta}{r^2} = -\frac{\sin\theta}{r}$$

$$\frac{\partial r}{\partial y} = \sin\theta \qquad \frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r}$$

$$\Rightarrow \quad \frac{\partial}{\partial x} = \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}$$

$$\frac{\partial}{\partial y} = \sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial \theta}$$

1.10 $$\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$$

$$= \left(\cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)\vec{e}_x + \left(\sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta}\right)\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$$

1.10 CONTINUED —

$$= \left(\vec{e}_x\cos\theta + \vec{e}_y\sin\theta\right)\frac{\partial}{\partial r} + \frac{1}{r}\left(-\vec{e}_x\sin\theta + \vec{e}_y\cos\theta\right)\frac{\partial}{\partial\theta} + \vec{e}_z\frac{\partial}{\partial z}$$

THUS: $$\nabla = \vec{e}_r\frac{\partial}{\partial r} + \frac{1}{r}\vec{e}_\theta\frac{\partial}{\partial\theta} + \vec{e}_z\frac{\partial}{\partial z}$$

1.11 $$\nabla p = \frac{\partial p}{\partial x}\vec{e}_x + \frac{\partial p}{\partial y}\vec{e}_y$$

$$\nabla p(a,b) = \rho_\infty V_\infty^2\left\{\left[\frac{1}{a}\cos 1\sin 1 + 2\right]\vec{e}_x + \frac{1}{b}\left(\sin 1\cos 1\right)\vec{e}_y\right\}$$

$$= \rho_\infty V_\infty^2\left\{\left[\frac{1}{a}\frac{\sin 2}{2} + 2\right]\vec{e}_x + \frac{1}{b}\left(\frac{\sin 2}{2}\right)\vec{e}_y\right\}$$

1.12 $$\nabla T(x,y) = T_0 e^{-1/4}\left[\frac{1}{a}\left(\cos\frac{x}{a}\cosh\frac{y}{b}\right)\vec{e}_x + \frac{1}{b}\left(\sin\frac{x}{a}\sinh\frac{y}{b}\right)\vec{e}_y\right]$$

$$\nabla T(a,b) = T_0 e^{-1/4}\left[\frac{1}{a}(\cos 1\cosh 1)\vec{e}_x + \frac{1}{b}(\sin 1\sinh 1)\vec{e}_y\right]$$

$$= T_0 e^{-1/4}\left[\frac{\cos 1\,(e+e^{-1})}{2a}\vec{e}_x + \frac{\sin 1\,(e-e^{-1})}{2b}\vec{e}_y\right]$$

$$= \frac{T_0 e^{-5/4}}{2}\left[\frac{\cos 1}{a}\left(1+e^{2}\right)\vec{e}_x + \frac{\sin 1}{b}\left(1-e^{2}\right)\vec{e}_y\right]$$

1.13 IN PROB 1.12 $T(x,y)$ IS DIMENSIONALLY HOMOGENEOUS (D.H.)

$P(x,y)$ IN PROB 1.11 WILL BE D.H. IF

$$P_0 \sim \frac{P}{V_0^2} \quad LB_F S^2/FT^4$$

OR USING THE CONVERSION FACTOR g_c

1.14 $\phi = 3x^2 y + 4y^2$

a) $\nabla\phi = (6xy)\vec{e}_x + (3x^2+8y)\vec{e}_y$

$$\nabla\phi\,(3,5) = 90\,\vec{e}_x + 67\,\vec{e}_y$$

b) $$\nabla\phi \cdot \vec{e}_s = \left[6xy\,\vec{e}_x + (3x^2+8y)\vec{e}_y\right] \cdot \left[\cos\theta\,\vec{e}_x + \sin\theta\,\vec{e}_y\right]$$

AT POINT (3,5)

$$\nabla\phi \cdot \vec{e}_s = (90\,\vec{e}_x + 67\vec{e}_y) \cdot \left[\cos(-60)\vec{e}_x + \sin(-60)\vec{e}_y\right]$$

$$= 45 - 58.02 = -13.02$$

1.15 FOR AN IDEAL GAS

$$P = \rho \frac{RT}{M}$$

FROM PROB. 1.3 $\rho = \dfrac{\rho_m(1-x)}{1-\dfrac{\rho_m}{\rho_s}x}$

$$\therefore P = \frac{\rho_m(1-x)}{1-\dfrac{\rho_m}{\rho_s}x}\,\frac{RT}{M}$$

1.16 $\psi = Ar\sin\theta\left(1-\dfrac{a^2}{r^2}\right)$

a) $$\nabla\psi = \frac{\partial\psi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\vec{e}_\theta$$

$$= A\sin\theta\left(1-a^2/r^2\right)\vec{e}_\theta$$

b) $$|\nabla\psi| = A\left[\sin^2\theta\left(1+a^2/r^2\right)^2 + \cos^2\theta\left(1-a^2/r^2\right)^2\right]^{1/2}$$

$|\nabla\psi|_{MAX}$ IS GIVEN BY $d|\nabla\psi| = 0$

OR $\dfrac{\partial}{\partial r}|\nabla\psi|\,dr + \dfrac{\partial}{\partial\theta}|\nabla\psi|\,d\theta = 0$

REQUIRING $\dfrac{\partial}{\partial r}|\nabla\psi| = \dfrac{\partial}{\partial\theta}|\nabla\psi| = 0$

FOR $\dfrac{\partial}{\partial r}|\nabla\psi| = 0$:

$$-\sin^2\theta\left(1+a^2/r^2\right) + \cos^2\theta\left(1-a^2/r^2\right) = 0 \qquad (1)$$

$\frac{1}{5}$ FOR $\dfrac{\partial}{\partial\theta}|\nabla\psi| = 0$

$$\sin\theta\cos\theta\left[\left(1+a^2/r^2\right)^2 - \left(1-a^2/r^2\right)^2\right] = 0 \qquad (2)$$

FROM EQ 2: $\sin\theta\cos\theta\;4a^2/r^2 = 0$

IF $a \neq 0, r \neq 0$ THEN $\sin\theta\cos\theta = 0$

FOR WHICH $\theta = 0, \pi/2$ (3)

SUBST INTO EQ. 1 . $\theta = 0,\; 1 - a^2/r^2 = 0$

GIVING $a = r$

FOR $\theta = \pi/2$ $\quad 1 + a^2/r^2 = 0$ ~ IMPOSSIBLE

THUS CONDITIONS FOR $|\nabla\psi|_{MAX}$ ARE

$$\theta = 0 \quad r = a$$

1.17 $P = P_0 + \frac{1}{2}\rho U_\infty^2\left[\frac{2xyz}{L^3} + 3\left(\frac{x}{L}\right)^2 + \frac{U_\infty t}{L}\right]$

$$\frac{\partial P}{\partial x}\vec{e}_x = \frac{1}{2}\rho U_\infty^2\left[\frac{2yz}{L^3} + \frac{6x}{L^2}\right]\vec{e}_x$$

$$\frac{\partial P}{\partial y}\vec{e}_y = \frac{1}{2}\rho U_\infty^2\left[\frac{2xz}{L^3}\right]\vec{e}_y$$

$$\frac{\partial P}{\partial z}\vec{e}_z = \frac{1}{2}\rho U_\infty^2\left[\frac{2xy}{L^3}\right]\vec{e}_y$$

$$\nabla P = \frac{1}{2}\rho U_\infty^2\left[\left(\frac{2xz}{L^3} + \frac{6x}{L^2}\right)\vec{e}_x + \frac{2xz}{L^3}\vec{e}_y + \frac{2xy}{L^3}\vec{e}_z\right]$$

1.18 VERTICAL CYLINDER $d = 10\,m$
$h = 6\,m$

$$V = \frac{\pi}{4}(10\,m)^2(6\,m) = 471.2\,m^3$$

@ $20°C$ $\rho_w = 998.2\,kg/m^3$

$$m = \rho_w V = (998.2)(471.2) = 470350\,kg$$

@ $80°C$ $\rho_w = 971.8\,kg/m^3$

$$m = (971.8)(471.2) = 457910\,kg$$

$$\Delta m = 12440\,kg$$

1.19 LIQUID – $V = 1200\,cm^3$ @ $1.25\,MPa$
$V = 1188\,cm^3$ @ $2.5\,MPa$

$$\beta = -V\left(\frac{\partial P}{\partial V}\right)_T \cong -V\frac{\Delta P}{\Delta V}$$

$$V = 1194\,cm^3 = 1.194\times10^{-3}\,m^3$$

$$\Delta V = -12\,cm^3 = -1.2\times10^{-7}\,m^3$$

$$\beta = -1.194\times10^{-3}\left[\frac{1.25\,MPa}{-1.2\times10^{-7}}\right]$$

$$= +12440\,MPa = +12.44\,MPa$$

1.20 $\beta = -V\left(\frac{\partial P}{\partial V}\right)_T \cong -V\frac{\Delta P}{\Delta V}$

$$V = 0.25\,m^3$$

$$\Delta V = -0.005\,m^3$$

$$\Delta P = 10\,MPa$$

$$\beta = -0.25\left[\frac{10}{-0.005}\right] = 500\,MPa$$

1.21 FOR H_2O – $\beta = 2.205\,GPa$

$$\frac{\Delta V}{V} = -0.0075$$

$$\beta \cong -V\frac{\Delta P}{\Delta V} \quad \text{OR} \quad \Delta P = \beta\frac{\Delta V}{V}$$

$$\Delta P = (2.205\,GPa)(0.0075)$$

$$= 0.0165\,GPa = 16.5\,MPa$$

1.22 H_2O: $P_1 = 100\,kPa$ $P_2 = 120\,MPa$
$\beta = 2.205\,GPa$

$$\beta \cong -V\frac{\Delta P}{\Delta V} \quad \text{OR} \quad \frac{\Delta V}{V} = \frac{\Delta P}{\beta}$$

$$\frac{\Delta V}{V} = \frac{\Delta P}{\beta} = \frac{(120000 - 100)\,kPa}{120\times10^6\,kPa}$$

$$= 0.999\times10^{-3}$$

$$= 0.0999\ \text{PERCENT}$$

1.23 H_2O @ 68°C (341 K)

$$\sigma = 0.123\left[1 - 0.00139(341)\right]$$

$$= 0.0647 \text{ N/m}$$

IN A CLEAN TUBE – $\theta = 0°$

$$h = \frac{2\sigma \cancel{\cos\theta}^{1}}{\rho g r}$$

$$= \frac{2(0.0647)}{979\,(9.81)(0.2875\times10^{-2}/2)}$$

$$= 9.37\times10^{-3}\text{ m} = 9.37 \text{ mm}$$

1.24 PARALLEL GLASS PLATES
- GAP = 1.625 mm

$$\sigma = 0.0735 \text{ N/m}$$

FOR A UNIT DEPTH:

SURFACE TENSION FORCE = $2(1)\sigma\cos\theta$

WEIGHT OF H_2O = $\rho g h(1)(1.625\times10^{-3})$

FOR CLEAN GLASS $\cos\theta = 1$

EQUATING FORCES:

$$2(1)(\sigma) = \rho g h(1)(1.625\times10^{-3})$$

$$h = \frac{2\,(0.0735)}{(1000)(9.81)(1.625\times10^{-3})}$$

$$= 0.00922 \text{ m} = 9.22 \text{ mm}$$

1.25 GLASS TUBE – $d_i = 0.25$ mm
$d_o = 0.35$ mm

$$\theta = 130°$$

SURFACE TENSION FORCE –

INSIDE – $2\pi r_i \sigma \cos\theta$

OUTSIDE – $2\pi r_o \sigma \cos\theta$

TOTAL UPWARD FORCE –

$$f = 2\pi\sigma\cos\theta\,(r_i + r_o)$$

$$= 2\pi\,(0.44)(\cos 130°)\left(\frac{0.25+0.35}{2}\times10^{-3}\right)$$

$$= 5.33\times10^{-4}\text{ N}$$

1.26 H_2O-AIR-GLASS INTERFACE @ 40°C

TUBE RADIUS = 1 mm

$$h = \frac{2\sigma\cancel{\cos\theta}^{1}}{\rho g r}$$

$$\sigma = 0.123\left[1 - 0.00139(313)\right] = 0.0695 \text{ N/m}$$

$$h = \frac{2(0.0695)}{(993)(9.81)(1\times10^{-3})}$$

$$= 0.0143 \text{ m} \quad (1.43 \text{ cm})$$

1.27 SOAP BUBBLE – T = 20°C $d = 4$ mm
$\sigma = 0.025$ N/m (TABLE 1.2)

FORCE BALANCE FOR BUBBLE:

$$\pi r^2 \Delta p = 2\pi r\sigma$$

$$\Delta p = \frac{2\sigma}{r} = \frac{2(0.025)}{2\times10^{-3}}$$

$$= 25 \text{ N/m}^2 = 25 \text{ Pa}$$

1.28 GLASS TUBE IN Hg (S.G. = 13.6)

FOR Hg / GLASS – $\sigma = 0.44$ N/m

$\theta = 130^\circ$

$$h = \frac{2\sigma}{\rho g r} \qquad r = 3\text{mm}$$

$$= \frac{2(0.44)}{13.6(1000)(1.5\times10^{-3})}$$

$$= -0.0277 \text{ m}$$

$$= 2.77 \text{ cm DEPRESSION}$$

1.29 @ 60°C $\sigma_{H_2O} = 0.0662$ N/m

$\sigma_{Hg} = 0.44$

TUBE DIAMETER = 0.55 MM

$$h = \frac{2\sigma\cos\theta}{\rho g r}$$

FOR H_2O:

$$h = \frac{2(0.0662)(\cancel{\cos 0})^{1}}{983(9.81)(0.55\times10^{-3}/2)}$$

$$= 0.0499 \text{ m} \quad (4.99 \text{ cm RISE})$$

FOR Hg:

$$h = \frac{2(0.44)(\cos 130^\circ)}{13.6(983)(9.81)\left(\frac{0.55\times10^{-3}}{2}\right)}$$

$$= -0.0157 \text{ m}$$

(1.57 cm DEPRESSION)

1.30 H_2O / GLASS INTERFACE

$T = 30^\circ C$

$$\sigma = 0.123\left[1 - 0.00139(303)\right]$$

$$= 0.0712 \text{ N/m}$$

$$\rho = 996 \text{ kg/m}^3$$

$$h \le 1 \text{ mm}$$

$$h = \frac{2\sigma\,\cancel{\cos\theta}^{\,1}}{\rho g r}$$

$$r = 2\sigma / \rho g h$$

$$= \frac{2(0.0712)}{996(9.81)(1\times10^{-3})}$$

$$= 0.0146 \text{ m} \quad (1.457 \text{ cm})$$

$$d = 2r = 2.915 \text{ cm}$$

2.1 ASSUME IDEAL GAS BEHAVIOR:

$$\frac{dP}{dy} = -\rho g = -\frac{Pg}{RT}$$

FOR $T = a + by$

$$\Rightarrow \quad T = 530 - 24\, y/h$$

$$\frac{dP}{P} = -\frac{g}{R}\,\frac{dy}{530 - 24(y/h)}$$

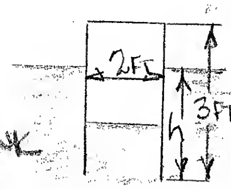
$$\int_{P_0}^{P} \frac{dP}{P} = \frac{gh}{24R}\int_0^1 \frac{-24\; d(y/h)}{530 - 24(y/h)}$$

$$\ln\frac{P}{P_0} = \frac{gh}{24R}\ln\frac{506}{530}$$

WITH $P = 10.6$ PSIA, $P_0 = 30.1$ inHg

$$h = 9192 \text{ FT}$$

2.2



$\sum F_y = 0$ ON TANK

$$P\frac{\pi d^2}{4} - P_{atm}\frac{\pi d^2}{4} - 250 = 0 \qquad (1)$$

AT H_2O LEVEL IN TANK:

$$P = P_{ATM} + \rho_w g (h - y) \qquad (2)$$

FROM (1) & (2) $\quad h - y = 1.275$ FT $\quad$ (3)

FOR ISOTHERMAL COMPRESSION OF AIR

$$P_{ATM} V_{TANK} = P(V_{AIR})$$

$$P = \frac{3}{3-y} P_{ATM} \qquad (4)$$

COMBINING (1) & (4)

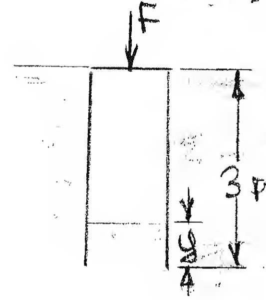
$$y = 0.12 \text{ FT}$$

$$h = 1.395 \text{ FT.}$$

2.2 CONT.

FOR TOP OF TANK FLUSH WITH H_2O LEVEL

$\sum F_y = 0$



$$P = P_{ATM} + \frac{250 + F}{\pi d^2/4}$$

AT H_2O LEVEL IN TANK:

$$P = P_{ATM} + \rho_w g (3 - y)$$

COMBINING EQUATIONS:

$$F = 196(3 - y) - 250$$

FOR ISOTHERMAL COMPRESSION OF AIR: (AS IN PART (a))

$$3 - y = 2.8 \text{ FT}$$

$$\Rightarrow \quad F = 196(2.8) - 250 = \underline{293.6 \text{ lbf}}$$

2.3 WHEN NET FORCE ON TANK = 0

WT = BUOYANT FORCE = 250 LBF

V_W DISPLACED $= 250/\rho_w g = 4.01 \text{ FT}^3$

ASSUMING ISOTHERMAL COMPRESSION

$$P_{ATM} A(3\text{FT}) = P(4.01 \text{ FT}^3)$$

$$= (P_{atm} + \rho g y)(4.01)$$

$$y = 45.88 \text{ FT}$$

TOP IS AT LEVEL: $\quad y - \dfrac{4.01}{\pi d^2/4}$

OR AT 44.6 FT BELOW SURFACE

2.4.

$$\frac{dp}{dy} = \rho g = \rho_0 e^{\Delta p/\beta}$$

$$\int_0^{\Delta p} \frac{-\Delta p/\beta}{e^{\Delta p/\beta}} = \int_0^{y} -\frac{\rho_0 g \, \Delta y}{\beta}$$

$$e^{-\Delta p/\beta} = 1 - \frac{\rho_0 g y}{\beta g}$$

$$\Delta p = -\beta \ln\left(1 - \frac{\rho_0 y}{\beta}\right)$$

$$= 300{,}000 \ln(1 - 0.0462)$$

$$= \underline{14190 \text{ psi}}$$

DENSITY RATIO:

$$\frac{\rho}{\rho_0} = e^{-\Delta p/\beta} = 1.0484$$

SO $\underline{\rho = 1.0484 \rho_0}$

2.5. BUOYANT FORCE:

$$F_B = \rho V = \frac{pV}{RT}$$

FOR CONSTANT VOLUME:

F VARIES INVERSELY WITH T

2.6 SEA H_2O : $S.G. = 1.025$

AT DEPTH $y = 185$ m

$$p_g = 1.025 \rho_w g y$$

$$= 1.025(1000)(9.81)(185)$$

$$= 1.86 \times 10^6 \text{ Pa}$$

$$= 1.86 \text{ MPa}$$

2.7 r MEASURED FROM EARTH'S SURFACE

R = RADIUS OF EARTH

$$\frac{dp}{dr} = \rho g = \rho g_0 \frac{r}{R}$$

$$p - p_{ATM} = \frac{\rho g_0 r^2}{2R}$$

AT CENTER OF EARTH -- $r = R$

$$p_{CTR} - p_{ATM} = \frac{\rho g_0 R}{2}$$

SINCE $p_{CTR} >> p_{ATM}$

$$p_{CTR} \cong \frac{\rho g_0 R}{2} = \frac{(5.67)(1000)(9.81)(6330 \times 10^3)}{2}$$

$$= 176 \times 10^9 \text{ Pa}$$

$$\sim 176 \text{ MPa}$$

2.8

$$\frac{dp}{dy} = -\rho g$$

$$\int_{p_{ATM}}^{p} dp = -\rho g \int_0^{-h} dy$$

$$p - p_{ATM} = \rho g (+h)$$

$$= (1050)(9.81)(11034)$$

$$= 113.7 \text{ MPa}$$

$$\cong 1122 \text{ ATMOSPHERES}$$

2.9 AS IN PREVIOUS PROBLEM

$$p - p_{ATM} = \rho g h$$

FOR $p - p_{ATM} = 101.33$ kPa

$$h = 101.33/\rho g$$

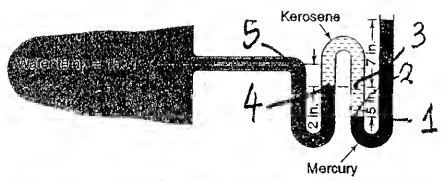
2.9 (CONT.)

FOR H_2O: $h = \frac{101.33}{(1000)(9.81)} = 10.33\,m$

SEA H_2O: $h = \frac{101.33}{(1.025)(1000)(9.81)} = 10.08\,m$

Hg $h = \frac{101.33}{13.6(1000)(9.81)} = 0.80\,M$

2.10



$P_1 = P_{atm} + \rho_{Hg}\, g\, (12'')$ $\quad P_1 = P_2$

$P_2 = P_3 + \rho_K\, g\, (5'')$ $\quad P_3 = P_4$

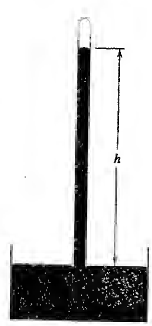
$P_4 = P_A + \rho_W\, g\, (2'')$ $\quad P_4 = P_5$

$P_{atm} + \rho_{Hg}\, g(12) = P_A + \rho_W g(2) + \rho_K g(5)$

$P_A = P_{ATM} + \rho_W g\left[(13.6)(12) - 2 - 0.75(5)\right]$

$= P_{atm} + 5.81\ PSI = \underline{5.81\ PSIG}$

2.11 FORCE BALANCE ON LIQUID COLUMN:



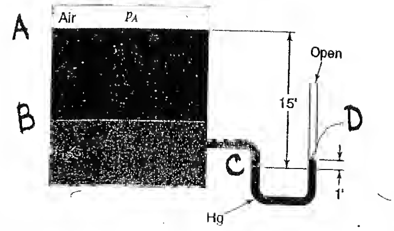
A = AREA OF TUBE

$-3A + 14.7A - \rho g h A = 0$

$h = \frac{11.7(144)}{62.4(12.2)}$

$= \underline{26.6\ IN.}$

2.12



$P_A = P_B - \rho_o g(10\,FT)$

$P_C = P_B + \rho_W g(5\,FT)$

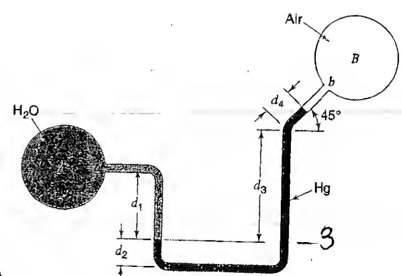
$P_D = P_C - \rho_{Hg} g(1\,FT)$

$P_A - P_D = \rho_{Hg} g(1) - \rho_W g(5) - \rho_o g(10)$

$P_A - P_{ATM} = \rho_W g\left(13.6 \times 1 - 5 - 0.8 \times 10 \times 1\right)$

$= 37.4\ LB_F/FT^2$

2.13



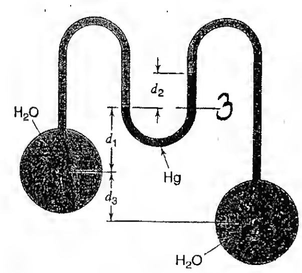
$P_3 = P_A - d_1 g \rho_W$

$= P_B + (\rho_{Hg}\, g) \times (d_3 + d_4 \sin 45)$

$P_A - P_B = \frac{(62.4)(32.2)}{32.2}\left[\left(\frac{2.4 + 4\sin 45}{12}\right)13.6 - 2\right]$

$= 245\ LB_F/FT^2 = 1.70\ PSI$

2.14



$P_3 = P_A - \rho_W g\, d_1$

$P_3 = P_B - \rho_W g(d_1 + d_2 + d_3) + \rho_{Hg} g\, d_2$

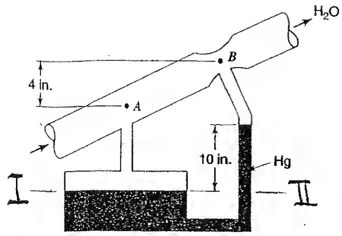
EQUATING:

$P_A - P_B = \rho_{Hg} g d_2 - \rho_W g\, (d_2 + d_3)$

$= \rho_W g\left[(13.6)(1/12) - 7.3/12\right]$

$= 32.8\ LB_F/FT^2 = 0.227\ PSI$

2.15



$P_I = P_A + \rho_w g\,(10'')$

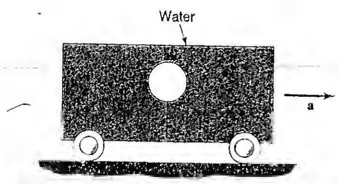
$P_{II} = P_B + \rho_w g\,(4'') + \rho_{Hg} g\,(10'')$

$P_I = P_{II}$

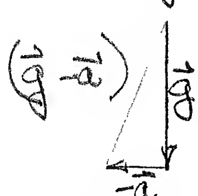
$P_A - P_B = \rho_w g\left[-6 + 13.6(10)\right]$

$= \underline{56.3 \text{ PSI}}$

2.16

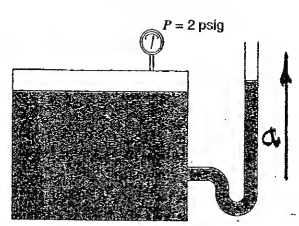


PRESSURE GRADIENT IS IN DIRECTION OF $\vec{g} - \vec{a}$ & ISOBARS ARE PERPENDICULAR TO $(\vec{g} - \vec{a})$



STRING WILL ASSUME THE $(\vec{g} - \vec{a})$ DIRECTION & BALLOON WILL MOVE FORWARD.

2.17



AT REST:

$P = \rho g y_o$

ACCELERATING:

$P = \rho|\vec{g} - \vec{a}| = \rho(g+a)y_a$

EQUATING: $y_a = \frac{g}{g+a}$ WHICH $< y_o$

LEVEL GOES DOWN

2.18 $F = P_{G.G.}A - P_{ATM}A - \rho g h(\pi r^2)$

$h = 2\text{m} \quad r = 0.3\text{m}$

$F = 5546 \text{ N}$

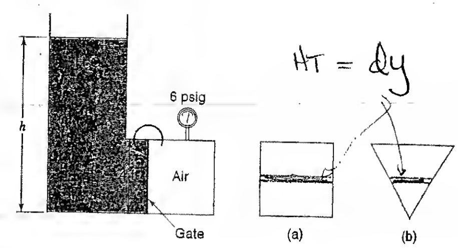
$y_{c.p.} = \bar{y} + I_{bb}/A\bar{y}$

FOR A CIRCLE: $I_{bb} = \pi r^4/4$

$y_{cp} = 2\text{m} + \frac{\pi(0.3\text{m})^4}{4\pi(0.3\text{m})^2(2\text{m})}$

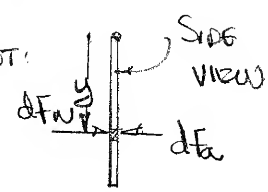
$= 2.011 \text{ m}$

2.19



HEIGHT OF H_2O COLUMN ABOVE DIFFERENTIAL ELEMENT:

$= h - 4 + y$



FOR (a) – RECTANGULAR GATE – $dA = 4\,dy$

$dF_w = \left[\rho_w g(h-4+y) + P_{ATM}\right] dA$

$dF_A = \left[P_{ATM} + (6\text{ psig})(144)\right] dA$

$\sum M_0 = \int_A y\,(dF_w - dF_A) = 0$

$\int_0^4 y\left[\rho g(h-4+y) - 864\right](4\,dy) = 0$

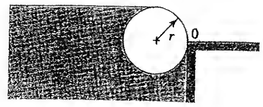
$h = 15.18 \text{ FT}$

FOR (b): $dA = (4-y)\,dy$

$\int_0^4 y\left[\rho g(h-4+y) - 864\right](4-y)\,dy = 0$

$h = 15.85 \text{ FT}$

2.20



PER UNIT DEPTH:

$\sum F_y = 0$

$F_y|_{up} = \rho_w g \pi r^2 / 2$ {BUOYANCY}

$F_y|_{down} = \rho g \pi r^2 + \rho_w g \left(r^2 - \pi r^2/4\right)$

EQUATING EQ.:

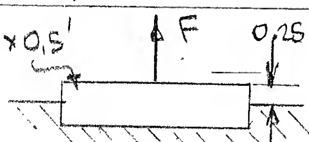
$$\frac{\rho_w g \pi r^2}{2} = \rho g \pi r^2 + \rho_w g r^2 \left(1 - \frac{\pi}{4}\right)$$

$$\rho = \rho_w \left(\frac{\pi}{2} - 1 + \frac{\pi}{4}\right) / \pi$$

$$= \rho_w \left(\frac{3}{4} - \frac{1}{\pi}\right) = 0.432\, \rho_w$$

$$= \underline{432\ \text{kg/m}^3}$$

2.21



a) TO LIFT BLOCK FROM BOTTOM

$F = \{\text{WT OF CONCRETE}\} + \{\text{WT OF } H_2O\}$

$= \rho_c g V + \left[\rho_w g (22.75') + P_{ATM}\right] A$

$= (150) g (3 \times 3 \times 0.5)$

$+ \left[62.4 g (22.75) + 14.7(144)\right] \times (3 \times 3)$

$= 675 + 31828 = \underline{32503\ \text{LB}_F}$

2.21 (CONT)

b) TO MAINTAIN BLOCK IN FREE POSITION.

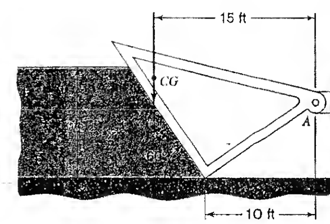
$F = \{\text{WT OF CONCRETE}\} - \{\text{BUOYANT FORCE OF } H_2O\}$

$= 675 - \rho_w g V$

$= 675 - \left[62.4 g (3 \times 3 \times 0.5)\right]$

$= 675 - 281 = \underline{394\ \text{LB}_F}$

2.22.



DISTANCE Z MEASURED ALONG GATE SURFACE FROM BOTTOM

$$\sum M_A = 500(15) - \int_0^{h/\sin 60} z \rho g (h - z \sin 60)\, dz = 0$$

$$\rho g \int_0^{h/\sin 60} (zh - z^2 \sin 60)\, dz = 7500$$

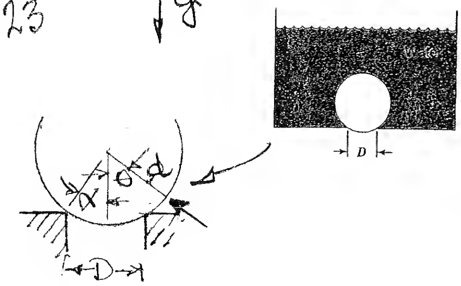
$$\rho g \left[h z^2/2 - \frac{z^3}{3} \sin 60 \right]_0^{h/\sin 60} = 7500$$

$$(62.4) g \left[\frac{h^3}{(\sin 60)^2} \left(\frac{1}{2} - \frac{1}{3} \right) \right] = 7500$$

$$h^3 = \frac{7500 (6)(\sin 60)^2}{62.4 g} = 541$$

$$\underline{h = 8.15\ \text{FT}}$$

2.23 $\downarrow y$



USING SPHERICAL COORDINATES FOR A RING AT $y =$ CONSTANT:

$$dA = 2\pi r^2 \sin\theta \, d\theta$$

$$p = \rho g\left[h - r\cos\alpha + r\cos\theta\right]$$

$$df_y = df \cos\theta$$

$$F_y = \int \rho g\left(h - r\cos\alpha + r\cos\theta\right)\left(2\pi r^2 \sin\theta\cos\theta \, d\theta\right)$$

$$= \underbrace{2\pi\rho g r^2}_{C}\int_\alpha^\pi \left(h - r\cos\alpha + r\cos\theta\right)\sin\theta\cos\theta \, d\theta$$

$$= C\left[\int_\alpha^\pi \left(h - r\cos\alpha\right)\sin\theta\cos\theta \, d\theta + r\int_\alpha^\pi \sin\theta\cos^2\theta \, d\theta\right]$$

$$= C\left[\left(h - r\cos\alpha\right)\sin^2\theta\Big|_\alpha^\pi + r\left(-\frac{1}{3}\cos^3\theta\right)\Big|_\alpha^\pi\right]$$

$$= C\left[\left(h - r\cos\alpha\right)\left(1 - \sin^2\alpha\right) - \frac{r}{3}\left(0 - \cos^3\alpha\right)\right]$$

NOW - FOR $F_y = 0$

$$\sin\alpha = \frac{D}{d} \quad \cos\alpha = \left[1 - \left(\frac{D}{d}\right)^2\right]^{1/2}$$

$$\& \; r = \frac{d}{2}$$

2.23 CONT.

$$0 = \left(h - \frac{d}{2}\cos\alpha\right)\left(\cancel{\cos^2\alpha}\right) + \frac{d}{6}\left(\cos^{\cancel{3}}\alpha\right)$$

$$= h - \frac{d}{2}\cos\alpha + \frac{d}{6}\cos\alpha$$

GIVING $h = \frac{d}{3}\cos\alpha$

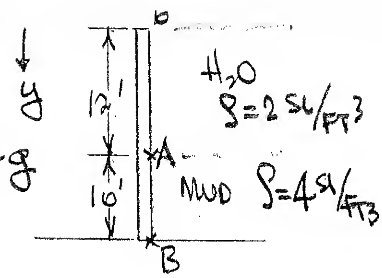
$$\frac{h}{d} = \frac{\cos\alpha}{3} = \frac{1}{3}\left[1 - \left(\frac{D}{d}\right)^2\right]^{1/2}$$

FOR $d = 0.6$ m

$$h = \frac{0.6}{3}\left[1 - \left(\frac{5D}{3}\right)^2\right]^{1/2}$$

$$= \frac{1}{5}\left[1 - \left(\frac{5D}{3}\right)^2\right]^{1/2}$$

2.24



$$P_A - P_{ATM} = \rho_w g(12) = 24g$$

$$P_B - P_{ATM} = 24g + 40g = 64g$$

BETWEEN 0 & A: $P - P_{ATM} = \rho_w g y$

" A & B $P = \rho_w g(12) + \rho_m g(y - 12)$

FOR UNIT DEPTH:

$$F = \int (P - P_{ATM}) \, dA$$

$$= \int_0^{12} \rho_w g y \, dy + \int_{12}^{22}\left[\rho_w g 12 + \rho_m g(y - 12)\right] dy$$

$$= \rho_w g(192) + \rho_m g(50)$$

$$= \underline{18790 \text{ lb}_F}$$

2.24 CONT.

FORCE LOCATION:

$$F\bar{y} = \int_0^{22} y(P - P_{ATM})\,dA$$

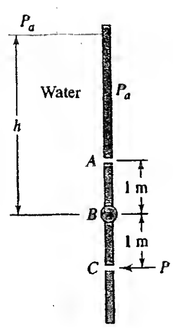
$$= \int_0^{12} \rho_w g y^2\,dy + \int_{12}^{22} \rho_w g\, 12y\,dy + \int_{12}^{22} \rho_m g (y^2 - 12y)\,dy$$

$$= \rho_w g (576 + 2040) + \rho_m g (2973 - 2040)$$

$$= 288\,400 \text{ FT LB}_F$$

$$\bar{y} = \frac{288400}{18790} = \underline{15.35 \text{ FT}}$$

2.25



FORCE ON GATE $= \rho g \bar{y} A$

$$= (1000)(9.81)(12)\frac{\pi}{4}(2)^2$$

$$= 369.8 \text{ kN}$$

$$y_{cp} = \frac{I_{oo}}{\bar{y}A} = \frac{(\pi/4)(1)^4}{(12)(\pi/4)(2)^2} = 0.0208 \text{ m}$$

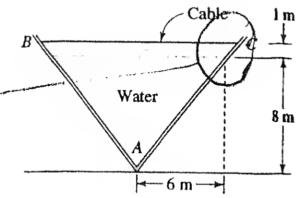
(BELOW AXIS B)

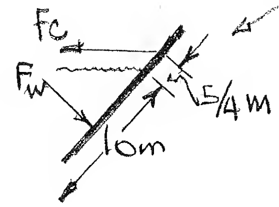
$$\Sigma M_B = 0$$

$$P(1) = (369.8 \times 10^3)(0.0208)$$

$$= .770 \text{ kN}$$

2.26





$$F_W = \rho g \bar{y} A$$

$$= (1000)(9.81)(4)(10)(1) = 392 \text{ kN}$$

y_{cp} IS $\frac{2}{3}$ DISTANCE FROM WATER LINE TO A

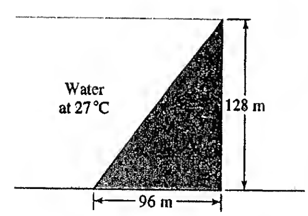
~ 6.66 m DOWN FROM H_2O LINE
3.33 m UP FROM A

$$\Sigma M_A = F_C(9) = 392(3.33)$$

$$\underline{F_C = 145.2 \text{ kN}}$$

2.27 WIDTH = 100m

H_2O @ 27°C $\rho = 997\ kg/m^3$



$$F = \rho g \bar{y} A$$

$$= (997)(9.81)(64) \times (160)(100)$$

$$= 10.016 \times 10^9 \text{ N} = \underline{10.02 \times 10^3 \text{ MN}}$$

FOR A FREE H_2O SURFACE

$$y_{cp} = \frac{2}{3}(128 \text{ m}) = 85.3 \text{ m}$$ {BELOW H_2O SURFACE}

$$= 106.7 \text{ m}$$ (MEASURED ALONG DAM SURFACE)

2.28 SPHERICAL FLOAT

UPWARD FORCES ~ $F + F_{BOOYANT}$

DOWNWARD " ~ W_T

$$W = \rho g V = \rho g\left(\frac{4}{3}\pi R^3\right)$$

$$F_b = \rho_w g V z = \rho_w g\left(\frac{4}{3}\pi R^3\right)z$$

z = FRACTION SUBMERGED

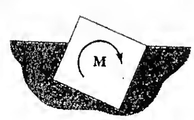
$$F = \rho g\left(\frac{4}{3}\pi R^3\right) - \rho_w g z\left(\frac{4}{3}\pi R^3\right)$$

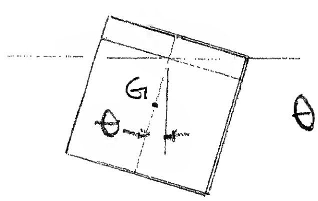
$$z = \frac{\rho g\left(\frac{4}{3}\pi r^3\right) - F}{\rho_w g\left(4/3\,\pi R^3\right)}$$

2.29

CUBE –

LENGTH OF SIDE = L





$$\theta = \tan^{-1}\frac{0.1}{0.5}$$

G IS CENTER OF MASS OF SOLID

$$\sum M_G = 2\left[\frac{1}{2}\left(\frac{L}{2}\right)(0.1L)(L)\rho g\left(\frac{2}{3}\frac{L}{2}\sin\theta\right) - (0.9L)(L)(L)\rho g(0.05L\sin\theta)\right] + M$$

{ PART OF ORIGINAL SUBMERGED VOLUME IS NOW OUT OF H_2O –

{ PART THAT WAS ORIGINALLY OUT IS NOW SUBMERGED }

2.29 (CONT.)

$$M = \rho g L^4 \sin\theta\left[-\frac{1}{60} + 0.045\right]$$

$$= \rho g L^4 \sin\theta\,(0.02833)$$

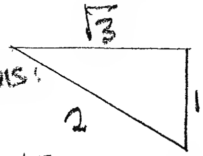
$$= \underline{0.00556\,\rho g L^4}$$

CHAPTER 4

4.1 $\vec{v} = 10\vec{e}_x + 7x\vec{e}_y$

At (2,2) $\vec{v} = 10\vec{e}_x + 14\vec{e}_y$

At -30° from x axis:



Unit vector: $\vec{e} = \frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y$

Along this direction the component is $\vec{e}\cdot\vec{v}$:

$$\vec{e}\cdot\vec{v} = \left(\frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y\right)\cdot\left(10\vec{e}_x + 14\vec{e}_y\right)$$

$$= 5\sqrt{3} - 7 = \underline{1.66 \text{ m/s}}$$

4.2 $\vec{v} = 10\vec{e}_x + 7x\vec{e}_y$

$$\frac{dy}{dx} = \frac{v_y}{v_x} = \frac{7x}{10}$$

$$10\int dy = 7\int x\,dx$$

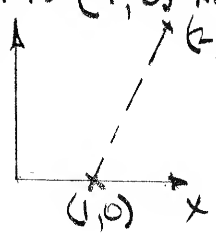
$$10y = 7\frac{x^2}{2} + C$$

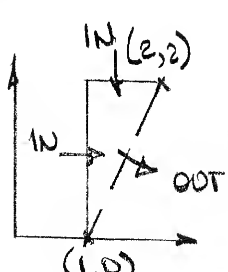
At (2,1) $C = 10 - 14 = -4$

Eqn is: $7\frac{x^2}{2} - 10y + C = 0$

or $x^2 - \frac{10}{7}y - 8/7 = 0$ (a)

Across the surface connecting points (1,0) and (2,2):





4.2 (cont)

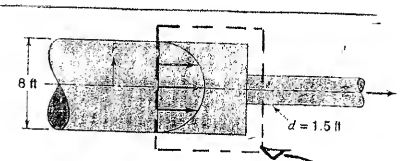
$$\dot{V} = \int_0^2 \vec{v}\cdot\vec{e}_x\,dy + \int_1^2 \vec{v}\cdot\vec{e}_y\,dx$$

$$= 10y\Big|_0^2 + 7\frac{x^2}{2}\Big|_1^2$$

$$= 20 + \frac{7}{2}(3) = 20 + 10.5$$

$$= \underline{30.5 \text{ m}^3/\text{s}} \qquad \text{(b)}$$

4.3



CONTROL VOLUME

$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA + \underbrace{\cancel{\frac{\partial}{\partial t}\iiint_{C.V.} \rho\,dV}}_{0} = 0$$

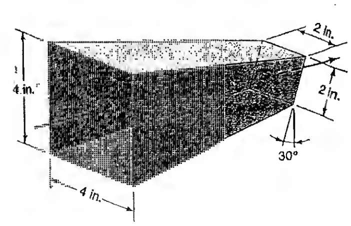
$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = \cancel{\rho} v_{2\,avg} A_2 - \int_0^R \cancel{\rho}\, 9\left(1 - \frac{r^2}{16}\right)2\pi r\,dr = 0$$

$$v_2 A_2 = 18\pi\int_0^4 \left(r - \frac{r^3}{16}\right)dr$$

$$= 18\pi\left[\frac{r^2}{2} - \frac{r^4}{64}\right]_0^4 = 72\pi$$

$$v_{2\,avg} = \frac{72\pi}{\pi(3/4)^2} = \underline{128 \text{ ft/s}}$$

4.4



$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = 0$$

$$= \int_{A_o} \rho v \cos 30\,dA - \int_{A_i} \rho v\,dA = 0$$

4.4 - CONTINUED

$$\rho_{IN} = \rho_{OUT} \qquad A_{IN} = 4 A_{OUT}$$

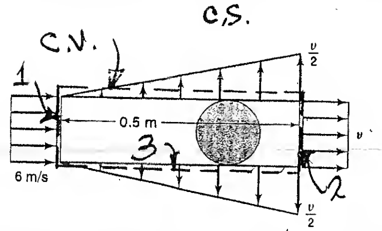
$$v_{OUT} = \frac{A_i v_i}{A_0 \cos 30^\circ}$$

$$= \frac{4(10)}{\cos 30^\circ} = \underline{\underline{46.2 \text{ FT/s}}}$$

$$\dot{V} = Av = 10(1/3 \times 1/3)$$

$$= \underline{\underline{1.11 \text{ FT}^3/\text{s}}}$$

4.5 STDY FLOW: $\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})dA = 0$



$$-\rho v A|_1 + \rho v A_2 + \int_{A_3} \rho(\vec{v}\cdot\vec{n})dA = 0$$

ρ = CONST:

$$v_1 A_1 = v_2 A_2 + \int_0^L \frac{v_2}{2} \frac{\pi D x}{L}\, dx$$

$$= v_2 A_2 + \frac{v_2}{2}\frac{\pi D}{L}\frac{x^2}{2}\Big|_0^L$$

$$= v_2 \frac{\pi D^2}{4} + \frac{v_2}{2}\frac{\pi D L}{2}$$

$$= \frac{v_2 \pi D}{4}(D+L)$$

$$6\frac{\pi}{4}(0.2)^2 = v_2 \frac{\pi}{4}\left[0.2(0.2+0.5)\right]$$

$$v_2 = \frac{6(0.04)}{(0.2)(0.7)}$$

$$= \underline{\underline{1.71 \text{ m/s}}}$$

4.6 FOR STEADY, INCOMPRESSIBLE FLOW:

$$\dot{V} = A v_{AVG} = \sum_i v_i \Delta A_i$$

FROM GIVEN DATA SET:

DIST FROM CENTER IN	v_i FT/S	ΔA_i IN^2	$v_i \Delta A_i$ FT^3/S
0	7.5	7.844	0.4084
3.16	7.10	37.64	1.856
4.45	6.75	31.96	1.498
5.48	6.42	32.10	1.431
6.33	6.15	31.48	1.344
7.07	5.81	29.85	1.204
7.75	5.47	33.22	1.262
8.37	5.10	33.22	1.176
8.94	4.50	31.44	0.982
9.49	3.82	31.57	0.838
10.–	2.40	15.82	0.264
$\Sigma \rightarrow$		316.14	18.263

$$\sum \Delta A_i = 316.14 \text{ IN}^2 \quad \left\{ \begin{matrix}\text{EXACT AREA} \\ = 314.16 \text{ IN}^2\end{matrix} \right\}$$

$$= 2.195 \text{ FT}^2$$

$$\dot{V} = \sum v_i \Delta A_i = \underline{18.26 \text{ FT}^3/\text{s}}$$

$$v_{AVG} = \dot{V}/A = \frac{18.26}{2.195} = \underline{\underline{8.32 \text{ FT/s}}}$$

4.7 INFLOW: $\dot{V} = 2$ gal/m $= 19.2$ LB/m

OUTFLOW: $\dot{V} = 19.2$ LBm/m

~ STEADY FLOW ~

FOR TOTAL FLOW: $\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})dA = \dot{m}_{OUT} - \dot{m}_{IN} = 0$

TOTAL MASS IN TANK = M

MASS OF SALT IN TANK = S

FOR SALT - $\dot{m}_{OUT} = 19.2\,(S/M)$ lbm/m

$\dot{m}_{IN} = 2(1.92)$ "

4.7 – CONTINUED

FOR SALT: CONS. OF MASS

$$\frac{19.2\,S}{M} - 3.84 + \frac{dS}{dt} = 0$$

$$\frac{dS}{dt} = 3.84 - \frac{19.2}{M} S$$

$$= A - BS$$

$$A = 3.84$$

$$B = \frac{19.2}{M} = \frac{(19.2)(7.48)}{(100)(62.4)}$$

$$= 0.0230$$

$$\int_0^S \frac{dS}{A - BS} = \int_0^t dt$$

$$-\frac{1}{B} \ln \frac{A - BS}{A} = t$$

$$\ln\left[1 - \frac{B}{A} S\right] = -Bt$$

OR $$S = \frac{A}{B}\left[1 - e^{-Bt}\right]$$

FOR $t = 100$ m

$$S = \frac{3.84}{0.0230}\left(1 - e^{-2.30}\right)$$

$$\cong \underline{150 \text{ LBm}} \quad (a)$$

FOR $t = \infty$ $\quad \underline{S = 167 \text{ LBm}}$ (b)

FOR $S = 100$ LBm

$$t = \frac{1}{0.023} \ln\left[1 - \frac{0.023}{3.84}(100)\right]$$

4.7 – CONTINUED

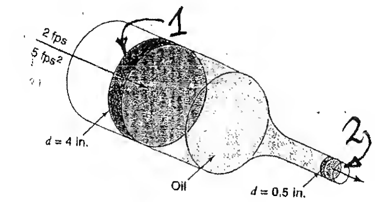
$t = 39.8$ m $\quad$ FOR $S = 100$ LBm

FROM (a) $t = 100$ m FOR $S \cong 150$ lbm

FOR S FROM 100 TO 150

$$\Delta t = 100 - 39.8 = \underline{60.2 \text{ M}} \quad (c)$$

4.8



FOR PISTON & CYLINDER SHOWN:

AT 1 $\quad v = v_1 = 2$ FT/s $\quad a = a_1 = 5$ FT/s²

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1$$

$$= \left(\frac{4}{0.5}\right)^2 (2) = \underline{128 \text{ FT/s}}$$

$$a_2 = a_1 \left(\frac{d_1}{d_2}\right)^2 = 5\left(\frac{4}{0.5}\right)^2 = \underline{320 \text{ FT/s}^2}$$

4.9 FOR STEADY FLOW:

$$\iint_{c.s.} \rho(\vec{v} \cdot \vec{n})\,dA = 0$$

OR $\rho v A = $ CONSTANT

$$\frac{d(\rho v A)}{\rho v A} = \frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0$$

Q.E.D.

4.10 $$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = \iint_{C.S.} d\dot{m}$$

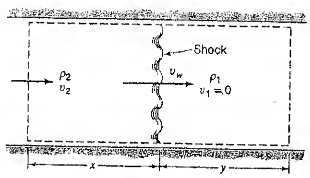
$$\rho(\vec{v}\cdot\vec{n})\,dA = d\dot{m}$$

$$\frac{\partial}{\partial t}\iiint_{C.V.} \rho\, dV = \frac{\partial}{\partial t} M$$

$$\therefore \quad \frac{\partial M}{\partial t} + \iint_{C.S} d\dot{m} = 0$$

Q.E.D.

4.11



FOR THE C.V. SHOWN

$$\iint_{CS} \rho(\vec{v}\cdot\vec{n})\,dA + \cancel{\frac{\partial}{\partial t}\iiint_{C.V.} \rho\, dV} = 0$$

0 - STY FLOW

C.V. MOVES TO RIGHT WITH $v = v_W$

THUS:

$$-\rho_1 A v_W + \rho_2 A (v_w - v_2) = 0$$

$$v_2 = v_w\left(1 - \rho_1/\rho_2\right)$$

4.12 $$v_{AVG} = \frac{1}{A}\int_A v\, dA$$

$$= \frac{v_{MAX}}{\pi R^2}\int_0^R 2\pi r\left[1 - \frac{r}{R}\right]^{1/7} dr$$

FOR $z = r/R \qquad dz = dr/R$

$$v_{AVG} = 2 v_{MAX}\int_0^1 z(1-z)^{1/7}\, dz$$

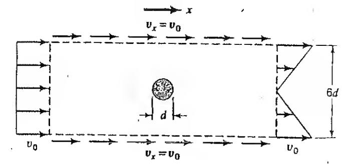
FOR $\xi = 1 - z \qquad d\xi = -dz$

$$v_{AVG} = -2 v_{MAX}\int_1^0 (1-\xi)\,\xi^{1/7}\, d\xi$$

4.12 - CONTINUED

$$v_{AVG} = \frac{49}{60} v_{MAX} = 0.817\, v_{MAX}$$

4.13



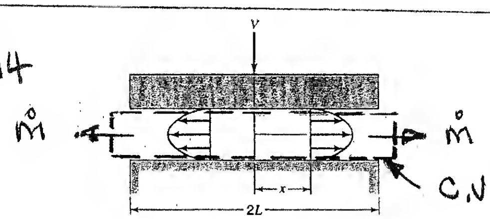
$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA + \cancel{\frac{\partial}{\partial t}\iiint_{C.V} \rho\, dV} = 0$$

0 - STY FLOW

$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = -\rho v_0 (6d) + \dot{m}_{HORIZ} + 2\int_0^{3d} \rho \frac{v_0}{3d} y\, dy$$

$$\dot{m}_{HORIZ} = \rho v_0 (6d) - \rho v_0 (3d) = \rho v_0 (3d)$$

4.14



$$\frac{\partial M}{\partial t} + \int d\dot{m} = 0$$

$$M = \rho(2L)(b)(1) \qquad \frac{\partial M}{\partial t} = 2\rho L \dot{b}$$

WHERE $\dot{b} = -V$

$$\int d\dot{m} = 2\dot{m}_{SIDE} = 2\int_0^b \rho v (1)\, dy$$

GIVING: $$-2\rho L V + 2\rho\int_0^b v\, dy = 0$$

OR $$L V = \int_0^b v\, dy$$

4.14 — CONTINUED

(a) FOR $v = V_{AVG}$ (A CONSTANT)

$$LV = v_{AVG}\, b$$

$$\underline{\underline{v_{AVG} = \frac{L}{b} V}}$$

(b) $v = ay + by^2$

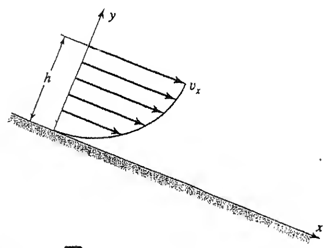
WITH $v(b) = 0 \quad v\left(\frac{b}{2}\right) = v_{MAX}$

$$v = 4 v_{MAX}\left[\frac{y}{b} - \left(\frac{y}{b}\right)^2\right]$$

$$LV = 4 v_{MAX} \int \left[\frac{y}{b} - \left(\frac{y}{b}\right)^2\right] dy$$

$$\underline{\underline{v_{MAX} = \frac{3}{2}\frac{LV}{b}}}$$

4.15



$$\dot{V} = \int_A v_x \, dA$$

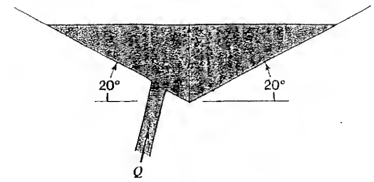
$$= w \int_0^h v_0\left(2\frac{y}{h} - \frac{y^2}{h^2}\right) dy$$

$$= \frac{2}{3} w v_0 h$$

$$2000 \frac{cm^3}{m} = \frac{2}{3}(10)\, v_0 (2)$$

$$\underline{\underline{v_0 = 150 \text{ cm/m} = 2.5 \text{ cm/s}}}$$

4.16



$$V = wA = w h^2 \cot 20$$

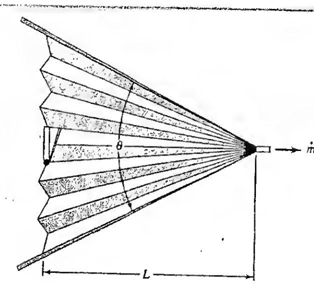
$$\dot{V} = w \cot 20 \; dh^2/dt$$

$$\int_{h_1}^{h_2} dh^2 = \frac{\dot{V} \tan 20}{w} \int_0^t dt$$

$$h^2 \Big|_{h_1}^{h_2} = \frac{\dot{V} \tan 20}{w} t$$

$$\underline{\underline{t = (h_2^2 - h_1^2)\left[\frac{w}{\dot{V}} \cot 20\right]}}$$

4.17



$$V = wA = wL^2 \tan \theta/2$$

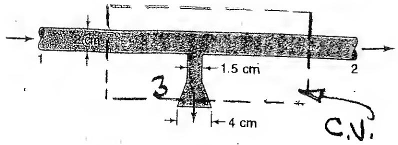
$$\dot{m} = \rho \dot{V} = \rho w L^2 \frac{d}{dt}\left(\tan \frac{\theta}{2}\right)$$

$$= \rho w L^2 \sec^2 \frac{\theta}{2} \; \frac{\dot{\theta}}{2}$$

$$\cos^2 \frac{\theta}{2} = \frac{1 + \cos\theta}{2}$$

$$\underline{\underline{\dot{m} = \frac{\rho w L^2 \dot{\theta}}{1 + \cos\theta} = \frac{\rho w L^2 \dot{\theta}}{2 \cos^2 \theta/2}}}$$

4.18



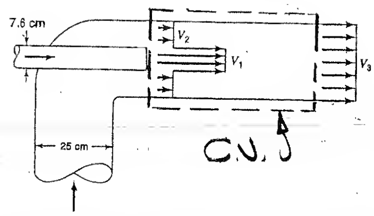
STEADY FLOW – $\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = 0$

$$\cancel{\rho}\dot{V}_1 = \cancel{\rho}A_2 v_2 + \cancel{\rho}A_3 v_3$$

$$1.3\times10^{-3} = \frac{\pi}{4}(0.02)^2(2.1) + (100)\frac{\pi}{4}(10^{-3})^2 v_3$$

$$\underline{v_3 = 8.15 \text{ m/s}}$$

4.19



STEADY FLOW – $\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = 0$

$$\cancel{\rho}A_3 v_3 - \cancel{\rho}A_1 v_1 - \cancel{\rho}A_2 v_2 = 0$$

$$v_3 = \frac{A_1 v_1 + A_2 v_2}{A_3}$$

$$= \frac{\left[\frac{\pi}{4}(0.076)^2(40) + \frac{\pi}{4}(0.25^2 - 0.076^2)(3)\right]}{\frac{\pi}{4}(0.25^2)}$$

$$\underline{\underline{v_3 = 5.15 \text{ m/s}}}$$

4.20 VOLUME DISPLACED BY PLUNGER

$$\dot{V} = A_p v_p = \frac{\pi}{4} d_p^2 V$$

VOLUME OF H_2O MOVING PAST P:

$$\dot{V} = (A - A_p)v = \frac{\pi}{4}(D^2 - d_p^2)v$$

IN STEADY STATE OPERATION THESE MUST BE EQUAL:

$$\cancel{\frac{\pi}{4}} d_p^2 V = \cancel{\frac{\pi}{4}}(D^2 - d_p^2)v$$

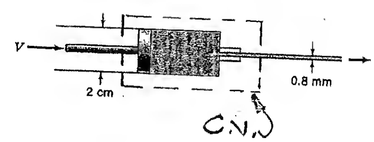
$$\underline{\underline{v = V\frac{d_p^2}{D^2 - d_p^2}}} \quad (a)$$

RELATIVE TO PLUNGER –

$$v_R = v + V$$

$$\underline{\underline{= V\left[\frac{d_p^2}{D^2 - d_p^2} + 1\right]}} \quad (b)$$

4.21



CONS. OF MASS – CONSTANT ρ

$$\dot{V}_{OUT} = 6 \text{ cm}^3/\text{s} \quad \text{– CONSTANT}$$

FOR NO LEAKAGE

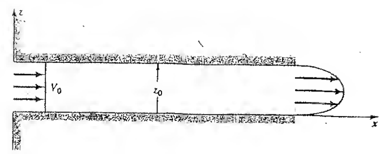
$$\dot{V} = A_p V = \frac{\pi}{4}(2)^2 V = 6$$

$$\underline{\underline{V = 1.91 \text{ cm/s}}}$$

FOR LEAKAGE – $\dot{V} = 6 + 0.6$

$$\dot{V} = \frac{6.6}{\pi/4\,(2^2)} = \underline{\underline{2.1 \text{ cm/s}}}$$

4.22



PARALLEL PLATES –

INCOMPRESSIBLE, STEADY FLOW –

$\dot{V}$ = CONSTANT

$$V_0 Z_0 = \int v \, dA$$

$$= a\int_0^{Z_0} Z(Z_0 - Z)\, dZ$$

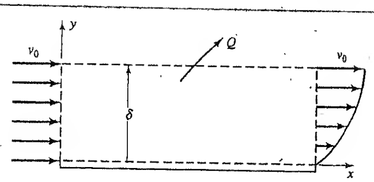
$$= a\frac{Z_0^3}{6} \Rightarrow a = \frac{6V_0}{Z_0^2}$$

v IS MAX AT $Z = Z_0/2$

$$v = 6\frac{V_0}{Z_0^2}\left[Z_0 - \frac{Z_0}{2}\right]\frac{Z_0}{2}$$

$$= 6V_0/4 = \underline{\underline{12 \text{ cm/s}}}$$

4.23



FOR STEADY INCOMPRESSIBLE FLOW:

$$\dot{V}_{OUT} - \dot{V}_{IN} = 0$$

$$Q + b\int_0^{\delta} V_0\left(\frac{3\eta - \eta^3}{2}\right) dy = V_0 \delta b$$

$$\int_0^{\delta} \frac{3\eta - \eta^3}{2}\, dy = \frac{\delta}{2}\int_0^1 \frac{3\eta - \eta^3}{2}\, d\eta$$

$$= 5/8\, \delta$$

$$Q = V_0 \delta b(1 - 5/8) = \underline{\underline{\frac{3}{8} V_0 b\delta}}$$

4.24 SEE SKETCH FOR PROB 4.14

PLATES ARE CIRCULAR

$$\frac{\partial M}{\partial t} + \int d\dot{m} = 0$$

$$M = \rho b \pi L^2$$

$$\frac{\partial M}{\partial t} = \rho \pi L^2 \dot{b} = -\rho \pi L^2 V$$

$$\dot{m}_{EXIT} = \rho\, 2\pi L b\, v_{EXIT} = \rho \pi L^2 V$$

$$\Rightarrow \underline{\underline{v_{EXIT} = LV/2b}} \quad (a)$$

AS IN PROB 4.14 – PARABOLIC EXIT PROFILE

$$v_{EXIT} = ay + by^2$$

$$= 4 v_{MAX}\left[\frac{y}{b} - \left(\frac{y}{b}\right)^2\right]$$

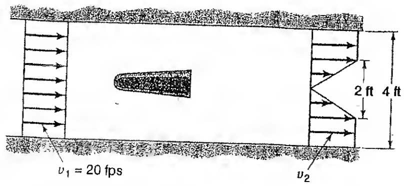
$$\dot{m}_{EXIT} = \rho \int_0^b v_{EXIT}\, 2\pi L\, dy$$

$$= \rho \frac{4}{3} \pi L b\, v_{MAX}$$

$$\therefore \underline{\underline{v_{MAX} = \frac{3}{4}\frac{L}{b}V}}$$

CHAPTER 5

5.1



CONS. OF MASS: $\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})dA = 0$

$$\cancel{\rho}v_1A_1 = 2\left[\int_0^1 \rho v_2 y\,dy + \int_1^2 \rho v_2\,dy\right]$$

$$4v_1 = 2v_2\left[\frac{y^2}{2}\Big|_0^1 + v_2 y\Big|_1^2\right]$$

$$= 3v_2$$

$$v_2 = \frac{4}{3}v_1 = \underline{26.7 \text{ FT/S}}$$

5.2 SYSTEM SHOWN IN PROB 5.1

$$\sum F_x = \iint_{C.S.} v_x\rho(\vec{v}\cdot\vec{n})\,dA$$

— ASSUMING UNIT DEPTH —

$$F_x + (P_1-P_2)4$$

$$= 2\rho\left[\int_0^1 (v_2 y)^2 dy + \int_1^2 v_2^2\,dy - v_1^2\int_0^2 dy\right]$$

$$= 2\rho\left[v_2^2\frac{y^3}{3}\Big|_0^1 + v_2^2 y\Big|_1^2 - v_1^2 y\Big|_0^2\right]$$

$$= 2\rho v_2^2\left[\left(\frac{1}{3}+1\right)\right] - 4\rho v_1^2$$

FROM 5.1 - $v_2 = \frac{4}{3}v_1$

5.2 - CONTINUED -

$$F_x + (P_1-P_2)4 = \frac{20}{27}\rho v_1^2$$

$$F_x = -800 \text{ N/m} = 52.8 \text{ lbf/FT}$$

$$P_1-P_2 = \frac{1}{4}\left[\frac{20}{27}\rho v_1^2 + 52.8\right]$$

$$= 157 \text{ lbf/FT}^2$$

$$\cong \underline{7500 \text{ Pa} = 7.5 \text{ kPa}}$$

5.3 SAME GENERAL CONFIGURATION EXCEPT EXIT VELOCITY DISTRIBUTION IS $v = v_2\left(1-\cos\frac{\pi y}{4}\right)$

AS IN 5.1 THE EXPRESSION TO BE USED IS:

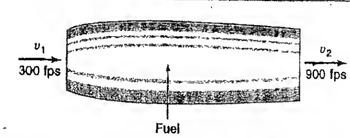
$$v_1A_1 = 2\left[\int_0^2 v_2\left(1-\cos\frac{\pi y}{4}\right)dy\right]$$

$$4v_1 = 2v_2\left[2-4/\pi\right]$$

$$v_2 = \frac{2v_1}{2-4/\pi} = \frac{v_1}{1-2/\pi}$$

$$= 55 \text{ FT/S}$$

5.4



STEADY FLOW

$$F_x = \iint_{C.S.} v_x\rho(\vec{v}\cdot\vec{n})dA$$

$$= \rho_2 v_2^2 A_2 - \rho_1 v_1^2 A_1$$

$$= \dot{m}_2 v_2 - \dot{m}_1 v_1$$

$$= \dot{m}_1(1.02\,v_2 - v_1)$$

5.4 - CONTINUED -

$$\dot{m} = \rho_1 A_1 v_1$$

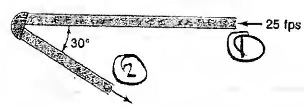
$$= \left(0.0805 \frac{\text{lbm}}{\text{ft}^3}\right)\left(10.8 \text{ ft}^2\right)\left(300 \frac{\text{ft}}{\text{s}}\right)$$

$$= 260.8 \text{ lbm/s}$$

$$F_x = \left(260.8 \frac{\text{lbm}}{\text{s}}\right)\left[1.02\,(900 \text{ ft/s}) - 300 \text{ ft/s}\right]$$

$$= \underline{5005 \text{ lbf}}$$

5.5



STEADY INCOMPRESSIBLE FLOW:

$$F_x = \iint_{C.S.} v_x \rho (\vec{v}\cdot\vec{n})\,dA$$

$$F_y = \iint_{C.S.} v_y \rho (\vec{v}\cdot\vec{n})\,dA$$

IN X-DIRECTION:

$$F_x = (\rho v A)\left[v_2 \cos(-30) - (-v_1)\right]$$

$$= \dot{m}\left[v(0.866) + 1\right]$$

$$\dot{m} = \rho A v = 62.4 \frac{\text{lbm}}{\text{ft}^3}\left(3 \text{ ft}^3/\text{s}\right)$$

$$= 187.2 \text{ lbm/s}$$

$$F_x = \frac{(187.2 \text{ lbm/s})(25 \text{ ft/s})(1.866)}{32.2 \text{ lbm ft/s}^2 \text{lbf}}$$

$$= \underline{271.2 \text{ lbf}}$$ — FORCE ON BLADE IS IN $(-x)$

5.5 - CONTINUED

$$F_y = \dot{m}\, v_2 \sin(-30)$$

$$= \frac{(187.2)(25)(-0.5)}{32.2}$$

$$= -72.7 \text{ lbf}$$

{FORCE ON BLADE IS IN +y DIRECTION}

PART (b) - BLADE MOVES TO RIGHT AT 15 FT/S

RELATIVE TO BLADE: $v_1 = -40$ FT/S

$v_2 = 40$ FT/S @ $-30°$

ABSOLUTE VELOCITY OF LEAVING JET

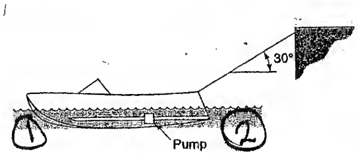
$$= \vec{v}\Big|_{\substack{\text{RELATIVE}\\ \text{TO BLADE}}} + \vec{v}_{\text{BLADE}}$$

$$= (34.64\,\vec{e}_x - 20\,\vec{e}_y) + 40\,\vec{e}_x$$

$$= \underline{74.64\,\vec{e}_x - 20\,\vec{e}_y}$$

$$|v_{\text{EXIT}}| \cong \underline{77.3 \text{ ft/s}}$$

5.6



C.V. AROUND BOAT - STEADY INCOMPRESSIBLE FLOW

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v}\cdot\vec{n})\,dA$$

$$= \dot{m}(v_2 - v_1)$$

5.6 - CONTINUED

$$F_x = \mathring{m}\,\mathring{V}\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

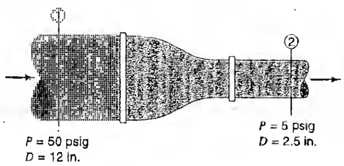
$$= \frac{(62.4)(6)^2\left(\frac{1}{0.15} - \frac{1}{0.25}\right)}{32.2}$$

$$= 186 \ \text{LB}_F$$

$$\text{TENSION IN ROPE} = F_x / \cos 30^\circ$$

$$\underline{= 215 \ \text{LB}_F}$$

5.7



FLOW IS STEADY, INCOMPRESSIBLE

$$\sum F_x = \iint_{C.S.} v_x \rho(\vec{v}\cdot\vec{n})\,dA$$

$$= \mathring{m}\,(v_2 - v_1)$$

$$\mathring{m} = \rho \mathring{V} = 0.8(62.4)(3 \ \text{FT}^3/\text{s})$$

$$= 149.8 \ \text{LB}_M/\text{s}$$

$$\sum F_x = F_x + P_1A_1 - P_2A_2$$

{ATMOSPHERIC PRESSURE CANCELS}

EQUATING:

$$F_x + P_1\frac{\pi D_1^2}{4} - P_2\frac{\pi D_2^2}{4} = \mathring{m}\,\mathring{V}\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

$$F_x = \frac{\pi}{4}\left(P_2D_2^2 - P_1D_1^2\right) + \mathring{m}\,\mathring{V}\,\frac{4}{\pi}\left(\frac{1}{D_2^2} - \frac{1}{D_1^2}\right)$$

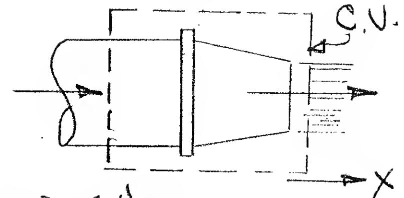
5.7 - CONTINUED

$$P_1 = 50 \ \text{PSIG} \qquad P_2 = 5 \ \text{PSIG}$$

$$F_x = -5630 + 392$$

$$\underline{= -5238 \ \text{LB}_F}$$

5.8



FLUID IS H_2O

$$P_1 = 60 \ \text{PSIG} \qquad P_2 = 14.7 \ \text{PSIA}$$

$$D_1 = 0.25 \ \text{FT} \qquad D_2 = \frac{1.5}{12} \ \text{FT}$$

$$\mathring{V} = 400 \ \text{gal/m} = 0.892 \ \text{FT}^3/\text{s}$$

STEADY, INCOMPRESSIBLE FLOW

$$\iint_{C.S.} v_x \rho(\vec{v}\cdot\vec{n})\,dA = 0$$

$$v_1 = \frac{\mathring{V}}{A_1} = \frac{0.892}{\frac{\pi}{4}(0.25)^2} = 18.17 \ \text{FT/s}$$

$$v_2 = \frac{v_1 D_1^2}{D_2^2} = 18.17\left(\frac{0.25(12)}{1.5}\right)^2 = 72.7 \ \text{FT/s}$$

$$F_x + P_1A_1 - P_2A_2 = \mathring{m}(v_2 - v_1)$$

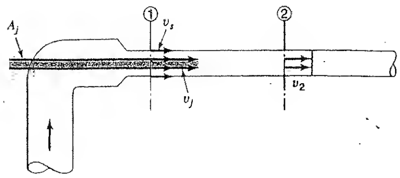
$$F_x = \rho\mathring{V}(v_2 - v_1) - P_1A_1 + P_2A_2$$

$$= \left[62.4(0.892)(72.7 - 18.17)\right]/32.2$$
$$- (60 + 14.7)(144)\frac{\pi}{4}(0.25)^2$$
$$+ (14.7)(144)\pi/4\left(\frac{1.5}{12}\right)^2$$

$$= 94.3 - 528.0 + 25.4$$

$$\underline{= -408 \ \text{LB}_F}$$

5.9



H_2O -- FLOW IS STEADY, INCOMPRESSIBLE

$$\sum F_x = \iint_{C.S.} \rho v_x (\vec{v}\cdot\vec{n})\,dA$$

$$\sum F_x = P_1 A_1 - P_2 A_2$$

$$\iint_{C.S.} \sim = \rho A_2 v_2^2 - \rho\left(A_s v_s^2 + A_j v_j^2\right)$$

EQUATING:

$$P_1 - P_2 = \rho\left[v_2^2 - \frac{A_s}{A} v_s^2 - \frac{A_j}{A} v_j^2\right]$$

BY CONSERVATION OF MASS:

$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA = 0$$

$$A_2 v_2 - A_s v_s - A_j v_j = 0$$

$$v_2 = \frac{A_s}{A_2} v_s + \frac{A_j}{A_2} v_j$$

$$= \frac{0.54}{0.60}(10) + \frac{0.06}{0.60}(90)$$

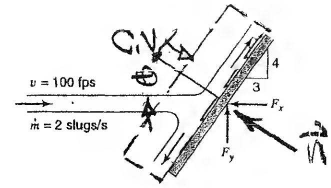
$$= 18 \text{ FT/S} \qquad (a)$$

$$P_1 - P_2 = \frac{62.4}{32.2}\left[(18)^2 - \frac{0.54}{0.6}(10)^2 - \frac{0.06}{0.6}(90)^2\right]$$

$$= -1116 \text{ LBF/FT}^2 = -7.75 \text{ PSI}$$

$$\underline{P_2 - P_1 = 7.75 \text{ PSI}}$$

5.10



FLOW IS STEADY, INCOMPRESSIBLE, FRICTIONLESS

FOR FRICTIONLESS FLOW --
NO DRAG ON PLATE --

$$\therefore \sum F_n = \iint_{C.S.} v_n \rho (v\cdot\vec{n})\,dA = 0$$

$$F_n = -v_j \rho (v\cos\theta) A_j$$

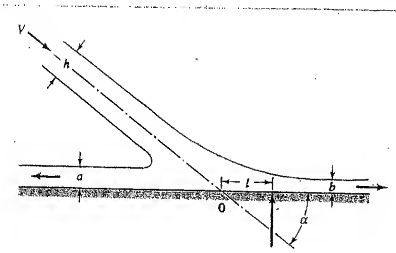
$$= -\dot{m}\, v\cos\theta$$

$$= -2(100)(4/5) = -160$$

$$F_x = (-160)(4/5) = \underline{-128 \text{ LBF}}$$

$$F_y = (-160)(3/5) = \underline{-96 \text{ LBF}}$$

5.11



STEADY, INCOMPRESSIBLE FRICTIONLESS FLOW

IN X-DIRECTION:

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v}\cdot\vec{n})\,dA$$

$$= \rho v^2 b - \rho v^2 a - \rho v^2 h\cos\alpha = 0$$

CONS. OF MASS: $\rho v h = \rho v(a+b)$

$$h = a + b$$

$$a = \frac{h}{2}(1-\cos\alpha) \qquad b = \frac{h}{2}(1+\cos\alpha)$$

5.11 - CONTINUED

$$\sum F_y = \iint_{C.S.} \rho v_y (\vec{v} \circ \vec{n})\, dA$$

$$= \rho v^2 h \sin\alpha \qquad (a)$$

PART (b):

$$\sum M_z = F_y \ell = \iint_{C.S.} (\vec{r} \times \vec{v})_z \rho (\vec{v} . \vec{n})\, dA$$

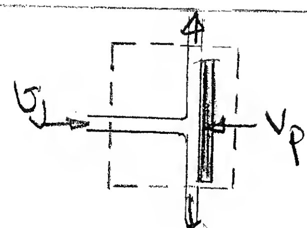
$$F_y \ell = \frac{a}{2} v(\rho v a) - \frac{b}{2} v(\rho v b)$$

$$\rho v^2 h (\sin\alpha) \ell = a^2 \frac{\rho}{2} v^2 - b^2 \frac{\rho v^2}{2}$$

$$\ell = \frac{a^2 - b^2}{2 h \sin\alpha}$$

$$= \frac{h^2 \cos\alpha}{2 h \sin\alpha} = \underline{\underline{\frac{h \cot\alpha}{2}}}$$

5.12



FLOW IS STEADY, INCOMPRESSIBLE, FRICTIONLESS -

ATMOSPHERIC PRESSURE CANCELS

C.V. MOVES TO LEFT WITH VELOCITY, V_p

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v} \circ \vec{n})\, dA$$

$$F_x = \rho A_J (v_J + V_p)^2$$

$$= \frac{(62.4)}{32.2} \frac{3}{30} (5+30)^2$$

$$= \underline{237.4 \text{ lbf}}$$

FOR MOVING PLATE

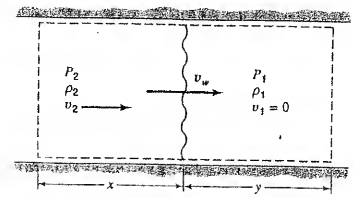
5.12 - CONTINUED

FOR $V_p = 0$

$$F_x = \frac{62.4}{32.2} \frac{3}{30} (30)^2$$

$$= 174.4 \text{ lbf}$$

5.13



CONS. OF MASS: FOR UNIT CROSS SECTION

$$\frac{\partial M}{\partial t} + \iint_{C.S.} d\dot{m} = 0$$

$$M = \rho_2 x + \rho_1 y \qquad \int d\dot{m} = -\rho_2 v_2$$

$$\rho_2 \dot{x} + \rho_1 \dot{y} - \rho_2 v_2 = 0$$

SINCE $\dot{x} = v_w \quad \dot{y} = -v_w$

$$\rho_2 (v_w - v_2) - \rho_1 v_w = 0 \qquad (1)$$

X-MOMENTUM:

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v} . \vec{n})\, dA + \frac{\partial}{\partial t} \iiint_{C.V.} v_x \rho\, dV$$

$$P_2 - P_1 = -v_2 \rho_2 v_2 + \frac{\partial}{\partial t} v_2 \rho_2 x$$

$$= -\rho_2 v_2^2 + \rho_2 v_2 v_w$$

$$= \rho_2 v_2 [v_w - v_2]$$

FROM (1): $\rho_2 v_2 (v_w - v_2) = \rho_1 v_2 v_w$

GIVING $\underline{\underline{P_2 - P_1 = \rho_1 v_2 v_w}}$

Q.E.D.

5.14 FOR SITUATION CONSIDERED IN PROB 5.13

(a) AIR $\quad v_w = 1130$ FT/s

$\rho = 0.00238$ SLUG/FT^3

$$P_2 - P_1 = \rho_1 v_2 v_w$$

$$= (0.00238)(10)(1130)$$

$$= \underline{\underline{26.9 \text{ PSF} = 0.187 \text{ PSI}}}$$

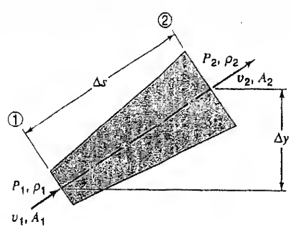
(b) H_2O $\quad v_w = 4700$ FT/s

$\rho = 1.938$ SLUGS/FT^3

$$\Delta P = (1.938)(10)(4700)$$

$$= \underline{\underline{91{,}080 \text{ PSF} = 633 \text{ PSI}}}$$

5.15.



CONS. OF MASS:

TECHNIQUE IS TO LET

$$P_2 = P_1 + \frac{\partial P}{\partial s}\Delta s$$

$$v_2 = v_1 + \frac{\partial v}{\partial s}\Delta s$$

ETC

BY CONSERVATION OF MASS - $\left.\right\}$ $\Delta s = \Delta y \to 0$

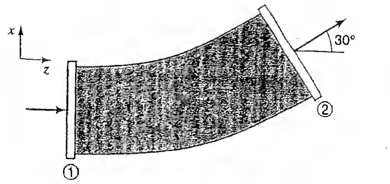
$$\frac{\partial}{\partial s}(\rho A v) = 0$$

BY MOMENTUM THEOREM, USING CONS. OF MASS RESULT:

$$dP + \rho v\, dv + g\, dy = 0$$

— MESSY —

5.16



$D_1 = 0.3$ m $\qquad D_2 = 0.38$ m

$v_1 = 12$ m/s $\qquad P_2 = 145$ kPag

$P_1 = 128$ kPag $\qquad V_2 = 7.48$ m/s

$$A_1 = \frac{\pi}{4}(0.3\text{m})^2 = 0.0707\text{ m}^2 \qquad A_2 = \sim = 0.1134\text{ m}^2$$

$$\dot{V} = A_1 v_1 = (0.0707)(12) = 0.8484\text{ m}^3/\text{s}$$

IN X DIRECTION: $\sum F_x = \iint_{c.s.} v_x \rho(\vec{v}\cdot\vec{n})\,dA$

$$F_x + P_1A_1 - P_2A_2\cos\theta = \dot{m}(v_2\cos\theta - v_1)$$

$$F_x = (1000)(0.8484)\left[7.48(\cos 30^\circ) - 12\right]$$

$$-(1000)\left[(128)(0.0707) + (145)(0.1134)\cos 30^\circ\right]$$

$$= \underline{-505.5 \text{ N}}$$

IN y DIRECTION: $\sum F_y = \iint_{c.s.} v_y \rho(\vec{v}\cdot\vec{n})\,dA$

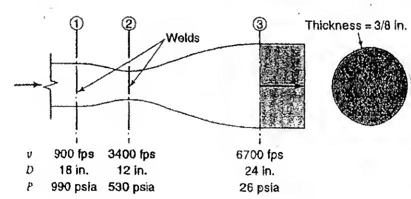
$$F_y - P_2A_2\sin\theta = \dot{m}(v_2\sin\theta)$$

$$F_y = (1000)(0.8484)(7.48\sin 30^\circ)$$

$$+ (1000)(145)(0.1134)(\sin 30^\circ)$$

$$= \underline{\underline{11395 \text{ N} = 11.395 \text{ kN}}}$$

5.17



STEADY INCOMPRESSIBLE FLOW:

$$\sum F_x = \iint_{c.s.} v_x \rho(\vec{v}\cdot\vec{n})\,dA$$

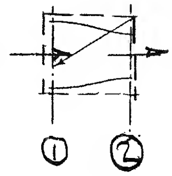
5.17 - CONTINUED -

$A_1 = \pi/4 (1.5)^2 = 1.767 \text{ ft}^2$

$A_2 = \pi/4 (1)^2 = 0.785$ "

$A_3 = \pi/4 (2)^2 = 3.142$ "

FOR C.V. BETWEEN ① & ②:



$$\sum F_x = \iint_{CS} \rho v_x (\vec{v} \cdot \vec{n}) dA$$

$$F_x + F_2 + P_1 A_1 - P_2 A_2 = \dot{m} (v_2 - v_1)$$

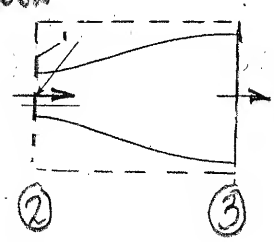
$$F_x = \frac{770(3400-900)}{32.2}$$

$$+ (530-14.7)(144)(0.785)$$

$$- (990-14.7)(144)(1.767) - F_2$$

$$= -130{,}000 \text{ LBF} - F_2$$

FOR C.V. BETWEEN ② & ③:



$$F_x + P_2 A_2 - P_3 A_3 = \dot{m} (v_3 - v_2)$$

$$F_x = \frac{770}{32.2}(6700 - 3400)$$

$$+ (26-14.7)(144)(3.14)$$

$$- (530-14.7)(144)(0.785)$$

$$= 25777 \text{ LBF}$$

5.17 - CONTINUED -

STRESS AT 2:

$$\sigma = \frac{F}{A} = \frac{25777 \text{ LBF}}{\pi (12)(3/8)} = \underline{\underline{1823 \text{ PSI}}}$$

COMPRESSION

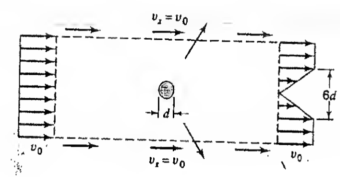
AT 1 $\quad F_1 = -130{,}000 + 25777$

$$= -104220 \text{ LBF}$$

$$\sigma = \frac{104220}{\pi (18)(3/8)} = \underline{\underline{4915 \text{ PSI}}}$$

TENSION

5.18



FLOW IS STEADY & INCOMPRESSIBLE

NO NET PRESSURE FORCE

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v} \cdot \vec{n}) dA$$

$$F_x = \int_{OUT} \rho v_x^2 dA - \rho v_0^2 A_{IN}$$

$$= 2 \int_0^{3d} \rho v_0^2 \left(\frac{y}{3d}\right)^2 dy$$

$$+ \underset{\uparrow}{\rho v_0^2 (3d)} - \rho v_0^2 (6d)$$

MOMENTUM OUT TOP & BOTTOM

$$F_y = 2 \rho v_0^2 \frac{1}{9d^2} \left. \frac{y^3}{3} \right|_0^{3d} + \rho v_0^2 (3d - 6d)$$

$$= \underline{-\rho v_0^2 d}$$

FORCE ON CYLINDER $= \underline{\underline{\rho v_0^2 d}}$

5.19 FLUID IS H_2O

$$V_{SONIC} = 1433 \text{ m/s}$$

THIS IS JUST LIKE PROB 5.13

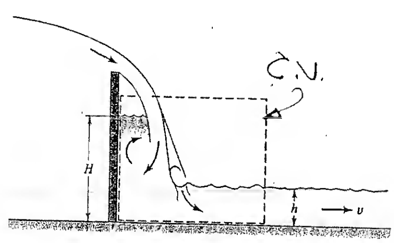
FOR AN OBSERVER MOVING WITH H_2O STREAM: $v = 3$ m/s

THEN $\Delta P = \rho v_w \Delta v$

$$= (1000)(1433-3)(3)$$

$$= \underline{4287 \text{ kPa}}$$

5.20



STATIC PRESSURE OF H_2O:

ON LEFT – $P = \frac{\rho g H}{2}$

ON RIGHT – $P = \frac{\rho g h}{2}$

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v} \cdot \vec{n})\, dA$$

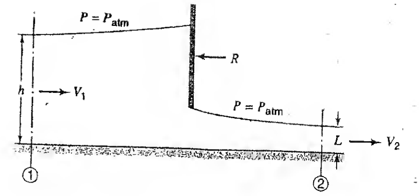
$$\frac{\rho g H^2}{2} - \frac{\rho g h^2}{2} = (\rho v h) v$$

$$H^2 = \frac{2}{\cancel{\rho} g}\left[\cancel{\rho} h v^2 + \frac{\cancel{\rho} g h^2}{2}\right]$$

$$= \frac{2hv^2}{g} + h^2$$

$$\underline{H = \left[2hv^2/g + h^2\right]^{1/2}}$$

5.21



CONSERVATION OF MASS:

$$\iint_{C.S.} \rho (\vec{v} \cdot \vec{n})\, dA = 0$$

$$-\rho h V_1 + \rho L V_2 = 0$$

$$\underline{V_2 = \frac{h V_1}{L}} \quad (a)$$

X – MOMENTUM:

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v} \cdot \vec{n})\, dA$$

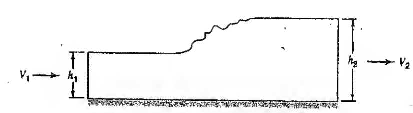
$$P_1 A_1 - P_2 A_2 + F_x = \dot{m}(v_2 - v_1)$$

$$F_x = \dot{m}(v_2 - v_1) + P_2 A_2 - P_1 A_1$$

$$= \rho v_1 h (v_2 - v_1) + \frac{\rho g L^2}{2} - \frac{\rho g h^2}{2}$$

$$\underline{= \rho v_1^2 h (h/L - 1) + \frac{\rho g}{2}(L^2 - h^2)} \quad (b)$$

5.22



CONSERVATION OF MASS:

$$v_1 h_1 = v_2 h_2$$

MOMENTUM THM:

$$\sum F_x = \iint_{C.S.} \rho v_x (\vec{v} \cdot \vec{n})\, dA$$

$$P_1 h_1 - P_2 h_2 = \dot{m}(v_2 - v_1)$$

$$P_1 = \frac{\rho g h_1}{2} \qquad P_2 = \frac{\rho g h^2}{2}$$

5.22 - CONTINUED

$$\frac{\rho g h_1^2}{2} - \frac{\rho g h_2^2}{2} = \rho v_1 h_1 (v_2 - v_1)$$

FROM CONS. OF MASS: $v_2 = v_1 h_1 / h_2$

$$\frac{g}{2}(h_1^2 - h_2^2) = v_1^2 h_1 \frac{h_1 - h_2}{h_2}$$

FACTORING & CANCELLING $h_1 - h_2$

$$\frac{g h_2}{2}(h_1 + h_2) = v_1^2 h_1$$

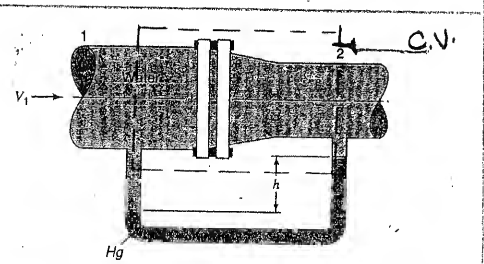
$$h_2^2 + h_1 h_2 - \frac{2 v_1^2 h_1}{g} = 0$$

$$h_2 = \frac{h_1}{2}\left[\sqrt{1 + \frac{8 v_1^2}{g h_1}} - 1\right]$$

& FROM CONTINUITY

$$v_2 = \frac{g h_1}{4 v_1}\left[1 + \sqrt{1 + \frac{8 v_1^2}{g h_1}}\right]$$

5.23



$D_1 = 8$ cm $\quad D_2 = 5$ cm

$v_1 = 5$ m/s $\quad P_2 = 1$ ATM

$h = 58$ cm

$$A_1 = \frac{\pi}{4}(8\text{cm})^2 = 50.3 \text{ cm}^2$$

$$A_2 = \frac{\pi}{4}(5\text{cm})^2 = 19.6$$

$$v_2 = 5 \text{ m/s}\left(\frac{50.3}{19.6}\right) = 12.83 \text{ m/s}$$

5.23 - CONTINUED

X - MOMENTUM: $\sum F_x = \iint_{C.S.} v_x \rho (\vec{v}\cdot\vec{n})\, dA$

$$F_x + P_1 A_1 - P_2 A_2 = \rho \dot{V}(v_2 - v_1)$$

$$P_1 - P_2 = \rho_w g h [13.6 - 1]$$

$$= (1000)(9.81)(0.58)(12.6)$$

$$= 71.69 \text{ kPa}$$

SINCE $P_2 = 1$ ATM

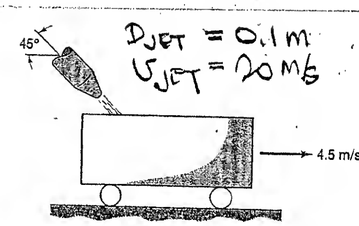
$P_1 = 71.69$ kPa GAGE

$$F_x + P_1 A_1 = (1000)(50.3\times10^{-4})(5)(12.83 - 5)$$

$$F_x = 197 - 71.69(50.3\times10^{-4})(1000)$$

$$= -163.7 \text{ N}$$

5.24



$D_{JET} = 0.1$ m

$v_{JET} = 20$ m/s

X MOMENTUM:

$$\sum F_x = \iint_{C.S.} v_x \rho (\vec{v}\cdot\vec{n})\, dA + \frac{\partial}{\partial t}\iiint v_x \rho\, dV$$

$$F_x = -\rho_w A_J v_J (v_J \cos\theta) + \rho_w A_J v_J v_c$$

$$= \rho_w A_J v_J (v_c - v_J \cos\theta)$$

$$= 1000\left(\frac{\pi}{4}\right)(0.1)^2(20)\left[4.5 - 20\cos 45^\circ\right]$$

$$= -1515 \text{ N}$$

FORCE ON CAR BY JET: $F_x = 1515$ N

524 – CONTINUED –

y MOMENTUM:

$$\sum F_y = \iint_{C.S.} v_y \rho(\vec{v}\cdot\vec{n})dA + \frac{\partial}{\partial t}\iiint_{C.V.} v_y \rho dV$$

$$F_y = -v_j \sin\theta\, \rho\,(v_j A_j) + 0$$

$$= -(20\sin 45)(1000)(-20) \times \pi/4 (0.1)^2$$

$$= +2220 \text{ N}$$

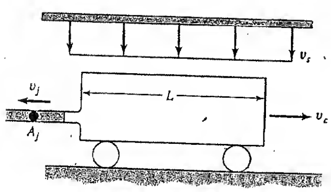
FORCE EXERTED BY H_2O:

$$F_y = -2220 \text{ N}$$

TOTAL FORCE –

$$\vec{F} = 1515\,\vec{e}_x - 2220\,\vec{e}_y \text{ N}$$

5.25



COORDINATES FIXED TO CART ~ MOVING TO RIGHT AT V_C

MOMENTUM THM IN X-DIRECTION

$$\sum F_x = \rho A_j v_j(-v_j) - \rho A_s v_s(-v_c)$$

$$F_x = \underline{\rho A_s v_s v_c - \rho A_j v_j^2}$$

IN y- DIRECTION

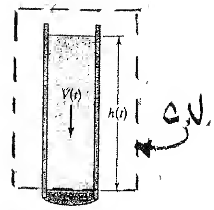
$$F_y = \rho A_j v_j(0) - \rho A_s v_s(-v_s)$$

$$= \underline{\rho A_s v_s^2}$$

FORCE OF FLUID ON CAR IS NEGATIVE OF THESE.

5.26

FOR C.V. SHOWN:



$$\int_{C.S.} d\dot{m} + \frac{\partial}{\partial t} M = 0$$

$$M = \rho A h$$

$$d\dot{m} = \rho A \dot{h}$$

$$\sum F_y = \iint_{C.S.} v_y \rho(\vec{v}\cdot\vec{n})dA + \frac{\partial}{\partial t}\iiint \rho v_y dV$$

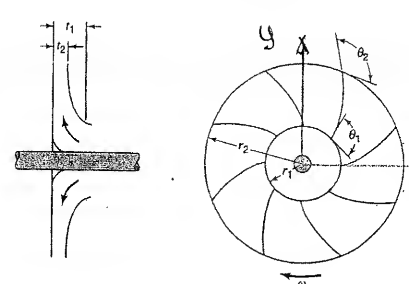
$$-\rho g A h = -\rho A \dot{h}^2 + \rho A \frac{\partial}{\partial t}(h\dot{h})$$

$$-gh = -\dot{h}^2 + \frac{\partial}{\partial t}(h\dot{h})$$

$$\Rightarrow \quad \underline{\ddot{h} = -g}$$

5.27

$\omega = 1180$ rpm, $t_2 = 0.6$ in.
$r_1 = 2$ in., $\theta_2 = 135°$
$r_2 = 8$ in., $t_1 = 0.8$ in.



STEADY
IN COMP.
FLOW

ROTATION IS ABOUT Z-AXIS –

$$\sum M_z = \iint_{C.S.} (\vec{r}\times\vec{v})_z \rho(\vec{v}\cdot\vec{n})dA$$

$$= \begin{vmatrix} \vec{e}_r & \vec{e}_\theta & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \dot{m}_{OUT}$$

$$= (r_x v_y - r_y v_x)\dot{m}_{OUT}$$

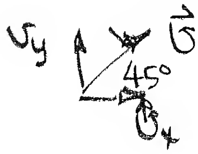
AT POSITION ON X-AXIS – $r_x = r_2$, $r_y = 0$

$$\dot{V} = 800\left(\frac{1}{7.48}\right)\left(\frac{1}{60}\right) = 1.783 \text{ FT}^3/\text{S}$$

$$v_x = \frac{1.783}{\pi(8/12)(2)(0.6/12)} = 8.51 \text{ FT/S}$$

5.27 – CONTINUED

AT THIS LOCATION –



$$v_{tan} = v_y = v_x = 8.51 \text{ FT/S}$$

ABS. VELOCITY @ r_2

$$v_y = -r\omega + v_{tan}$$

$$= -\left(\tfrac{8}{12}\right)\left(\frac{1180\times 2\pi}{60}\right) + v_t$$

$$= -82.38 + 8.51 = -73.87 \text{ FT/S}$$

NOW – IN MOMENTUM EXPRESSION:

$$M_z = (r_2 v_y)\rho\dot{V}$$

$$= \frac{8}{12}\frac{(-73.87)(64)(1.783)}{32.2}$$

$$= 174 \text{ FT LB}_F$$

$$\text{POWER} = M_z\omega$$

$$= 174\left(\frac{1180\times 2\pi}{60}\right)\left(\frac{1}{550}\right)$$

$$= \underline{\underline{39.1 \text{ Hp}}}$$

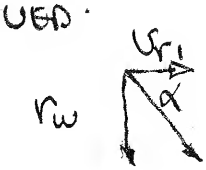
5.28 FOR CONFIGURATION OF PROB 5.28

$$\dot{V} = 1.783 \text{ FT}^3/\text{S}$$

$$\text{AT INLET} - v_r = \frac{\dot{V}}{2\pi r_1 t_1}$$

$$= \frac{1.783}{2\pi\left(\tfrac{2}{12}\right)\left(\tfrac{0.8}{12}\right)} = 25.54 \text{ FT/S}$$

5.28 – CONTINUED.



$$r\omega = -r_1\omega = -\left(\tfrac{2}{12}\right)\left(\frac{1180\times 2\pi}{60}\right)$$

$$= -20.6 \text{ FT/S}$$

$$\alpha = \tan^{-1}\frac{r\omega}{v_{r_1}} = \tan^{-1}\frac{20.6}{25.54} = \underline{\underline{38.9^\circ}}$$

PART (B): $\sum F_x = \iint_{C.S.} v_x \rho(\vec{v}\cdot\vec{n})\,dA$

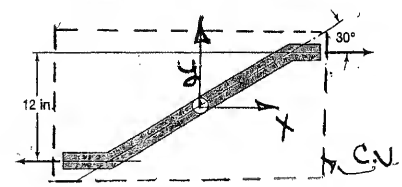
$$F_x = -\rho v_z(-v_z)A_1$$

$$= \dot{m}v_z = \rho\dot{V}\frac{\dot{V}}{A_1}$$

$$= \frac{64(1.783)^2}{\pi\left(\tfrac{2}{12}\right)^2(32.2)}$$

$$= \underline{\underline{70.6 \text{ LB}_F}}$$

5.29



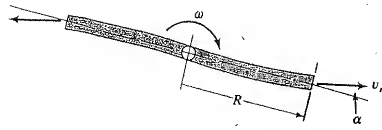
FOR C.V. SHOWN: $\sum M_z = \iint_{C.S.} (\vec{r}\times\vec{v})_z \rho(\vec{v}\cdot\vec{n})\,dA$

$$M_z = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix}_z \dot{m}$$

$$= 2\dot{m}(v_x r_y)$$

$$= \frac{2(62.4)\left(\tfrac{\pi}{4}\right)\left(\tfrac{0.5}{12}\right)^2(20)^2\left(\tfrac{6}{12}\right)}{32.2}$$

$$= \underline{\underline{1.057 \text{ FT LB}_F}}$$

5.30 

$$\sum M_z = \iint_{C.S.} (\vec{r}\times\vec{v})_z \rho(\vec{v}\cdot\vec{n})dA$$

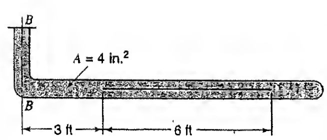
$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} (\dot{m})$$

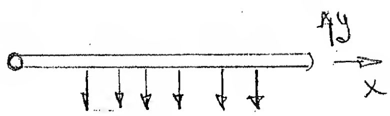
$$M_z = 2(-r_y v_x)\dot{m}$$

$$r_y = R\sin\alpha$$
$$v_x = v_r\sin\alpha - R\omega$$

$$M_z = 2\dot{m}\left[-R\sin\alpha(v_r\sin\alpha - r\omega)\right]$$

$$\omega = \frac{M_z}{2\dot{m}r^2\sin\alpha} + \frac{v_r\sin\alpha}{r}$$

5.31 

TOP VIEW 

$$\sum M_z = \iint_{C.S.} (\vec{r}\times\vec{v})_z \rho(\vec{v}\cdot\vec{n})dA$$

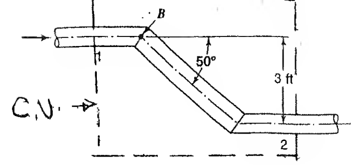
$$= \int_3^9 \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \rho v_y t\,dx$$

$$= \int_3^9 -\rho t v^2 x\,dx = -\rho v^2 t\left(\frac{x^2}{2}\right)\Big|_3^9$$

$$v = \frac{8}{6(0.25/12)} = 64 \text{ FT/s}$$

$$M = -\frac{62.4(64)^2\left(\frac{0.25}{12}\right)\left(\frac{81-9}{2}\right)}{32.2}$$

$$= \underline{-5950 \text{ FT LBF}}$$

5.32 

$$\sum M_B = \iint_{C.S.} (\vec{r}\times\vec{v})_z \rho(\vec{v}\cdot\vec{n})dA$$

$$M_B = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} \dot{m} + r_2 P_2 A_2$$

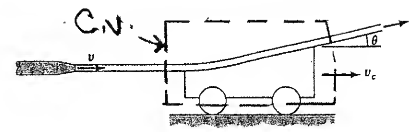
$$= (\cancel{r_x v_y} - r_y v_x)\dot{m} + r_2 P_2 A_2$$

$$\dot{V} = \left(30\ \frac{\text{gal}}{\text{m}}\right)\left(\frac{1}{7.48\times 60}\right) = 0.0668\ \text{Ft}^3/\text{s}$$

$$v_2 = \frac{\dot{V}}{A_2} = \frac{0.00668}{\frac{\pi}{4}\left(\frac{0.75}{12}\right)^2} = 21.79 \text{ FT/s}$$

$$M_B = +\frac{3(21.79)(62.4)(0.0668)}{32.2} + (3)(24)\left(\frac{\pi}{4}\right)(0.75)^2$$

$$= 8.46 + 31.81 = \underline{40.3 \text{ FT LBF}}$$

5.33 

LINEAR MOMENTUM: COORDINATE SYSTEM MOVES WITH CART

$$\sum F_x = \iint_{C.S.} v_x \rho(\vec{v}\cdot\vec{n})dA$$

$$F_x = \rho A\left[(v-v_c)\cos\theta\right](v-v_c) + \rho A(v-v_c)^2$$

$$P = v_c F_x = \rho A(v-v_c)^2\left[\cos\theta - 1\right]v_c$$

FOR $m = \frac{v_c}{v}$ $\quad P = \rho A\,[\ ?\ ]\,v^3 m(1-m)^2$

FOR $P = P_{max}$ $\quad \frac{dP}{dm} = 0$

$$\therefore \quad \frac{dP}{dm} = \rho A[\sim]\,V^3 \frac{d}{dm}\left(m - 2m^2 + m^3\right)$$

$$= \{\ \}\left(1 - 4m + 3m^2\right) = 0$$

$$m = 1,\ 1/3$$

$m = 1$ — MINIMUM

$m = 1/3$ MAXIMUM

$$\therefore \quad m = \frac{V_c}{V} = 1/3$$

PART (b) ROTATION ABOUT Z-AXIS –

$$M_z = \iint_{C.S.} (\vec{r} \times \vec{V})_z\, \rho(\vec{V} \cdot \vec{n})\, dA$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ r_x & r_y & r_z \\ V_x & V_y & V_z \end{vmatrix}_z \dot{m} \quad \text{OUT}$$

$$- \left|\quad\right|_z \dot{m} \quad \text{IN}$$

$$= r\dot{m}\left[(V - V_c)\cos\theta + V_c - V\right]$$

$$= r\dot{m}(\cos\theta - 1)(V - V_c)$$

$$P = M_z \omega = M_z \frac{V_c}{r}$$

$$= \left[\dot{m}(\cos\theta - 1)\right] V_c (V - V_c)$$

FOR $m = \frac{V_c}{V}$

$$P = [\ ]\, m V^2 (1 - m)$$

$$\frac{dP}{dm} = \left[\ \right] V^2 (1 - 2m) = 0$$

OR P_{max} OCCURS WHEN

$$m = \frac{V_c}{V} = \frac{1}{2}$$

CHAPTER 6

6.1. For $V = A + Br$

$$V(r_o) = 0 = A + Br_o$$

$$V(r_i) = \frac{\omega d}{2} = A + Br_i$$

$$A = -Br_o = \frac{\omega d}{2} - Br_i$$

$$B(r_o - r_i) = -\frac{\omega d}{2}$$

$$B = -\frac{1}{r_o - r_i}\frac{\omega d}{2}$$

$$A = \frac{r_o}{r_o - r_i}\frac{\omega d}{2}$$

$$\underline{\underline{V = \frac{r_o - r}{r_o - r_i}\frac{\omega d}{2}}}$$

6.2 Steady flow $\frac{\delta Q}{dt} = \frac{\delta W_\mu}{dt} = 0$

$$-\frac{\delta W}{dt} = \iint_{c.s.}\left(e + \frac{P}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA$$

$$-\frac{\delta W_s}{dt} = \dot{m}\left[(u_2 - u_1) + \frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)\right]$$

$$\dot{m} = \rho\dot{V} = 1025(.21) = 215.25\ \mathrm{kg/s}$$

$$v_1 = \frac{\dot{V}}{A_1} = 4.278\ \mathrm{m/s}$$

$$v_2 = \frac{\dot{V}}{A_2} = 11.573\ \text{"}$$

Since $\Delta T = 0$ $\quad u_2 - u_1 = 0$

$$z_2 - z_1 = 1.8\ \mathrm{m}$$

$$P_2 = 175\ \mathrm{kPa}$$

$$P_1 = -0.15\ \mathrm{m\ Hg}$$

$$= -19.9\ \mathrm{kPa}$$

6.2 - Continued -

$$\frac{P_2 - P_1}{\rho} = \frac{175 + 19.9}{1.025} = 190\ \mathrm{m^2/s^2}$$

$$\frac{v_2^2 - v_1^2}{2} = \frac{(4.28)^2 - (11.57)^2}{2} = -57.7\ \text{"}$$

$$g(z_2 - z_1) = 9.81(1.8) = 17.7\ \text{"}$$

$$-\dot{W} = (190 - 57.7 + 17.7)(215.3)$$

$$= 32{,}295\ \mathrm{W} = \underline{\underline{32.3\ \mathrm{kW}}}$$

6.3 $\overset{0}{\cancel{\frac{\delta Q}{dt}}} - \overset{0}{\cancel{\frac{\delta W_s}{dt}}} - \overset{0}{\cancel{\frac{\delta W_\mu}{dt}}} = \iint_{c.s.}\left(e + \frac{P}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA + \frac{\partial}{\partial t}\iiint e\rho\, dV$

$$-\dot{m}\left[h_1 + \frac{v_1^2}{2} + \underbrace{\cancel{gz_1}}_{\text{NEG}}\right] + \frac{\partial}{\partial t}\left[mu\right]_{c.v.} = 0$$

$$\cancel{\rho} V c_v \frac{dT}{dt} = \cancel{\rho} A v\left(\frac{v^2}{2}\right)$$

$$\frac{dT}{dt} = \frac{Av}{Vc_v}\frac{v^2}{2}$$

$$= \frac{\frac{\pi}{4}\left(\frac{8}{12}\right)^2 \mathrm{ft}^2 \left(110\frac{\mathrm{ft}}{\mathrm{s}}\right)^3}{2\left(10\ \mathrm{ft}^3\right)\left(0.17\frac{\mathrm{Btu}}{\mathrm{lb_m F}}\right)\left(\frac{778\ \mathrm{ft\,lb_f}}{\mathrm{Btu}}\right)\left(\frac{32.2\ \mathrm{lb_m ft}}{\mathrm{s^2\,lb_f}}\right)}$$

$$= \underline{\underline{21.8\ \mathrm{F/s}}}$$

6.4 ENERGY EQUATION REDUCES TO

$$\iint_{C.S.} \left(e+\frac{P}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA = 0$$

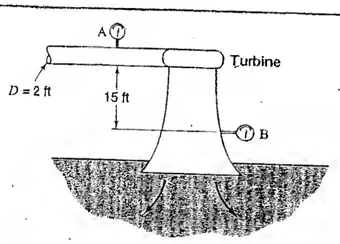
$$u_2 - u_1 + \frac{P_2-P_1}{\rho} + \underbrace{\cancel{\frac{v_2^2-v_1^2}{2}}}_{0} + g\underbrace{\cancel{(y_2-y_1)}}_{0} = 0$$

$$\Delta u = C\Delta T = \frac{P_1-P_2}{\rho}$$

$$\Delta T = \frac{\Delta P}{C\rho} = \frac{10(144)}{(1)(62.4)(778)}$$

$$= \underline{0.0297\ F}$$

6.5.



BETWEEN A & B:

$$-\frac{\delta W_s}{dt} = \iint_{C.S.} \left(e+\frac{P}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA$$

$$\frac{\delta W_s}{dt} = \frac{600(550)}{0.82} = 402\,000\ \frac{\text{ft lb}_f}{s}$$

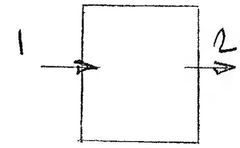
$$\iint \sim = \dot{m}\left[\underbrace{\cancel{\frac{v_B^2-v_A^2}{2}}}_{0} + \frac{P_B-P_A}{\rho} + g(y_B-y_A) + \underbrace{\cancel{\Delta u}}_{0}\right]$$

$$\frac{P_B-P_A}{\rho} = \left\{-\frac{\delta W_s}{dt} + g(y_A-y_B) + \frac{v_A^2}{2}\right\}$$

$$P_B = P_A + \rho\{\ \}$$

$$= \underline{46.5\ \text{PSIA}}$$

6.6.



$\dot{V} = 4\ \text{ft}^3/s$

$P_1 = -6$ PSIG $\qquad P_2 = 40$ PSIG

$$v_1 = \frac{4}{\frac{\pi(1)^2}{4}} = 5.10\ \text{ft}/s \qquad v_2 = \frac{4}{\frac{\pi}{4}\left(\frac{10}{12}\right)^2} = 7.33\ \text{ft}/s$$

$$y_2 - y_1 = 5\ \text{ft}$$

ENERGY EQN. REDUCES TO:

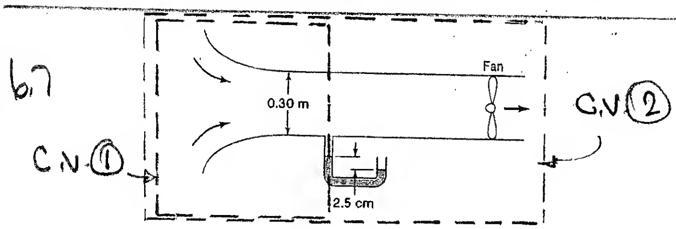
$$-\frac{\delta W_s}{dt} = \dot{m}\left[\frac{\Delta P}{\rho} + \frac{v_2^2-v_1^2}{2} + g(y_2-y_1)\right]$$

$$= \dot{m}g\left[\frac{\Delta P}{\rho g} + \frac{v_2^2-v_1^2}{2g} + y_2 - y_1\right]$$

$$= 27850\ \text{ft lb}_f/s$$

$$= \underline{50.6\ \text{Hp}}$$

6.7



FOR C.V① – ENERGY EQN. REDUCES TO

$$0 = \frac{v_2^2 - \cancel{v_1^2}^{\,0}}{2} + \frac{P_2-P_1}{\rho} + g\underbrace{\cancel{(z_2-z_1)}}_{0}$$

$$\frac{v_2^2}{2} = \frac{P_1-P_2}{\rho} = \frac{2.5\ \text{cm}\ H_2O}{\rho}$$

$$v_2 = \left[2\frac{\Delta P}{\rho}\right]^{1/2} = 20\ \text{m/s}$$

$$\dot{V} = Av_2 = \frac{\pi}{4}(0.3)^2(20) = \underline{1.417\ \text{m}^3/s}$$

6.7 - CONTINUED

FOR C.V. (2) ENERGY EQN IS:

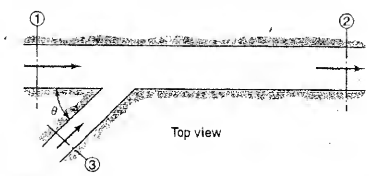
$$-\frac{\delta W_s}{dt} = \dot{m}\left[\cancel{\Delta u}^{\,0} + \frac{V_2^2 - \cancel{V_1^2}^{\,0}}{2} + \cancel{\frac{\Delta p}{\rho}}^{\,0} + \cancel{g\Delta y}^{\,0}\right]$$

$$= \rho \dot{V} \frac{V_2^2}{2}$$

$$= (1.22)(1.417)(20)^2/2$$

$$= \underline{\underline{346 \text{ W}}}$$

6.8.



STEADY FLOW ENERGY EQN:

$$\dot{m}_1\left(u_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho}\right) + \dot{m}_3\left(u_3 + \frac{V_3^2}{2} + \frac{p_3}{\rho}\right) = \dot{m}_2\left(u_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}\right)$$

CONS. OF MASS:

$$V_1A_1 + V_3A_3 = V_2A_2 \qquad (1)$$

ENERGY EQN CAN BE WRITTEN

$$A_1V_1\left[c_vT_1 + \frac{V_1^2}{2} + \frac{p_1}{\rho}\right] + A_3V_3\left[c_vT_3 + \frac{V_3^2}{2} + \frac{p_3}{\rho}\right] = A_2V_2\left[c_vT_2 + \frac{V_2^2}{2} + \frac{p_2}{\rho}\right] \qquad (2)$$

MOMENTUM:

$$(p_1 - p_2)A_1 = \rho V_2^2 A_1 - \rho V_1^2 A_1 - \rho V_3^2 A_3 \cos\theta \qquad (3)$$

6.8 - CONTINUED

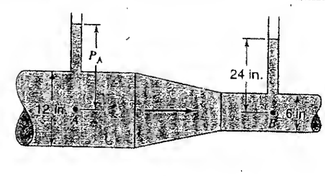
FOR $u = c_vT$, $T_1 = T_3$, $p_1 = p_3$

& LOTS OF ALGEBRA

$$c_v(T_2 - T_1) = \frac{V_1^2}{2}\left[1 + \left(\frac{A_3V_3}{A_1V_1}\right)^2 - 1\right] \times$$

$$\times \left[\frac{1 + 2\frac{A_3V_3}{A_1V_1}}{1 + \frac{A_3V_3}{A_1V_1}}\right] + \frac{V_3^2}{2}\left[\frac{\frac{A_3V_3}{A_1V_1}}{1 + \frac{A_3V_3}{A_1V_1}} - 2\frac{A_3}{A_1}\cos\right.$$

6.9



$\dot{V} = 3 \text{ ft}^3/\text{s}$

BETWEEN A & B - ENERGY EQN IS:

$$\iint_{C.S.}\left(e + \frac{p}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA = 0$$

$$\frac{V_B^2 - V_A^2}{2} + u_B - u_A + \frac{p_B - p_A}{\rho} = 0$$

$$V_A = \frac{3}{(\pi/4)(1)^2} = 3.82 \text{ ft/s}$$

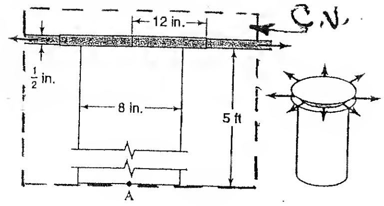
$$V_B = \frac{3}{\pi/4(1/2)^2} = 15.28 \text{ ft/s}$$

$$\frac{p_A - p_B}{\rho g} = \frac{(15.28)^2 - (3.82)^2}{2g} + 0.45$$

$$= 2.15 \text{ ft } H_2O$$

$$\underline{\underline{p_A = 2.15 + 2 = 4.15 \text{ ft of } H_2O}}$$

6.10



$P_A = 10$ PSIG

$$\sum F_y = \iint_{C.S} v_y \rho (\vec{v} \cdot \vec{n}) dA$$

FLOW RATE MUST BE DETERMINED

ENERGY EQN FOR C.V. SHOWN:

$$\cancel{\Delta u}^{0} + \frac{\Delta P}{\rho} + \Delta \frac{v^2}{2} + g\Delta z = 0$$

$$\frac{\Delta P}{\rho} = \frac{10(144)(32.2)}{62.4} = 743 \ \text{FT}^2/\text{s}^2$$

$$v_A = \frac{\dot{V}}{\pi/4 (2/3)^2} = 2.865 \dot{V}$$

$$v_B = \frac{\dot{V}}{\pi(2)(0.5/12)} = 3.82 \dot{V}$$

$$\Delta \frac{v^2}{2} = (2.865^2 - 3.82^2)\frac{\dot{V}^2}{2} = -3.19 \dot{V}^2$$

$$g\Delta y = 32.2(-5) = -161 \ \text{FT}^2/\text{s}^2$$

$$743 - 3.19 \dot{V}^2 - 161 = 0$$

$$\dot{V}^2 = \frac{682}{3.19} \qquad \dot{V} = 14.6 \ \text{FT}^3/\text{s}$$

$$v_A = 41.9 \ \text{FT/s} \qquad V_B = 55.9 \ \text{FT/s}$$

$$F_y + P_A A_A - \rho g V = \dot{m}(-v_A)$$

$$P_A A_A = 10\left(\frac{\pi}{4}\right)(8)^2 = 502 \ \text{LB}_F$$

$$\underset{(\text{WT})}{\rho g V} = \frac{(62.4)(32.2)\left(\frac{\pi}{4}\right)\left(\frac{8}{12}\right)^2(5)}{32.2}$$

$$= 109 \ \text{LB}_F$$

6.10 - CONTINUED

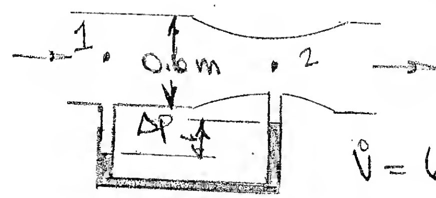
$$\dot{m}(-v_A) = \frac{-62.4(14.6)(41.9)}{32.2} = -1185 \ \text{LB}_F$$

$$F_y = -502 + 109 - 1185$$

$$= \underline{-1578 \ \text{LB}_F}$$

FORCE ON LID IS 1578 LB_F ↑

6.11



$$\dot{V} = 6 \ \text{m}^3/\text{s}$$

$$\Delta P = 0.10 \ \text{m ALCOHOL (S.G.} = 0.8)$$

$$= 0.08 \ \text{m} \ H_2O = 785 \ \text{Pa}$$

$$A_1 = \frac{\pi}{4}(0.6)^2 = 0.283 \ \text{m}^2$$

$$v_1 = \frac{\dot{V}}{A_1} = \frac{6}{0.283} = 21.2 \ \text{m/s}$$

ENERGY EQN. REDUCES TO:

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + \cancel{g\Delta y}^{0} = 0$$

$$\frac{P_1 - P_2}{\rho} = \frac{v_2^2 - v_1^2}{2} = \frac{\dot{V}^2}{2}\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right]$$

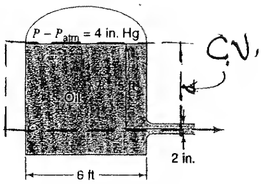
$$= \frac{v_1^2}{2}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$\frac{P_1 - P_2}{\rho} = \frac{785}{1.226} = 640 \ \text{m}^2/\text{s}^2$$

$$640 = \frac{(21.2)^2}{2}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$A_2 = 0.144 \ \text{m}^2 \qquad \underline{D_2 = 0.428 \ \text{m}}$$

6.12



ENERGY EQN. REDUCES TO:

$$\Delta u + \frac{\Delta P}{\rho} + \Delta \frac{v^2}{2} + g\Delta y = 0$$

$\dot{\xi}$ FOR $\Delta u = v_1 = 0$

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2}{2} + g(y_2 - y_1) = 0$$

$$v_2 = \left[2\left(\frac{P_1 - P_2}{\rho}\right) + g\Delta y\right]^{1/2}$$

BY CONSERVATION OF MASS:

$$A_{TANK}\left(-\frac{dy}{dt}\right) = A_{JET}\, v_2$$

$$-\frac{A_t}{A_j}\frac{dy}{dt} = [\ \ ]^{1/2}$$

$$-\frac{A_t}{A_j}\int_{y_0}^{y_0-2}\frac{dy}{(K_1 + K_2 y)^{1/2}} = \int_0^t dt$$

$$K_1 = \frac{2\Delta P}{\rho} \qquad K_2 = 2g$$

$$t = -\frac{A_t}{A_j}\left[\frac{2}{K_2}(K_1 + K_2 y)^{1/2}\right]_{y_0}^{y_0-2}$$

$$K_1 = 344\ \text{ft}^2/\text{s}^2$$

$$[K_1 + K_2(y_0 - 2)]^{1/2} = 23.2\ \text{ft/s}$$

$$[K_1 + K_2(y_0)]^{1/2} = 25.8\ \text{"}$$

$$\underline{\underline{t = 105\ \text{s}}}$$

6.13. ENERGY EQN. REDUCES TO

$$\frac{P_1}{\rho_1} + \frac{v_1^2}{2} = \frac{P_2}{\rho} + \frac{v_2^2}{2}$$

$$P_1 = P_{ATM} = 29\ \text{"Hg}\left(\frac{14.7}{29.92}\right) = 14.25\ \text{psi}$$

$P_2 = ?$

$$v_1^2 = \left[\left(85\ \text{mi/hr}\right)\left(\frac{5280}{3600}\right)\right]^2 = (124.7)^2$$

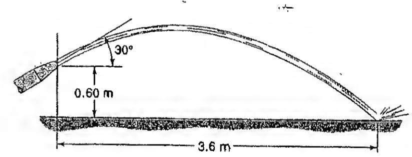
$$v_2^2 = 120^2$$

$$P_2 = 14.25 + \frac{\rho}{2}\left[(124.7)^2 - (120)^2\right]$$

$$\rho = P/RT = \frac{14.25(144)}{53.3(500)} = 0.0770\ \text{lbm/ft}^3$$

$$P_2 = 14.25 + 1.37 = \underline{15.6\ \text{psi}}$$

6.14



ENERGY EQN: $\frac{v_2^2 - v_1^2}{2} + g(y_2 - y_1) = 0$

IN X-DIRECTION: $v_0 \cos\theta = v_x = \frac{dx}{dt}$

IN y-DIRECTION $v_0 \sin\theta - gt = \frac{dy}{dt}$

$$x = (v_0\cos\theta)t$$

$$y = (v_0 \sin\theta)t - gt^2/2$$

COMBINING: $y = x\tan\theta - \frac{g}{2}\frac{x^2}{(v_0\cos\theta)^2}$

6.14 – CONTINUED

$y = 0.6\ m \qquad \tan\theta = 0.577$

$x = 3.6\ m \qquad \cos\theta = 0.866$

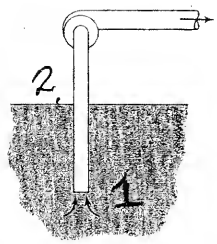
$$0.6 = 3.6(0.577) - \frac{9.81}{2}\frac{3.6^2}{(0.866 v_0)^2}$$

$$\underline{v_0 = 7.57\ ft/s}$$

$$\text{TOTAL HEAD} = 0.6 + \frac{v^2}{2g}$$

$$= \underline{3.52\ m}$$

6.15



$$\dot{V} = 550\ g/m = 1.225\ ft^3/s$$

$$v = \frac{\dot{V}}{A} = \frac{1.225}{\pi/4\,(5.95/12)^2} = 6.35\ ft/s$$

ENERGY EQN: $\underline{2}$ IS AT H_2O LEVEL OUTSIDE PIPE

$$\frac{P_1 - \cancel{P_2}^{\,0}}{\rho} + \frac{v_1^2 - \cancel{v_2^2}^{\,0}}{2} + g(y_1 - y_2) = 0$$

$$\frac{P_1}{\rho} = -\frac{v_1^2}{2} - g y_1$$

$$= -\frac{(6.35)^2}{2} - 32.2(6)$$

$$= -(20.16 + 193.2)$$

$$= -213.4\ ft^2/s^2$$

$$P_1 = -\frac{(62.4)(213.4)}{(144)32.2} = \underline{-2.87\ psig}$$

6.16 WITH REFERENCE TO PROB 16.15

BETWEEN H_2O SURFACE & PUMP INLET ENERGY EQN IS

$$\frac{\overset{P_{atm}}{P_1} - \overset{P_v}{P_2}}{\rho} + \frac{\cancel{v_1^2}_{\,0} - v_2^2}{2} + g(y_1 - y_2) = h_L$$

$$\frac{P_{atm} - P_v}{\rho g} - \frac{v_2^2}{2g} + y_1 - y_2 = h_L$$

$$\frac{v_2^2}{2g} = \frac{P_{atm} - P_v}{\rho g} - (y_2 - y_1) - h_L$$

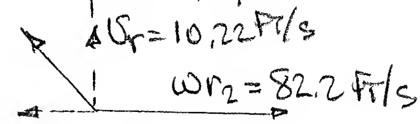
$$= \frac{(14.7 - 0.247)(144)(32.2)}{62.4(32.2)} - 4 - 4$$

$$= 25.35\ FT$$

$$v_2 = \left[2(32.2)(25.35)\right]^{1/2} = 40.4\ ft/s$$

$$\dot{V} = A v_2 = \frac{\pi}{4}\left(\frac{5.95}{12}\right)^2(40.4) = \underline{\underline{7.8\ ft^3/s}}$$

6.17 FROM PROB 5.27



$v_t = 10.22\ ft/s$

AT r_2 — $v_x = 82.2 - 10.22$

$v_y = 10.22$

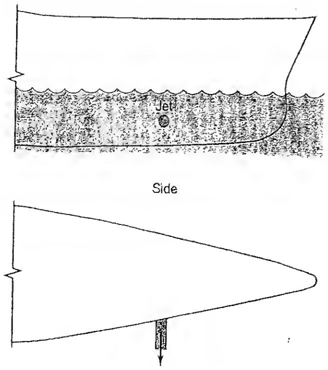
$$v = (v_x^2 + v_y^2)^{1/2} = 82.9\ ft/s$$

$$\text{HEAD} = v^2/2g = \underline{\underline{106.7\ FT}}$$

$$\Delta P = \rho \frac{v^2}{2} = 62.4(82.9)^2/32.2(2)$$

$$= \underline{\underline{6660\ lb_f/ft^2 = 46.2\ psi}}$$

6.18



FOR THE SITUATION SHOWN -

$$\text{THRUST} = F = \rho \dot{V} v$$

$$\text{POWER} = -\frac{\delta W_s}{dt} = \rho \dot{V} \frac{v^2}{2}$$

$$\frac{\text{POWER}}{\text{THRUST}} \sim \frac{\rho \dot{V} v^2/2}{\rho \dot{V} v} \cong v$$

$$\frac{\text{THRUST}}{\text{POWER}} \sim \frac{1}{v} \sim \frac{1}{h^{1/2}}$$

FAVORABLE CHOICE: $\left\{ \begin{array}{l} \text{HIGH VOLUME} \\ \text{LOW PRESSURE} \end{array} \right.$

6.19 FROM PROB 5.7:

$P_1 = 50$ PSIG $\quad P_2 = 5$ PSIG

$D_1 = 12$ IN $\quad D_2 = 2.5$ IN

$\dot{V} = 3\ \text{ft}^3/\text{s} \quad$ S.G. $= 0.8$

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g}$$

$$v_1 = \frac{3}{\frac{\pi}{4}(1)^2} = 3.82\ \text{ft/s}$$

$$v_2 = \frac{3}{\frac{\pi}{4}\left(\frac{2.5}{12}\right)^2} = 88\ \text{ft/s}$$

6.19 - CONTINUED

$$h_L = \frac{(50-5)144}{0.8(62.4)} + \frac{3.82^2 - 88^2}{2(32.2)}$$

$$= \underline{\underline{9.79\ \text{FT}}}$$

6.20 FOR A C.V. ENCLOSING THE FALLS:

$$\Delta u + \underbrace{\cancel{\frac{\Delta p}{\rho}}}_{0} + \underbrace{\cancel{\frac{\Delta v^2}{2}}}_{0} + g\Delta y = 0$$

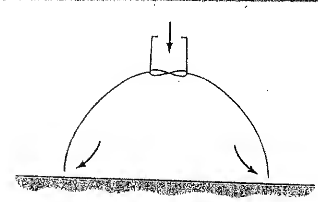
$$\Delta u = g\Delta y = 9.81(165) = 1620\ \text{m}^2/\text{s}^2$$

$$= 1620\ \text{m}^2/\text{s}^2 \left(1\ \frac{\text{s}^2 \cdot \text{N}}{\text{kg m}}\right) = 1620\ \text{J/kg}$$

FOR H_2O: $c_p = 4184\ \text{J/kg}\cdot\text{K}$

$$\Delta T = \frac{\Delta u}{c_p} = \frac{1620}{4184} \cong \underline{\underline{0.39^\circ\text{C}}}$$

6.21



ASSUME VERTICAL FORCES DO NOT INCLUDE MOMENTUM OF INCOMING AIR -

$$(P - P_{atm})A = Mg \quad \left\{ \begin{array}{c} \text{PRESSURE} \\ \text{FORCE} \end{array} \right\} = \text{WT}$$

ENERGY EQN BECOMES BERNOULLI EQN BETWEEN INSIDE & EXIT -

$$\frac{P - P_{atm}}{\rho} = \frac{v^2}{2}$$

OR $v^2 = 2\,\dfrac{Mg}{\rho A}$

6.21 CONTINUED

$$v^2 = 2\frac{(8100\,kg)(9.81\,m/s^2)}{(1.205\,kg/m^3)(27\,m^2)}$$

$$= 4885\,m^2/s^2 \qquad v = 69.9\,m/s$$

$$\dot{V} = 69.9(24)(0.03)$$

$$= 50.3\,m^3/s$$

$$\dot{m} = 60.6\,kg/s$$

ENERGY EQN:

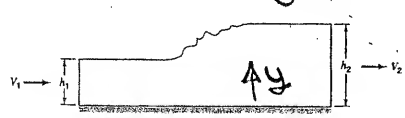
$$-\frac{\delta W_s}{dt} = \dot{m}\Big(\underset{0}{\cancel{\Delta u}} + \frac{\Delta v^2}{2} + \underset{0}{\frac{\Delta P}{\rho}} + \underset{NO}{g\Delta y}\Big)$$

$$= \dot{m}\frac{v^2}{2}$$

$$= 60.6\frac{(69.9)^2}{2} = \underline{\underline{148\ kW}}$$

6.22 FROM PROB 5.22

$$h_2 = \frac{h_1}{2}\left[\left(1+\frac{8v_1^2}{gh_1}\right)^{1/2}-1\right]$$



APPLIES TO

FOR BERNOULLI EQN. TO BE VALID — $h_L = 0$

ENERGY EQN FOR THIS CASE IS

$$h_L = \frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} + y_1 - y_2$$

& SINCE $P = P_{atm} + \rho g(h-y)$

6.22 - CONTINUED

$$h_L = \frac{v_1^2 - v_2^2}{2g} + h_1 - h_2$$

WRITING SOLN. TO PROB 5.22 AS

$$h_2 = \frac{h_1}{2}\left(\sqrt{1+B}-1\right) \quad \left\{B = \frac{8v_1^2}{gh_1}\right\}$$

& NOTE THAT — FOR $h_2 > h_1$ $B > 8$

BERNOULLI EQN APPLIES FOR $B = 8$

& OBVIOUSLY $h_L > 0$ FOR $B > 8$

6.23 ENERGY EQN APPLIES IN FORM:

$$-\frac{\delta W_s}{dt} = \dot{m}\frac{P - P_{atm}}{\rho} = \dot{V}\Delta P$$

$$= \eta_p \eta_m (\text{Power})$$

$$\left\{\text{PER PERSON}\right\} \quad \dot{V} = \frac{80}{(7.48)(24)(3600)} = 1.238\times10^{-4}\ ft^3/s$$

$$P = \frac{\dot{V}\Delta P}{\eta_p \eta_m}$$

$$= \frac{(1.238\times10^{-4})(60)(144)}{1.\ 0.75(0.9)}$$

$$= 1.584\ \frac{ft\,lb_f}{s}$$

$$= 2.148\ W$$

PER MONTH —

$$P = 2.148\ W\ (30)(24)$$

$$= 1547\ W{\cdot}h = \underline{\underline{1.547\ kWh}}$$

6.24 BERNOULLI EQN
BETWEEN FREE STREAM & A REFERENCE POINT (1) ON GAL

$$\frac{P_{ATM}}{\rho g} + \frac{(W+V)^2}{2g} = \frac{P_1}{\rho g} + \frac{(V-W)^2}{2g}$$

$$\frac{P_1 - P_{ATM}}{\rho} = \frac{P_{1g}}{\rho} = 2WV$$

$$\underline{P_{1g} = 2\rho W V}$$

6.25 ENERGY EQN IS

$$\frac{\delta Q}{dt} = \iint_{c.s.} \left(e + \frac{P}{\rho}\right)\rho(\vec{v}\cdot\vec{n})\,dA$$

$$\dot{Q} = \dot{m}\left[(u_2 - u_1) + \frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g(y_2 - y_1)\right]$$

$$\Delta u = 200 \text{ kJ/kg}$$

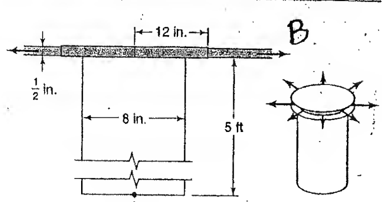
$$\frac{\Delta P}{\rho} = \frac{340\times 10^3}{1001} = 340 \text{ kJ/kg}$$

$$\frac{\Delta v^2}{2} = 0$$

$$g\,\Delta y = 9.81(15) = 0.147 \text{ kJ/kg}$$

$$\dot{Q} = 200 + 340 + 0.15 = \underline{540 \text{ kJ/kg}}$$

6.26



$$\dot{V} = v_A \frac{\pi}{4}\left(\frac{8}{12}\right)^2 = v_B(2\pi)(1)\left(\frac{0.5}{12}\right)$$

6.26 – CONTINUED

$v_A = 2.865\,\dot{V}$ $\qquad v_A^2 = 8.22\,\dot{V}^2$

$v_B = 3.82\,\dot{V}$ $\qquad v_B^2 = 14.6\,\dot{V}^2$

FOR NEGLIGIBLE FRICTION — BERNOULLI EQN APPLIES

$$\frac{P_A - P_B}{\rho} + \frac{v_A^2 - v_B^2}{2} + g(y_2 - y_1) = 0$$

$$\frac{v_B^2 - v_A^2}{2} = \dot{V}^2\,\frac{14.6 - 8.22}{2} = 3.32\,\dot{V}^2$$

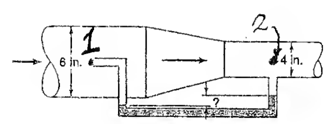
$$\frac{P_A - P_B}{\rho} = \frac{10(144)(32.2)}{62.4} = 743 \text{ ft}^2/\text{s}^2$$

$$g(y_A - y_B) = 32.2(-5) = -161$$

$$3.32\,\dot{V}^2 = 743 - 161 = 582$$

$$\underline{\dot{V} = 13.2 \text{ ft}^3/\text{s}}$$

6.27



BERNOULLI EQN BETWEEN 1 & 2:

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} = 0$$

$$v_1 = 1\Big/\frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 5.09 \text{ ft/s}$$

$$v_2 = 1\Big/\frac{\pi}{4}\left(\frac{4}{12}\right)^2 = 11.5 \text{ ft/s}$$

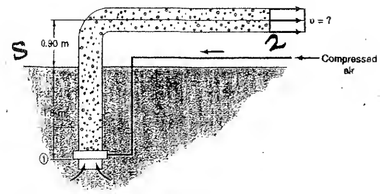
$$\frac{\Delta P}{\rho} = \frac{-\left[(11.5)^2 - (5.09)^2\right]}{2} = -1.65 \text{ ft } H_2O$$

MANOMETER RDG — $= -1.457$ "Hg

$$h\left[1 - 1/13.6\right] = 1.457$$

$$\underline{h = 1.63 \text{ INCHES}}$$

6,28



FOR A CONTROL VOLUME BETWEEN 1 & 2 (IN MIXTURE REGION)

$$\frac{P_2 - P_1}{\rho_M} + \cancel{\frac{v_2^2 - v_1^2}{2}} + g(y_2 - y_1) = 0$$

$$P_1 - P_{ATM} = \rho_m g \Delta y_1 \qquad (1)$$

MASS BALANCE AROUND MIXING CHBR.

$$\dot{m}_{AIR} + \dot{m}_W = \dot{m}_M$$

AS GIVEN: $\rho_m = \rho_W / 2$

$$\therefore \quad v_m = 2v_W + 2\frac{\rho A v|_A}{\rho A|_W} \qquad (2)$$

CONTROL VOLUME BETWEEN H_2O SURFACE & 1 (H_2O ONLY)

$$\frac{P_{ATM} - P_1}{\rho_W} + \frac{0 - v_W^2}{2} + g\Delta y_2 = 0$$

$$P_1 - P_{ATM} = \rho_W g \Delta y_2 - \rho_W \frac{v_W^2}{2} \qquad (3)$$

EQUATING (1) & (3):

$$\rho_m g \Delta y_1 = \rho_W \left(g\Delta y_2 - \frac{v_W^2}{2}\right)$$

$$\frac{v_W^2}{2} = g(\Delta y_2 - \Delta y_1/2)$$

$$v_W = [2g(0.45\,m)]^{1/2}$$

6,28 - CONTINUED

SUBSTITUTING EXPRESSION FOR v_W INTO (2)

$$v_m = 2[g(0.9m)]^{1/2} + \frac{2(\rho A v)_{AIR}}{\rho_W v_W}$$

SINCE $\frac{v_W}{v_A} \gg 1$ 2ND TERM IS SMALL

$$\therefore \quad v_m = 2[9.81(0.9)]^{1/2} = \underline{\underline{5.94\,m/s}}$$

6,29 FOR CONDITIONS OF PROB 6,28

CONTROL VOLUME AROUND MIXING CHBR.

$$\sum F_y = \iint_{C.S.} v_y \rho (\vec{v}\cdot\vec{n})\,dA$$

$$\Delta P A = \rho_m A v_m \left[v_m - \frac{\rho_m}{\rho_W} v_m\right]$$

THIS NEGLECTS MOMENTUM OF AIR

FROM PROB 6,28

ABOVE MIXER - $P = P_{ATM} + \rho_m g \Delta y_1$

BELOW " $P = P_{ATM} + \rho_W g \Delta y_2 - \rho_W \frac{v_W^2}{2}$

$$\Delta P = \rho_W g (y_2 - y_1) - \rho_W v_W^2$$

EQUATING WITH MOMENTUM EXPRESSION

$$\Delta P = \rho_m v_m^2 \left(1 - \frac{\rho_m}{\rho_W}\right)$$

$$= \rho_W \left[g(y_2 - y_1) - v_W^2\right]$$

$$\frac{v_m^2}{4} = g(y_2 - y_1) - v_W^2$$

FOR $v_W / v_m = 2$ $\quad \underline{\underline{v_m = 4.6\,m/s}}$

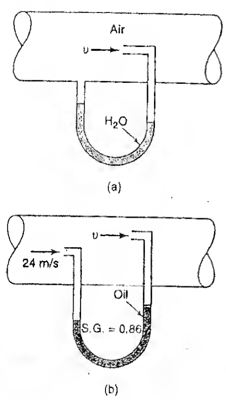
6.29 – CONTINUED

$$\Delta P = \rho_m v_m^2 (1 - 1/2)$$

$$= \frac{1}{2}(4.6)^2(1000/2)$$

$$\cong \underline{5.3\ kPa}$$

6.30



IN BOTH CASES – BERNOULLI EQN. IS

$$\frac{v_2^2 - v_1^2}{2} + \frac{P_2 - P_1}{\rho_2} = 0$$

(a) $$\Delta P = \frac{v^2}{2g} = \frac{15^2}{2(9.81)}$$

$$= 11.47\ m\ AIR$$

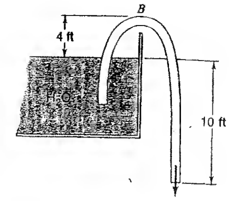
$$= \underline{\underline{1.39\ cm\ H_2O}}$$

(b) $$\Delta P = \frac{24^2 - 15^2}{2(9.81)}$$

$$= 17.9\ m\ AIR$$

$$= \underline{\underline{2.52\ cm\ OIL}}$$

6.31



BETWEEN LIQUID SURFACE & EXIT:

BERNOULLI EQN:

$$\frac{v^2}{2} = g\Delta y$$

$$v = (2g\Delta y)^{1/2}$$

$$= 2(32.2)(10) = 25.35\ ft/s$$

$$\dot{V} = \frac{\pi}{4}\left(\frac{1}{12}\right)^2(25.35) = \underline{\underline{1.66\ ft^3/s}}$$

BETWEEN POINT B & EXIT:

$$\frac{P_B - P_{ATM}}{\rho} + g(y_B - y_{EXIT}) = 0$$

$$P_B = P_{ATM} - \rho g \Delta y$$

$$= 14.7\ psi - \frac{62.4(32.2)(14)}{32.2(144)}$$

$$= \underline{\underline{8.63\ psi}}$$

BY CONTINUITY –

$$\dot{V}_{TANK} = A_{TANK}\left(-\frac{dy}{dt}\right)$$

$$= A_{PIPE}\sqrt{2gy}$$

$$-\frac{dy}{dt} = \frac{A_{PIPE}}{A_{TANK}}\sqrt{2g}\,y^{1/2}$$

6.31 CONTINUED

$$-\int_{10}^{7} y^{-1/2}\,dy = \frac{A_P}{A_t}\sqrt{2g}\int_0^t dt$$

$$2y^{1/2}\Big|_7^{10} = \frac{A_P}{A_t}\sqrt{2g}\;t$$

$$2\left[10^{1/2}-7^{1/2}\right] = \frac{(1/12)^2}{10^2}\sqrt{2(32.2)}\;t$$

$$t = \underline{1854\ s = 0.515\ h}$$

6.32 ENERGY EQN FOR THIS CASE:

$$\frac{v_2^2-v_1^2}{2} + g(y_2-y_1) + u_2 - u_1 = 0$$

$v_1 = 0$

$y_2 - y_1 = -10$ FT

$u_2 - u_1 = 3.2\, v^2/g$

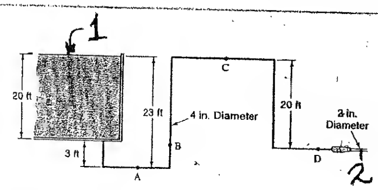
$$\frac{v_2^2}{2g} + (-10) + 3.2\frac{v_2^2}{g} = 0$$

$$\frac{3.2v^2}{g} = 10$$

$$v = 10.03\ \text{FT/s}$$

$$\dot{V} = \frac{\pi}{4}\left(\frac{1}{12}\right)^2(10.03) = \underline{0.0547\ \text{FT}^3/\text{s}}$$

6.33



BETWEEN 1 & 2

$$\frac{v_2^2-0}{2} + g(y_2-y_1) = 0$$

6.33 CONTINUED

$$v_2 = \left[2(32.2)(20)\right]^{1/2} = 35.9\ \text{FT/s}$$

$$\dot{V} = Av = \frac{\pi}{4}\left(\frac{2}{12}\right)^2(35.9) = \underline{0.783\ \text{FT}^3/\text{s}}$$

IN 4" LINE — $v = \frac{v_2}{4} = 8.975$ FT/s

$v^2 = 80.55\ \text{FT}^2/\text{s}^2$

BETWEEN 1 & A:

$$\frac{P_A - P_1}{\rho} + \frac{v_A^2 - v_1^2}{2} + g(y_A - y_1) = 0$$

$$P_A = P_{ATM} + \rho\left(-v_A^2/2\right) + \rho g(y_1 - y_A)$$

$$= P_{ATM} + \frac{62.4}{32.2}\left(-\frac{80.55}{2}\right) + 62.4(23)$$

$$= P_{ATM} + 1356\ \text{LB/FT}^2 = \underline{3475\ \text{PSF}}$$

(24.12 PSI)

$$v_A = \underline{8.975\ \text{FT/s}}$$

BETWEEN A & B:

$$\frac{P_A - P_B}{\rho} + \underbrace{\frac{v_A^2 - v_B^2}{2}}_{0} + g(y_A - y_B) = 0$$

$$P_B = P_A + \rho g(-3) = \underline{3290\ \text{PSF}}$$

(22.83 PSI)

$$v_B = \underline{8.975\ \text{FT/s}}$$

CONDITIONS AT D & B ARE EQUAL

$\therefore$ $P_D = \underline{3290\ \text{PSF}}$

$v_D = \underline{8.975\ \text{FT/s}}$

6.33 CONTINUED

BETWEEN B & C:

$$\frac{P_B - P_C}{\rho} + \frac{\cancel{v_B^2} - \cancel{v_C^2}}{2} + g(y_B - y_C) = 0$$

$$P_C = P_B + \rho g (y_B - y_C)$$

$$= P_B + 62.4(-20)$$

$$= \underline{2042 \text{ PSF}}$$

(14.18 PSI)

$$\underline{v_C = 8.975 \text{ FT/S}}$$

6.34 BETWEEN WATER LEVEL (1) & EXIT (2)

$$\frac{v_2^2 - \cancel{v_1^2}}{2} + g(y_2 - y_1) = 0$$

$$v_2 = \sqrt{2g\Delta y}$$

$$\dot{V} = \frac{\pi}{4} D_{EXIT}^2 [2gy]^{1/2}$$

H_2O IN TANK:

$$\dot{V} = \frac{\pi}{4} D_{TANK}^2 \left(-\frac{dy}{dt}\right)$$

$$\frac{dy}{dt} = \left(\frac{D_{EXIT}}{D_{TANK}}\right)^2 \sqrt{2g}\, y^{1/2}$$

$$\int y^{-1/2} dy = \left(\frac{D_E}{D_t}\right)^2 \sqrt{2g} \int dt$$

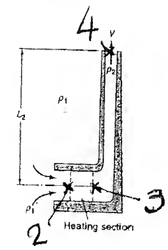
$$2y^{1/2} \Big|_4^{28} = \left(\frac{D_E}{D_t}\right)^2 \sqrt{2g}\, t$$

6.34 - CONTINUED

$$t = \frac{2(28^{1/2} - 4^{1/2})}{\left(\frac{2/12}{15}\right)^2 [2(32.2)]^{1/2}}$$

$$= \underline{6644 \text{ S} = 1.846 \text{ HOURS}}$$

6.35



FROM 1 TO 2

$$\frac{P_1 - P_2}{\rho_1} + \frac{v_2^2 - v_1^2}{2} + g(y_1 - y_2) = 0$$

$$\frac{P_1 - P_2}{\rho_1} + \frac{v_2^2}{2} = 0 \qquad (1)$$

FROM 3 TO 4:

$$\frac{P_3 - P_4}{\rho_2} + \frac{v_3^2 - v_4^2}{2} + g(y_3 - y_4) = 0$$

$$\frac{v_4^2}{2} = \frac{v_3^2}{2} + \frac{P_3 - P_4}{\rho_2} - gL \qquad (2)$$

NOTE THAT $P_4 + \rho_1 g L = P_1$, GIVING

$$\frac{v_4^2}{2} = \frac{v_3^2}{2} + \frac{P_3 - P_1 + \rho_1 g L}{\rho_2} \qquad (3)$$

FROM 2 TO 3

$$\frac{P_2}{\rho_1} - \frac{P_3}{\rho_2} + \frac{v_2^2 - v_3^2}{2} = 0$$

FOR v_2 & v_3 NEGLIGIBLE,

$$P_2 = P_3$$

& FROM (2

$$\frac{v^2}{2} = -gL + \frac{\rho_1}{\rho_2} gL = \underline{gL\left(\frac{\rho_1}{\rho_2} - 1\right)}$$

6.36 FROM PROB 6.28:

$$\frac{P_1-P_2}{\rho_1}+\frac{v_2^2}{2}=0$$

$$\frac{P_3-P_1}{\rho_2}=\frac{v^2}{2}+gL\frac{(\rho_2-\rho)}{\rho_2}-\frac{v_3^2}{2}$$

CONS. OF MASS:

$$\rho_1 v_2=\rho_2 v_3=\rho_2 v/R$$

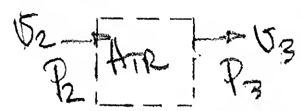
$$\frac{A_{HTR}}{A_{STACK}}=R$$

$$\therefore\ v_2^2=\left(\frac{\rho_2}{\rho_1}\right)^2\left(\frac{v}{R}\right)^2 \qquad v_3^2=\frac{v^2}{R^2}$$

GIVING $$P_2-P_1=-\frac{\rho_1}{2}\left(\frac{\rho_2}{\rho_1}\right)^2\frac{v^2}{R^2}$$

$$P_3-P_1=\rho_2\frac{v^2}{2}+gL(\rho_2-\rho_1)-\frac{\rho_2 v^2}{2R^2}$$

FROM MOMENTUM THEOREM—



$$F_x=\iint_{CS} v_x\rho(\vec{v}\cdot\vec{n})\,dA$$

$$(P_2-P_3)A=\rho_1 v_2 A(v_3-v_2)$$

$$P_2-P_3=\frac{\rho_2 v^2}{R^2}\left(1-\frac{\rho_2}{\rho_1}\right)$$

BERNOULLI EQN:

$$\frac{v^2}{R^2}\left(1-\frac{\rho_2}{\rho_1}\right)+\frac{\rho_1}{2\rho_2}\left(\frac{\rho_2}{\rho_1}\right)^2\frac{v^2}{R^2}+\frac{v^2}{2}+gL\left(1-\frac{\rho_1}{\rho_2}\right)-\frac{v^2}{2R^2}=0$$

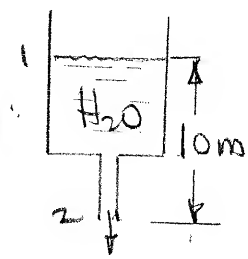
6.36 CONTINUED

DOING THE ALGEBRA:

$$v_2=\frac{2gL\left(\frac{\rho_1}{\rho_2}-1\right)}{1+\frac{1-\rho_2/\rho_1}{R^2}}$$

6.37

FRICTIONLESS FLOW:



FROM BERNOULLI

$$\frac{P_2-P_1}{\rho g}+\frac{v_2^2-v_1^2}{2g}+y_2-y_1=0$$

$$v_2^2=2g(y_1-y_2)$$

$$v=\left[2(9.81)(10)\right]^{1/2}=14\ \text{m/s}$$

$$\dot{m}=(1000)\left(\frac{\pi}{4}\right)(0.04)^2(14)=\underline{17.6\ \text{kg/s}}$$

WITH NOZZLE — $v=14$ m/s {STILL}

$$\dot{m}=(1000)\left(\frac{\pi}{4}\right)(0.01)^2(14)=\underline{1.10\ \text{kg/s}}$$

WITH $u_2-u_1=3v^2$

ENERGY EQN REDUCES TO

$$\frac{v_2^2}{2g}+\frac{3v^2}{g}=2(y_1-y_2)$$

$$v_2=\left[\frac{4}{7}(9.81)10\right]^{1/2}=7.49\ \text{m/s}$$

PIPE: $\dot{m}=\underline{9.42\ \text{kg/s}}$

NOZZLE: $\dot{m}=\underline{0.589\ \text{"}}$

6.38 SAME TANK AS IN PROB 6.37 BUT 2 EXIT PIPES —

PIPE 1 : $D = 0.04\,m$

$\Delta y = 10\,m$

PIPE 2 $D = 0.04\,m$

$\Delta y = 20\,m$

FRICTIONLESS FLOW !

PIPE 1 —

AS IN PROB 6.37

$v = \sqrt{2g\Delta y} = 14\,m/s$

$\dot{m} = \underline{\underline{17.6\,kg/s}}$

PIPE 2 : ALSO — $v = \sqrt{2g\Delta y}$

$v = \left[2(9.81)(20)\right]^{1/2}$

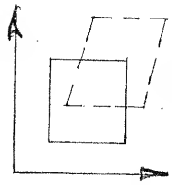
$= \underline{\underline{19.81\,kg/s}}$

$\dot{m} = (1000)\left(\frac{\pi}{4}\right)(0.04)^2(19.81)$

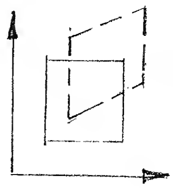
$= \underline{\underline{24.9\,kg/s}}$

CHAPTER 7

7.1

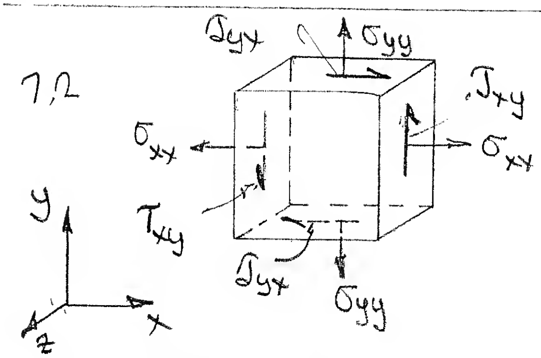


(a) $\dfrac{\partial v_x}{\partial y} \gg \dfrac{\partial v_y}{\partial x}$



(b) $\dfrac{\partial v_y}{\partial x} \gg \dfrac{\partial v_x}{\partial y}$

7.2



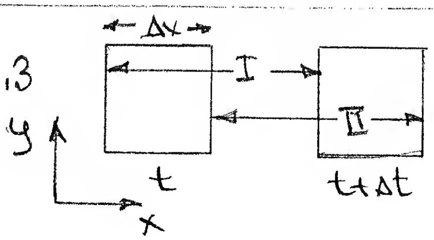
FOR 2-D FLOW - IN x, y

$v_z = 0 \qquad \sigma_{zz} = 0$

$\dfrac{\partial v_z}{\partial x} = 0 \qquad \tau_{zx} = \tau_{xz} = 0$

$\dfrac{\partial v_z}{\partial y} = 0 \qquad \tau_{zy} = \tau_{yz} = 0$

7.3



$\mathrm{I} = v_x(x)\Delta t$

$\mathrm{II} = v_x(x+\Delta x)\Delta t$

7.3 - CONTINUED

AXIAL STRAIN RATE:

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta t \to 0}} \frac{v_x(x+\Delta x)\Delta t - v_x(x)\Delta t}{\Delta x \Delta t} = \frac{\partial v_x}{\partial t}$$

RATE OF VOLUME CHANGE:

$$= \lim_{\substack{\Delta x \to 0 \\ \Delta t \to 0}} \frac{A\Delta x|_{t+\Delta t} - A\Delta x|_t}{A\Delta x \Delta t}$$

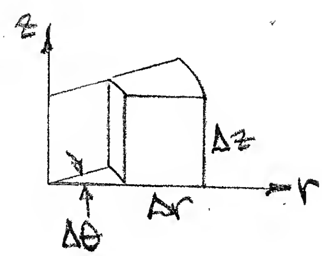
$$= \lim_{\substack{\Delta x \to 0 \\ \Delta t \to 0}} \frac{v_x(x+\Delta x)\Delta t - v_x(x)\Delta t}{\Delta x \Delta t} = \frac{\partial v_x}{\partial x}$$

IN 3 DIMENSIONS

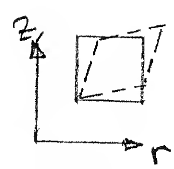
BOTH AXIAL STRAIN RATE AND VOLUME CHANGE RATE ARE GIVEN BY

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

7.4



IN r-z PLANE:



$$-\frac{d\delta}{dt} = -\lim_{\Delta t \to 0} \frac{\delta|_{t+\Delta t} - \delta|_t}{\Delta t}$$

$$= -\lim_{\Delta t \to 0} \left[\frac{\pi/2 - \tan^{-1}\left(\dfrac{v_r|_{z+\Delta z} - v_r|_z}{\Delta z}\right)\Delta t}{\Delta t} \right.$$

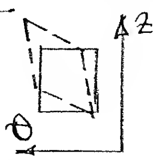
$$\left. \frac{- \tan^{-1} \dfrac{v_z|_{r+\Delta r} - v_z|_r}{\Delta r}\Delta t + \pi/2}{\Delta t} \right]$$

7.4 - CONTINUED

$$= \lim_{\substack{\Delta z \to 0 \\ \Delta r \to 0}} \left[\frac{v_r|_{z+\Delta z} - v_r|_z}{\Delta z} + \frac{v_z|_{r+\Delta r} - v_z|_r}{\Delta r} \right]$$

$$\therefore \quad \tau_{rz} = \tau_{zr} = \mu \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right]$$

IN THE θ-z PLANE —

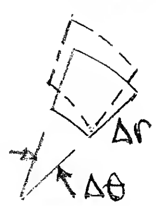


SAME PROCEDURE

$$\lim_{\substack{\Delta z \to 0 \\ \Delta \theta \to 0}} \left[\frac{v_\theta|_{z+\Delta z} - v_\theta|_z}{\Delta z} + \frac{1}{r} \frac{v_z|_{\theta+\Delta\theta} - v_z|_\theta}{\Delta \theta} \right]$$

$$\tau_{\theta z} = \tau_{z\theta} = \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]$$

& IN r-θ PLANE



$$\lim_{\substack{\Delta r \to 0 \\ \Delta \theta \to 0}} \left[\frac{v_r|_{\theta+\Delta\theta} - v_r|_\theta}{r\Delta\theta} + \frac{r\left(\frac{v_\theta}{r}\Big|_{r+\Delta r} - \frac{v_\theta}{r}\Big|_r\right)}{\Delta r} \right]$$

$$\tau_{r\theta} = \tau_{\theta r} = \mu \left[\frac{1}{r} \frac{\partial v_r}{\partial \theta} + r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) \right]$$

7.5 NITROGEN 175 K

$$\mu = 2.6693 \times 10^{-6} \frac{\sqrt{MT}}{\sigma^2 \Omega_\mu}$$

$T = 175$ K $\quad \sigma = 3.681$ Å

$M = 28 \quad \Omega_\mu = 1.1942$

$\epsilon_A/_K = 91.5$

$KT/_\epsilon = 1.91$

$$\mu \cong 11.55 \times 10^{-6} \text{ Pa.s.}$$

7.6 OXYGEN @ 350 K

EQN. 7.10 $\quad \mu = 2.6693 \times 10^{-6} \dfrac{\sqrt{MT}}{\sigma^2 \Omega_\mu}$

$$\frac{KT}{\epsilon} = \frac{T}{\epsilon/K} = \frac{350}{113} = 3.097$$

YIELDING $\Omega_\mu = 1.03$

$M = 32 \quad \sigma = 3.433$

$\mu = \underline{2.327 \times 10^{-5} \text{ Pa.S}}$

TABLE VALUE: $\mu = 2.318$ Pa.S.

7.7 FOR H_2O

$\mu|_{60°} = 0.76 \times 10^{-3}$ lbm/s.FT

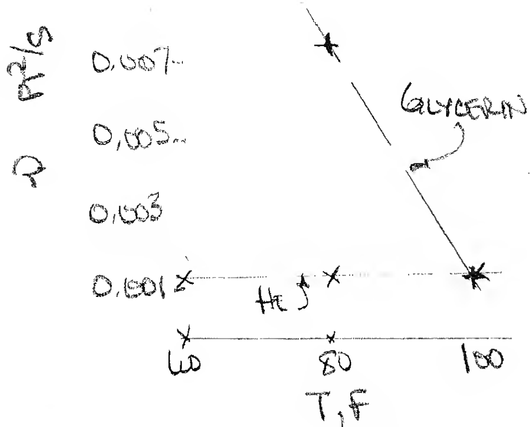
$\mu|_{120°} = 0.375 \times 10^{-3}$ "

$$\text{PERCENT CHANGE} = \frac{0.76 - 0.375}{0.76}$$

$= \underline{0.51}$ OR $\underline{51\%}$

7.8 PROPERTIES OF HELIUM, GLYCERIN FROM APPENDIX

T, F	ν, HE	ν, GLYCERIN
60	0.00125	0.0177
80	0.00132	0.00762
100	0.00141	0.00128



INTERSECTION IS VERY CLOSE TO 100°F

7.9 FOR H_2O $\dot{V} \sim 1/\mu$

@ 120F $\mu_w = 0.391 \times 10^{-3}$ Lbm/s.ft
@ 32F $= 1.2 \times 10^{-3}$ "

$$\frac{\dot{V}_{140}}{\dot{V}_{32}} = \frac{1.2\times10^{-3}}{0.391\times10^{-3}} = 3.07$$

PER CENT CHANGE

$$= \frac{1.2 - 0.391}{0.391}$$

$$= 3.07 - 1 = 2.07$$

OR $\underline{\underline{207\ \%}}$

7.10 FOR AIR:

@ 140°F $\mu = 1.34\times10^{-5}$ LBm/s.FT
32°F $\mu = 1.15\times10^{-5}$ "

FOR $\dot{V} \sim 1/\mu$

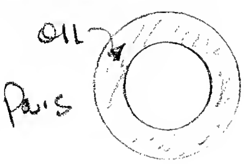
$$\frac{\dot{V}_{140}}{\dot{V}_{32}} = \frac{1.15}{1.34} = 0.852$$

$$\text{PER CENT CHANGE} = \frac{1.15 - 1.34}{1.34}$$

$$= 0.852 - 1 = -0.148$$

$$\underline{\underline{= -14.8\ \%}}$$

7.11



$\mu = 0.1$ Pa.s

$D_i = 3.175$ cm
$D_o = 3.183$ "

1ST LAW: $\frac{\delta Q}{dt} - \cancel{\frac{\delta W_s}{dt}} - \frac{\delta W_\mu}{dt} = 0$ {NO FLOW IN OR OUT}

$$\dot{Q} = \dot{W}_{VISCOUS}$$

$= \tau(A)v$ — AT MOVING BOUNDARY

$$\tau_i = \mu\frac{\partial v}{\partial r} \cong \mu\frac{r\omega}{t} \quad \{t = \text{GAP WIDTH}\}$$

$$\dot{Q} = \mu\frac{r\omega}{t}(\pi DL)(r\omega)$$

$$= \frac{\mu(r\omega)^2 \pi DL}{t}$$

$$\omega = 1700\left(\frac{2\pi}{60}\right) = 178 \text{ RAD/s}$$

$$\dot{Q} = \frac{(0.01)\left(\frac{0.03175}{2}\right)^2(178)^2(\pi)(0.03175)(0.028)}{4\times10^{-5}}$$

$$= \underline{\underline{5.58\ W}}$$

7.12 REFER TO PROB. 7.13

FOR $\omega_2 = 2\omega_1$

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2\omega_1)^2}{\omega_1^2} = 4$$

PER CENT INCREASE

$$= \frac{\dot{Q}_2 - \dot{Q}_1}{\dot{Q}_1} = 4 - 1 = 3$$

$$= \underline{\underline{300\%}}$$

7.13 SHIP 1 $\rightarrow v_1 = 4\,\text{m/s}$

SHIP 2 $\rightarrow v_2 = 3.1\,\text{m/s}$

CHOOSE CONTROL VOLUME ATTACHED TO SHIP 1

$$\sum F_x = \iint_{C.S} v_x \rho (\vec{v} \cdot \vec{n})\, dA$$

$$= \dot{m} v_{x1} - \dot{m} v_{x2}$$

RELATIVE TO MOVING SHIP

$$v_{x1} = 0 \quad v_{x2} = -0.9\,\text{m/s}$$

$$F_x = +\dot{m} v_{x2}$$

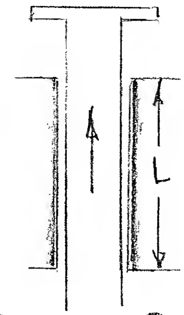
$$= 100\,\text{kg/s}\,(0.9\,\text{m/s})$$

$$= 90\,\text{N}$$

THIS IS FORCE APPLIED TO MAINTAIN STATED CONDITIONS.

FORCE EXERTED BY FLUID TRANSFER $= \underline{\underline{-90\,\text{N}}}$

7.14



$$t = \text{GAP} = \frac{D_{\text{OUTSIDE}} - D_{\text{INSIDE}}}{2}$$

$$= \frac{36.04 - 36.02}{2} = 0.01\,\text{cm}$$

$$F = \tau A = \tau \pi D L$$

$$\tau = \mu \frac{du}{dy} = \mu \frac{\Delta u}{\Delta y}$$ (ASSUMES LINEAR PROFILE)

$$\tau = \mu \frac{v}{t}$$

$$F = \frac{\mu V \pi D L}{t} = \frac{\rho \nu v \pi D L}{t}$$

$$= \frac{\left[0.85(1000)(3.7\times10^{-4})(0.15) \times \pi (0.3602)(3.14)\right]}{1\times10^{-4}}$$

$$= \underline{\underline{1676\,\text{N}}}$$

7.15 REFER TO CONDITIONS OF PROB 7.14

LOAD ON RAM $= 680\,\text{kg}$, $L = 2.44\,\text{m}$

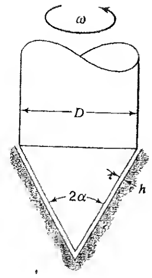
$$F = mg = \frac{\rho \nu v \pi D L}{t}$$

$$v = \frac{mgt}{\rho \nu \pi D L}$$

$$= \frac{680(9.81)(1\times10^{-4})}{0.85(1000)(3.7\times10^{-4})\pi(0.3602)(2.44)}$$

$$= \underline{\underline{0.768\,\text{m/s}}}$$

7.16



$M = \int r\,dF$

$dF = \tau\,dA$

dA IS ON THE CONICAL SURFACE

$= 2\pi r\,dL$

{ dL IS ALONG SLANTED SURF }

$dL = dr/\sin\alpha$

SO: $dF = \tau\,dA$

$$= \mu \frac{r\omega}{h}\, 2\pi r \frac{dr}{\sin\alpha}$$

$$\int_0^M dM = r\,dF$$

$$= \frac{2\pi\mu\omega}{h\sin\alpha}\int_0^{D/2} r^3\,dr$$

$$M = \frac{\pi\mu\omega D^4}{32 h \sin\alpha}$$

7.17 $v_x = v_{MAX}\left[1-\left(\frac{r}{R}\right)^2\right]$

$$= 2v_{AVG}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$$\tau = \mu \left.\frac{dv}{dr}\right|_{r=R}$$

$$\frac{dv}{dr} = 2v_{AVG}\left[-\frac{2r}{R^2}\right]$$

7.17 - CONTINUED

AT $r=R$ $\quad \frac{dv}{dr} = -4\frac{v_{AVG}}{R}$

$$\tau = -\frac{4\mu v_{AVG}}{R}$$

$\mu_w = 0.76\times10^{-3}$ LBm/s·FT @ 60°F

$$\tau = -\frac{4(0.76\times10^{-3})(2)}{(0.05/12)(32.2)}$$

$$= -0.0453 \text{ LBF/FT}^2$$

7.17 FOR CONDITIONS OF PROB 7.16

$$\tau = -\frac{4\mu v_{AVG}}{R}$$

$$F = \tau A = \tau \pi D L$$

$$= -\frac{4\mu v_{AVG}}{R}(\pi D L)$$

$$= (-0.0453)(\pi)\left(\frac{0.1}{12}\right)(1)$$

$$= \underline{0.00119 \text{ LBF}}$$

$$\Delta P = \frac{F}{\pi D^2/4}$$

$$= \frac{0.00119 \text{ LBF}}{\pi(0.1/12)^2/4}$$

$$= \underline{21.75 \text{ PSF}}$$

7.19 SHEAR WORK RATE $= \tau v$

$$\tau v = \mu v \frac{dv}{dr}$$

FOR PARABOLIC PROFILE -

$$v = v_{MAX}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$\tau v = \mu v_{MAX}^2\left[1 - \left(\frac{r}{R}\right)^2\right]\left[-\frac{2r}{R^2}\right]$$

$$= \mu v_{MAX}^2\left[-\frac{2r}{R^2} + 2\frac{r^3}{R^4}\right]$$

$$\frac{d}{dr}(\tau v) = \mu v_{MAX}^2\left[-\frac{2}{R^2} + 6\frac{r^2}{R^4}\right]$$

FOR $\frac{d}{dr}(\tau v) = 0$

$$6\frac{r^2}{R^4} = \frac{2}{R^2}$$

$$\frac{r}{R} = \frac{1}{\sqrt{3}}$$

CHAPTER 8

8.1 HAGEN – POISEUILLE EQN.

$$\frac{dP}{dx} = \frac{32\mu V_{AVG}}{D^2}$$

$$= \frac{32\mu\dot{V}}{D^2 A} = \frac{32\mu\ \dot{V}}{\pi/4\ \ D^4}$$

FOR $D = D_0$ $\quad \dot{V}_0 = \left[\left(-\frac{dP}{dx}\right)\frac{\pi/4}{32\mu}\right] D_0^4$

FOR $D_1 = 2D_0$ $\quad \dot{V}_1 = [\ \ ](2D_0)^4$

$$\Rightarrow \dot{V}_1 = 16\,\dot{V}_0$$

PER CENT CHANGE

$$= \frac{\dot{V}_1 - \dot{V}_0}{\dot{V}_0} = \frac{\dot{V}_1}{\dot{V}_0} - 1 = 15$$

$$= \underline{\underline{1500\ \%}}$$

8.2 FOR SINGLE PIPE:

$$\Delta P_0 = \left[\frac{32\mu}{D^2 A}\right]\dot{V}_0\,(40)$$

FOR SINGLE – PARALLEL COMBINATION

$\Delta P_1 = [\ \ ]\ \dot{V}_1\ (22)$ – {SINGLE BRANCH}

$\Delta P_2 = [\ \ ]\ \dot{V}_2\ (18)$ {PARALLEL BRANCH}

$$\Delta P_1 + \Delta P_2 = \Delta P_0 = 3.45 \times 10^6\ Pa$$

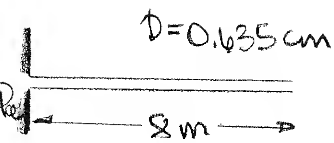
$$\dot{V}_1 = 2\dot{V}_2$$

CASE 2: $\Delta P_1 + \Delta P_2 = [\ \ ](22+9)\ \dot{V}_1$

$$\dot{V}_1 = \frac{40}{31}\dot{V}_0 = \frac{40}{31}(4000)$$

$$= \underline{\underline{5161\ BBL/DAY}}$$

8.3



FOR 1 — RESERVOIR
2 — PIPE ENTRANCE
3 — " EXIT

BETWEEN 1 & 2: $\frac{P_1 - P_2}{\rho} + \frac{\cancel{v_1^2} - v_2^2}{2} + g\cancel{(y_1 - y_2)} = 0$

$$\frac{P_1}{\rho} = \frac{P_2}{\rho} + \frac{v^2}{2}$$

BETWEEN 2 & 3:

$$\frac{P_2 - P_3}{\rho} + \cancel{\frac{v_2^2 - v_3^2}{2}} + g\cancel{(y_2 - y_3)} - \Delta u = 0$$

$$\frac{P_2}{\rho} = \frac{P_{ATM}}{\rho} + \Delta u$$

FOR INVISCID FLOW – $\Delta u = 0$

FOR LAMINAR, VISCOUS FLOW

$$\Delta u = \frac{\Delta P}{\rho}\bigg|_{FRICTION} = \frac{32\mu}{\rho D^2}v$$

INVISCID CASE:

$$\frac{v^2}{2} = \frac{P_1 - P_{ATM}}{\rho} = \frac{P_{1G}}{\rho}$$

ASSUMING FLUID IS HYDRAULIC FLUID @ 60 F – 15.9 K

$\rho = 849\ kg/m^3$ $\quad \mu = 0.0165\ Pa.s$

$$v = \left[\frac{2(207\,000)}{849}\right]^{1/2} = 22.08\ m/s$$

$$\dot{V} = Av = \frac{\pi}{4}(0.00635^2)(22.08)$$

$$\underline{\underline{\cong 7 \times 10^{-4}\ m^3/s}}$$

8.3 - CONTINUED

VISCOUS CASE:

$$\frac{P_1 - P_{ATM}}{\rho} = \frac{V^2}{2} + \Delta u$$

$$\frac{P_{1G}}{\rho} = \frac{V^2}{2} + \frac{32\mu V}{\rho D^2}$$

$$V^2 + \frac{64\mu}{\rho D^2} V - 2\frac{P_{1G}}{\rho} = 0$$

$$\frac{64\mu}{\rho D^2} = \frac{64(0.0165)}{849(0.00635)^2}$$

$$= 30.85 \ m^2/s^2$$

$$2\frac{P_{1G}}{\rho} = 481.6 \ \text{"}$$

$$V^2 + 30.85V - 481.6 = 0$$

$$V = \frac{-30.85 \pm \left[(30.85)^2 + 4(481.6)\right]^{1/2}}{2}$$

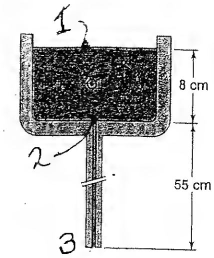
$$= 11.51 \ m/s$$

$$\dot{V} = \frac{\pi}{4}(0.00635)^2(11.51)$$

$$= \underline{\underline{3.645 \times 10^{-4} \ m/s}}$$

$$\frac{\dot{V}_{INVISCID}}{\dot{V}_{VISCOUS}} = \frac{7}{3.645} = \underline{1.92}$$

8.4



FROM 1 TO 2 (BERNOULLI)

$$\frac{P_2 - P_{ATM}}{\rho} + \frac{V_2^2 - \cancel{V_1^2}^{\,0}}{2} + g(y_2 - y_1) = 0$$

$$\frac{V_2^2}{2} = \frac{P_{ATM} - P_2}{\rho} + g(y_1 - y_2)$$

FROM 2 TO 3

$$\frac{P_2 - P_{ATM}}{\rho} + \frac{\cancel{V_2^2 - V_3^2}^{\,0}}{2} + g(y_2 - y_3) + \Delta u = 0$$

$$\Delta u = \frac{32\mu V L}{\rho D^2} = \frac{32 V L}{D^2}\nu$$

COMBINING EXPRESSIONS:

$$\frac{32VL}{D^2}\nu = g(y_1 - y_3) - \frac{V^2}{2}$$

$$V = \frac{4\dot{V}}{\pi D^2}$$

$$\nu = g\Delta y \frac{\pi D^4}{128 L \dot{V}} - \frac{\dot{V}}{16\pi L}$$

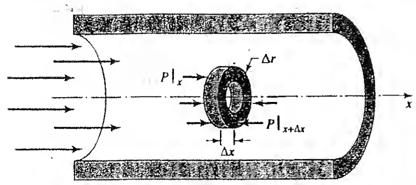
$$\frac{9.81(0.63)\pi(0.0018)^4}{128(0.55)(0.273\times10^{-6})} = 1.0605 \times 10^{-5}$$

$$\frac{0.273\times10^{-6}}{16(\pi)(0.55)} = 0.000987 \times 10^{-5}$$

$$\nu = (1.0605 - 0.001)\times 10^{-5}$$

$$= \underline{\underline{1.0595 \times 10^{-5} \ m^2/s}}$$

8.5



USING THE SAME DEVELOPMENT AS IN SECTION 8.1:

$$\frac{d}{dr}(r\tau) = r\frac{dp}{dx}$$

FOR AN ELEMENT OF LENGTH, L

$$\frac{d}{dr}(r\tau)\int_0^L dx = r\int_{P_1}^{P_2} dp$$

WHICH BECOMES

$$\frac{d}{dr}(r\tau) = r\frac{\Delta P}{L}$$

INTEGRATING:

$$r\tau = \frac{\Delta P}{L}\frac{r^2}{2} + C_1$$

$$\tau = \frac{\Delta P}{L}\frac{r}{2} + C_1/r$$

FOR LAMINAR FLOW, NEWTONIAN

$$\tau = \mu\frac{dv}{dr}$$

SO $$\mu\frac{dv}{dr} = \frac{\Delta P}{L}\frac{r}{2} + \frac{C_1}{r}$$

$$dv = \frac{\Delta P}{2\mu L}r\,dr + \frac{C_1}{\mu}\frac{dr}{r}$$

INTEGRATING:

$$v = \frac{\Delta P}{4\mu L}r^2 + \frac{C_1}{\mu}\ln r + C_2$$

BOUNDARY CONDITIONS:

$v(r=R)=0$

$v(r=kR)=0 \qquad k<1$

8.5 - CONTINUED -

CONSIDERABLE ALGEBRA YIELDS

$$C_1 = -\frac{\Delta P}{4L}\frac{R^2(1-k^2)}{\ln 1/k}$$

$$C_2 = -\frac{\Delta P}{4\mu L}R^2\left[1-(1-k^2)\frac{\ln R}{\ln 1/k}\right]$$

& WITH SUBSTITUTION & SIMPLIFICATION:

$$v = -\frac{\Delta P R^2}{4\mu L}\left[1-\frac{r^2}{R^2}-\frac{1-k^2}{\ln 1/k}\ln\frac{R}{r}\right]$$

8.6 THIS IS SAME CONFIGURATION AS SHOWN IN PROB 8.5

$$\therefore \quad \frac{d}{dr}(r\tau) - \frac{dp}{dx}r = 0$$

INTEGRATING: $\tau_{rx} - \frac{dp}{dx}\frac{r}{2} = \frac{C_1}{r}$

& FOR LAMINAR FLOW, NEWTONIAN FLUID:

$$\tau_{rx} = \mu\frac{dv}{dr}$$

$$\frac{dv}{dr} - \frac{r}{2\mu}\frac{dp}{dx} = \frac{C_1}{\mu r}$$

INTEGRATING:

$$v_x - \frac{1}{4\mu}\frac{dp}{dx}r^2 = \frac{C_1}{\mu}\ln r + c^2$$

BOUNDARY CONDITIONS:

$v(r=D/2) = 0$

$v(r=d/2) = V$

MORE ALGEBRA:

$$C_1 = -\frac{\mu}{\ln D/d}\left[V + \frac{1}{16\mu}\frac{dp}{dx}(D^2-d^2)\right]$$

8.6 – CONTINUED

$$C_2 = -\frac{1}{4\mu}\frac{dp}{dx}\frac{D^2}{4} - \frac{C_1}{\mu}\ln\frac{D}{2}$$

DRAG FORCE PER UNIT LENGTH:

$$F = \tau A = \tau(\pi d)(1)$$

$$= \mu \left.\frac{dv}{dr}\right|_{r=d/2}(\pi d)$$

GIVING:

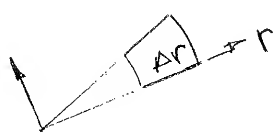
$$F = \pi d\mu\left[\frac{c_1}{\mu r} + \frac{r}{2\mu}\frac{dp}{dx}\right]_{r=\frac{d}{2}}$$

$$= \pi d\mu\left[\frac{2c_1}{\mu d} + \frac{d}{4\mu}\frac{dp}{dx}\right]$$

FOR THE CASE WITH $\frac{dp}{dx} = 0$

$$F = -\frac{2\pi\mu V}{\ln D/d}$$

8.7



IN θ – DIRECTION:

$$\sum F_\theta = -r\Delta\theta\Delta z\,\tau_{r\theta}\Big|_r + r\Delta\theta\Delta z\,\tau_{r\theta}\Big|_{r+\Delta r}$$

$$+ \underbrace{\Delta r\Delta z\,\tau_{\theta r}\Delta\theta}_{\substack{\theta \text{ COMPONENT OF} \\ \text{FORCE ON } (+\theta) \text{ FACE}}}$$

DIVIDE BY $r\Delta r\Delta\theta\Delta z$ & TAKE LIMIT AS $\Delta r \to 0$:

$$\frac{d}{dr}(r\tau_{r\theta}) + \tau_{r\theta} = 0$$

$$r\frac{d\tau}{dr} + 2\tau = 0$$

8.7 – CONTINUED –

$$\frac{d\tau}{\tau} + 2\frac{dr}{r} = 0$$

$$\ln\tau + 2\ln r = \ln(\text{CONSTANT})$$

$$r^2\tau = \text{CONSTANT}$$

$$\tau = \mu r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)$$

$$r^2\tau = \mu r^3\frac{d}{dr}\left(\frac{v_\theta}{r}\right) = \text{CONSTANT } (c)$$

$$d\left(\frac{v_\theta}{r}\right) = \frac{c}{\mu}\frac{dr}{r^3}$$

$$\frac{v_\theta}{r} = c_1\left(-\frac{1}{r^2}\right) + c_2$$

$$v_\theta = -\frac{c_1}{r} + rc_2$$

BOUNDARY CONDITIONS:

$v_\theta(R) = 0 \quad - \quad 0 = \frac{c_1}{R} + Rc_2$

$v_\theta(kR) = V \quad - \quad V = -\frac{c_1}{kR} + kRc_2$

ALGEBRA

$$v_\theta = \frac{VR}{k - 1/k}\left(\frac{r}{R^2} - \frac{1}{r}\right)$$

IF PROFILE IS LINEAR:

$$v_\theta = ar + b$$

$$v_\theta = \frac{V}{k-1}\left(\frac{r}{R} - 1\right)$$

8.7 CONTINUED

$$\text{PERCENT ERROR} = \frac{\Delta v}{v}$$

$$\Delta v = v_{ACTUAL} - v_{LINEAR}$$

$$= \frac{VRk}{K^2-1}\left(\frac{r}{R^2} - \frac{1}{r}\right) - \frac{V}{R(K-1)}(r-R)$$

$$\frac{d}{dr}\Delta v = \frac{VRK}{K^2-1}\left(\frac{1}{R^2} + \frac{1}{r^2}\right) - \frac{V}{R(K-1)}$$

$$= \frac{V}{K-1}\left[\frac{RK}{K+1}\left(\frac{1}{R^2} + \frac{1}{r^2}\right) - \frac{1}{R}\right]$$

$$= 0$$

ΔV_{MAX} OCCURS AT $\frac{r}{R} = \sqrt{K}$

$$\left.\frac{\Delta V}{V}\right|_{MAX} = 1 - \frac{(\sqrt{K}-1)(K+1)\sqrt{K}}{K(K-1)}$$

$$= 0.01$$

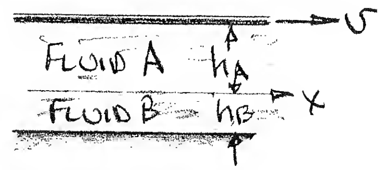
RESULTING IN

$$\frac{(\sqrt{K}-1)(K+1)}{\sqrt{K}\,(K-1)} = 0.99$$

$$\Downarrow$$

$$\underline{\underline{K = 0.96}}$$

8.8 FOR FLOW BETWEEN 2 HORIZONTAL PLATES —



GOVERNING D.E. —

$$\frac{d}{dy}(\tau_{yx}) - \frac{dp}{dy} = 0$$

LAMINAR, STEADY NEWTONIAN

$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

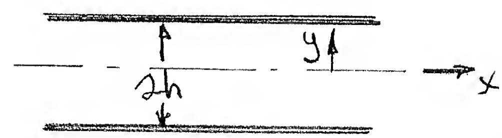
B.C. — AT INTERFACE $(y=0)$

(1) $v_{xA} = v_{xB}$

(2) $\tau_{yxA} = \tau_{yxB}$

(3) $v_x(-h_B) = v_x(h_A) = 0$

8.9



FULLY DEVELOPED, STEADY, LAMINAR FLOW; NEWTONIAN FLUID —

$$\frac{d}{dy}\tau_{yx} - \frac{dp}{dx} = 0$$

$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

B.C. $v_x = 0$ FOR $y = \pm h$

$$\mu \frac{d^2 v}{dy^2} = \frac{dp}{dx}$$

$$\frac{dv}{dy} = \frac{1}{\mu}\frac{dp}{dx}\,y + c_1$$

$$v = \frac{1}{2\mu}\frac{dp}{dx}\,y^2 + c_1 y + c_2$$

8.9 - CONTINUED

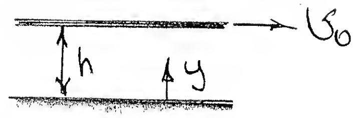
APPLYING BOUNDARY CONDITIONS

$$C_1 = 0$$

$$C_2 = -\frac{1}{2\mu}\frac{dP}{dx}h^2$$

GIVING:

$$\underline{\underline{v_x = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - h^2\right)}}$$

8.10 

GOVERNING D.E. IS

$$\frac{d}{dy}\tau_{yx} - \frac{dP}{dx} = 0$$

INTEGRATING: $\tau_{yx} - \frac{dP}{dx}y = C_1$

LAMINAR FLOW, NEWTONIAN FLUID:

$$\tau_{yx} = \mu\frac{dv_x}{dy}$$

$$\mu\frac{dv_x}{dy} - \frac{dP}{dx}y = C_1$$

FOR $\tau_{yx}(0) = 0$ $C_1 = 0$

$$\frac{dv_x}{dy} - \frac{1}{\mu}\frac{dP}{dx}y = 0$$

$$v_x - \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} = C_2$$

$$v_x @ y = h = v_0$$

$$C_2 = v_0 - \frac{1}{\mu}\frac{dP}{dx}\frac{h^2}{2}$$

8.10 - CONTINUED -

ALSO - $v_x @ y = 0 = 0$ $\therefore C_2 = 0$

GIVING $\underline{\underline{\frac{dP}{dx} = \frac{2\mu V_0}{h^2}}}$

8.11 FOR HORIZONTAL PIPE FLOW:

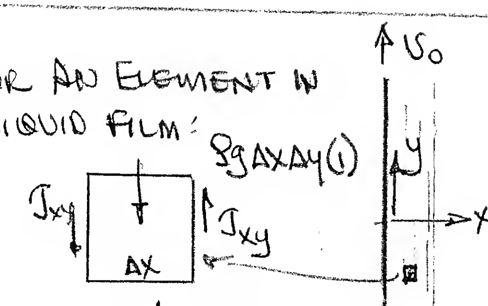
DEVELOPMENT IN SECTION 8.1 RESULTS IN

$$v_x = \left(\frac{dP}{dx}\right)\frac{r^2}{4\mu} + C_2$$

FOR $\mu = 0$ $\frac{dP}{dx}\frac{r^2}{4}$ MUST $= 0$ FOR ALL r

$\therefore$ $\underline{\underline{v_x = \text{CONSTANT} = V}}$

8.12 FOR AN ELEMENT IN LIQUID FILM:



$$\tau_{xy}\Delta y\big|_{x+\Delta x} - \tau_{xy}\Delta y\big|_x - \rho g\Delta x\Delta y = 0$$

$$\frac{\tau_{xy}\big|_{x+\Delta x} - \tau_{xy}\big|_x}{\Delta x} - \rho g = 0$$

IN LIMIT AS $\Delta x \to 0$

$$\frac{d}{dx}\tau_{xy} - \rho g = 0$$

8.12- CONTINUED

$$\tau_{xy} = \mu \frac{dv_y}{dx}$$

$$\frac{dv_y^2}{dx^2} - \frac{\rho g}{\mu} = 0$$

$$\frac{dv}{dx} - \frac{\rho g}{\mu} x = C_1$$

$$v - \frac{\rho g}{2\mu} x^2 = C_1 x + C_2$$

BOUNDARY CONDITIONS:

$$v(0) = v_0 \qquad \tau(h) = 0$$

$$C_1 = -\frac{\rho g}{\mu} h \qquad C_2 = v_0$$

GIVING

$$v = v_0 - \frac{\rho g}{2\mu}\left(2hx - x^2\right)$$

$$= v_0 - \frac{\rho g h^2}{2\mu}\left[2\frac{x}{h} - \left(\frac{x}{h}\right)^2\right]$$

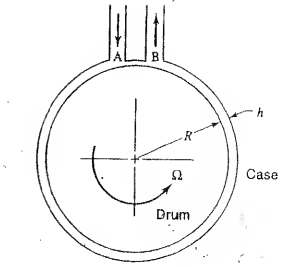
$$\dot{V} = \int_0^h v \, dy$$

$$= \int_0^h v_0 - \frac{\rho g h^2}{2\mu}\left[2\frac{x}{h} - \left(\frac{x}{h}\right)^2\right] dy$$

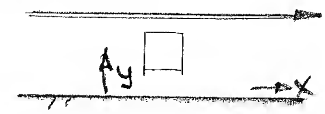
$$= v_0 h - \frac{\rho g}{2\mu}\left(h^3 - h^3/3\right)$$

$$= v_0 h - \frac{\rho g h^3}{3\mu}$$

8.13



TREAT FLUID LAYER AS A THIN LINEAR LAYER:



IN THE USUAL WAY:

$$\frac{d\tau}{dy} - \frac{dp}{dx} = 0$$

TREAT $\frac{dp}{dx}$ CONSTANT ~ $\frac{dp}{dx} = \frac{\Delta p}{L}$

& $\tau = \mu \frac{dv_x}{dy}$

GIVING $v = \frac{\Delta p}{L}\frac{y^2}{2\mu} + C_1 y + C_2$

BOUNDARY CONDITIONS:

$$v(0) = R\Omega$$

$$v(h) = 0$$

GIVING $R\Omega = C_2$

$$0 = \frac{\Delta p}{L}\frac{h^2}{2\mu} + C_1 h + R\Omega$$

$$C_1 = -\frac{\Delta p}{L}\frac{h}{2\mu} - \frac{R\Omega}{h}$$

$$\therefore \quad v = R\Omega\left(1 - \frac{y}{h}\right) - \frac{\Delta p h^2}{2\mu L}\left[\left(\frac{y}{h}\right) - \left(\frac{y}{h}\right)^2\right]$$

FLOW RATE $= \int_0^h v \, dy$

8.13 – CONTINUED

$$\dot{V} = \int_0^h \{\text{EXPRESSION FOR } v\}\, dy$$

$$= \frac{R\Omega h}{2} - \frac{\Delta p\, h^3}{12\mu L}$$

GIVING: $$\Delta p = \frac{12\mu L}{h^3}\left[\frac{R\Omega h}{2} - \dot{V}\right]$$

$$\text{EFFICIENCY} = \frac{\text{POWER OUT}}{\text{POWER IN}}$$

$$\equiv \frac{\dot{m}\,\Delta p/\rho}{R\Omega\tilde{\tau}}$$

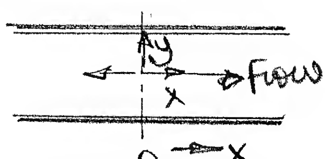
$\tilde{\tau}$ EVALUATED AT $R\,(y=0)$

$$\tilde{\tau} = \mu\frac{R\Omega}{h} + \frac{\Delta p}{L}\frac{h}{2}$$

AFTER DOING THE ALGEBRA:

$$\eta = \frac{12\dot{V}}{R\Omega h}\;\frac{R\Omega h/2 - \dot{V}}{4R\Omega h - 6\dot{V}}$$

8.14



FLUID ENTERS AT $x=0$ & FLOWS EQUALLY IN $+x$ & $-x$ DIRECTIONS, EXITING AT $x=L/2$ WHERE $p=p_{ATM}$.

WORKING WITH THE R.H. FLOW (IN $+x$ DIRECTION)

THE APPLICABLE D.E. IS

$$\frac{d\tau}{dy} = \frac{dp}{dx}$$

8.14 – CONTINUED –

& AS USUAL – $\tau = \mu\dfrac{dv}{dy}$

GIVING $$\frac{d^2v}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

INTEGRATING: $$\frac{dv}{dy} = \frac{1}{\mu}\frac{dp}{dx}y + C_1$$

BOUNDARY COND: $\dfrac{dv}{dy}(0) = 0 \;\therefore C_1 = 0$

AGAIN $$v = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_2$$

BOUNDARY COND: $v\left(\frac{b}{2}\right) = 0$

SO $$C_2 = \left(-\frac{dp}{dx}\right)\frac{b^2}{8\mu}$$

VELOCITY EXPRESSION IS:

$$v = \frac{1}{2\mu}\left(-\frac{dp}{dx}\right)\left(\frac{b^2}{4} - y^2\right)$$

$$\dot{V} = 2\int_0^{b/2} v\, dy$$

$$= \frac{1}{\mu}\left(-\frac{dp}{dx}\right)\int_0^{b/2}\left(\frac{b^2}{4} - y^2\right) dy$$

$$= \frac{1}{\mu}\left(-\frac{dp}{dx}\right)\frac{b^3}{12}$$

SO THE EXPRESSION FOR $-\dfrac{dp}{dx}$ IS:

$$-\frac{dp}{dx} = \frac{12\mu\dot{V}}{b^3}$$

$$\&\; \int_{p_0}^{p_{ATM}} dp = \frac{12\mu\dot{V}}{b^3}\int_0^{L/2} dx$$

$$p_0 - p_{ATM} = \frac{6\mu\dot{V}L}{b^3}$$

& FOR THE PLATE OF TOTAL LENGTH, L,

$$F_y = (p_0 - p_{ATM})2L = \frac{12\mu\dot{V}L^2}{b^3}$$

8.15 LIQUID FLOWING DOWN THE OUTSIDE OF A CYLINDER:

GOVERNING D.E. IS

$$\frac{1}{r}\frac{d}{dr}(r\tau)+\rho g=0$$

& , AS USUAL $\tau=\mu\frac{dv}{dr}$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{dv}{dr}\right)+\rho g=0$$

$$r\frac{dv}{dr}+\frac{\rho g}{\mu}\frac{r^2}{2}=C_1$$

B.C. $\frac{dv}{dr}=0$ @ $r=R+h$

$$\frac{r\,dv}{dr}=\frac{\rho g}{2\mu}\left[(R+h)^2-r^2\right]$$

AND AGAIN:

$$v=\frac{\rho g}{2\mu}\left[(R+h)^2\ln r-\frac{r^2}{2}\right]+C_2$$

B.C. $v(R)=0$

GIVING:

$$v=\frac{\rho g}{2\mu}(R+h)^2\ln\frac{r}{R}+\frac{\rho g R^2}{4\mu}\left(1-\frac{r^2}{R^2}\right)$$

8.16 FOR RESULT OF PROB. 8.15

v_{MAX} OCCURS WHERE $\frac{dv}{dr}=0$

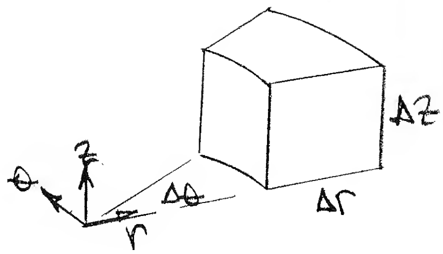
WHICH IS AT $r=R+h$

$\therefore$

$$v_{MAX}=\frac{\rho g R^2}{4\mu}\left[2\left(1+\frac{h}{R}\right)\ln\left(1+\frac{h}{R}\right)-\frac{h^2}{R^2}-\frac{2h}{R}\right]$$

CHAPTER 9

9.1



$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})dA + \frac{\partial}{\partial t}\iiint_{C.V.} \rho\, dV = 0$$

$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})dA = \rho v_r \Delta z\, r\Delta\theta\big|_{r+\Delta r}$$
$$-\rho v_r \Delta z\, r\Delta\theta\big|_r + \rho v_\theta \Delta r\Delta z\big|_{\theta+\Delta\theta}$$
$$-\rho v_\theta \Delta r \Delta z\big|_\theta + \rho v_z\, r\Delta\theta\Delta r\big|_{z+\Delta z}$$
$$-\rho v_z\, r\Delta\theta\Delta z\big|_z$$

$$\frac{\partial}{\partial t}\iiint_{C.V.} \rho\, dV = \frac{\partial}{\partial t}\rho\,\Delta r\, r\Delta\theta\Delta z$$

SUBSTITUTING INTO C.V. RELATIONSHIP & EVALUATING IN LIMIT AS $\left.\begin{matrix}\Delta r\\ \Delta\theta\\ \Delta z\end{matrix}\right\}\to 0$

$$\underline{\underline{\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_z}{\partial z} = 0}}$$

9.2 $\vec{v} = v_x\vec{e}_x + v_y\vec{e}_y + v_z\vec{e}_z$

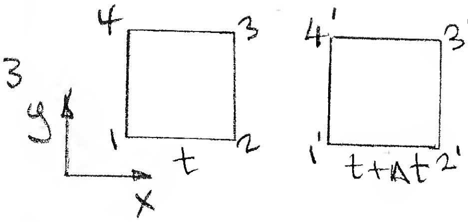
$$\nabla = \frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z$$

$$\vec{v}\cdot\vec{\nabla} = v_x\frac{\partial}{\partial x}(\vec{e}_x\cdot\vec{e}_x) + v_y\frac{\partial}{\partial y}(\vec{e}_y\cdot\vec{e}_y)$$
$$+ v_z\frac{\partial}{\partial z}(\vec{e}_z\cdot\vec{e}_z)$$

NOTE: $\vec{e}_i\cdot\vec{e}_j = 1$ FOR $j=i$
$= 0$ FOR $j\neq i$

$$\therefore\ \underline{\underline{\vec{v}\cdot\nabla = v_x\frac{\partial}{\partial x} + v_y\frac{\partial}{\partial y} + v_z\frac{\partial}{\partial z}}}$$

9.3



FOR 2-DIMENSIONAL FLOW:

VOLUME CHANGE $= (\overline{1'2'})(\overline{3'2'}) - (\overline{12})(\overline{32})$

$\overline{12} = \Delta x$; $\overline{32} = \Delta y$

$\overline{1'2'} = \Delta x + \left[v_x(x+\Delta x, y) - v_x(x,y)\right]\Delta t$

$\overline{3'2'} = \Delta y + \left[v_y(x+\Delta x, y+\Delta y) - v_y(x+\Delta x, y)\right]\Delta t$

$(\overline{12})(\overline{32}) = \Delta x\Delta y$

$(\overline{1'2'})(\overline{3'2'}) = \Delta x\Delta y + \left[v_y(x+\Delta x, y+\Delta y) - v_y(x+\Delta x, y)\right]\Delta x\Delta t$
$+ \left[v_x(x+\Delta x, y) - v_x(x,y)\right]\Delta y\Delta t$
$+ [\ \ ]\Delta t^2$

DIVIDING BY $\Delta x\Delta y\Delta t$ & EVALUATING IN LIMIT AS $\Delta x, \Delta y, \Delta t \to 0$

VOLUME CHANGE $= \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_x}{\partial y}$
$= \nabla\cdot\vec{v}$

BUT, FROM CONTINUITY $-\nabla\cdot\vec{v} = 0$

9.4 $\vec{v} = v_r\vec{e}_r + v_\theta\vec{e}_\theta$

$$d\vec{v} = \frac{\partial\vec{v}}{\partial r}dr + \frac{\partial\vec{v}}{\partial\theta}d\theta + \frac{\partial\vec{v}}{\partial t}dt$$

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial r}\frac{dr}{dt} + \frac{\partial\vec{v}}{\partial\theta}\frac{d\theta}{dt} + \frac{\partial\vec{v}}{\partial t}$$

9.4 CONTINUED -

$$\frac{\partial \vec{v}}{\partial r} = \frac{\partial v_r}{\partial r}\vec{e}_r + \frac{\partial \vec{v}}{\partial \theta}e_\theta + v_r\frac{\partial \vec{e}_r}{\partial r} + v_\theta\frac{\partial \vec{e}_\theta}{\partial r}$$

$$\frac{\partial \vec{v}}{\partial \theta} = \frac{\partial v_r}{\partial \theta}\vec{e}_r + \frac{\partial v_\theta}{\partial r}\vec{e}_\theta + v_r\frac{\partial \vec{e}_r}{\partial \theta} + v_\theta\frac{\partial \vec{e}_\theta}{\partial \theta}$$

$$\vec{e}_r = \vec{e}_x\cos\theta + \vec{e}_y\sin\theta$$
$$\vec{e}_\theta = -e_x\sin\theta + \vec{e}_y\cos\theta$$

$$\frac{\partial \vec{e}_r}{\partial r} = \frac{\partial \vec{e}_r}{\partial \theta}\frac{\partial \theta}{\partial r} = \vec{e}_\theta\frac{\partial \theta}{\partial r} = 0$$

$$\frac{\partial e_r}{\partial \theta} = -\vec{e}_x\sin\theta + \vec{e}_y\cos\theta = \vec{e}_\theta$$

IN SIMILAR FASHION

$$\frac{\partial e_\theta}{\partial r} = 0 \qquad \frac{\partial e_\theta}{\partial \theta} = -\vec{e}_r$$

GIVING:

$$\frac{\partial \vec{v}}{\partial r} = \frac{\partial v_r}{\partial r}\vec{e}_r + \frac{\partial v_\theta}{\partial \theta}\vec{e}_\theta$$

$$\frac{\partial \vec{v}}{\partial \theta} = \left(\frac{\partial v_r}{\partial \theta} - v_\theta\right)\vec{e}_r + \left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)\vec{e}_\theta$$

FOR $\frac{d\vec{v}}{dt}$ TO BE $\frac{D\vec{v}}{Dt}$ $\quad \frac{dr}{dt} = v_r$

$$\text{\&}\ \frac{d\theta}{dt} = \omega = \frac{v_\theta}{r}$$

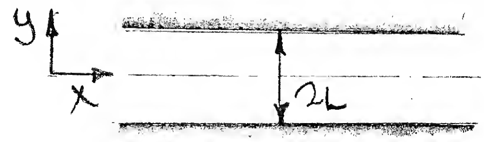
$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \left(v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r}\right)\vec{e}_r + \left(v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r}\right)\vec{e}_\theta$$

9.5 NAVIER-STOKES EQN - INCOMPRESSIBLE FORM:

$$\frac{D\vec{v}}{Dt} = \vec{g} - \frac{\nabla P}{\rho} + \nu\nabla^2\vec{v}$$

a) FOR $\vec{v}$ SMALL - ALL TERMS INVOLVING $\vec{v}$ $\left(\sim \frac{\partial \vec{v}}{\partial t}\ \text{\&}\ \nu\nabla^2\vec{v}\right)$ ARE SMALL RELATIVE TO OTHER TERMS.

b) FOR ν SMALL & $\vec{v}$ LARGE THE PRODUCT $\nu\vec{v}$ CANNOT BE CONSIDERED SMALL RELATIVE TO OTHER TERMS

9.6 

INCOMPRESSIBLE N.S. EQN. IN X DIRECTION

$$\cancel{\frac{\partial v_x}{\partial t}}^{0} + v_x\cancel{\frac{\partial v_x}{\partial x}}^{0} + \cancel{v_y}^{0}\frac{\partial v_x}{\partial y} = g_x - \frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\nabla^2 v_x$$

$$\Rightarrow \nabla^2 v_x = \frac{d^2 v_x}{dy^2} = \frac{1}{\mu}\frac{dP}{dx}$$

$$\frac{dv_x}{dy} = \frac{1}{\mu}\frac{dP}{dx}y + C_1$$

$$v_x = \frac{1}{\mu}\frac{dP}{dx}\frac{y^2}{2} + C_1 y + C_2$$

B.C. $\quad v_x = 0 \ @\ y = \pm L$

$$C_1 = 0 \qquad C_2 = -\frac{1}{2\mu}\frac{dP}{dx}L^2$$

$$v_x = \frac{1}{2\mu}\frac{dP}{dx}\left(y^2 - L^2\right)$$

9.7 $\vec{v} = \frac{\omega R^2}{r}\vec{e}_\theta$

$$\nabla\cdot\vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial\theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla\cdot v = \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\omega R^2}{r}\right)$$

$$= \frac{\omega R^2}{r}\frac{\partial}{\partial\theta}\left(\frac{1}{r}\right) = 0$$

AND CONTINUITY IS SATISFIED

9.8 $\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + v_y\frac{\partial\rho}{\partial y} = -v\frac{\partial\rho}{\partial y}$

$$= -v\frac{\partial}{\partial y}\rho_0 e^{-y/\beta} = \frac{\rho_0 v e^{-y/\beta}}{\beta}$$

AT $y = 100{,}000$ FT $\quad v = 20{,}000$ FT/S

$$\frac{D\rho}{Dt} = \frac{20{,}000}{22{,}000}\rho_0 e^{-4.545}$$

$$= 0.0096\,\rho_0 \ \mathrm{s}^{-1}$$

9.9 $\nabla P = \rho\left(\vec{g} - \frac{D\vec{v}}{Dt}\right)$

$$\vec{v} = 400\left[(y/L)^2\vec{e}_x + (x/L)^2\vec{e}_y\right]$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial\vec{v}}{\partial t} + v_x\frac{\partial\vec{v}}{\partial x} + v_y\frac{\partial\vec{v}}{\partial y}$$

$$= 400\left(\frac{y}{L}\right)^2 800\frac{x}{L^2}\vec{e}_y$$

$$+ 400\left(\frac{x}{L}\right)^2 800\frac{y}{L^2}\vec{e}_x$$

$$= \frac{32\times10^4}{L^4}\left[x^2 y\,\vec{e}_x + x y^2\,\vec{e}_y\right]$$

9.9 CONTINUED –

EVALUATED AT $(L, 2L)$ WE GET

$$\nabla P = -\frac{128\times10^4}{L}\vec{e}_x$$

$$-\left[2g + \frac{256\times10^4}{L}\right]\vec{e}_y \qquad \frac{\mathrm{LB_F/FT^2}}{\mathrm{FT}}$$

9.10 IN X-DIRECTION:

$$\rho\frac{Dv_x}{Dt} = \rho g_x - \frac{\partial P}{\partial x} - \frac{\partial}{\partial x}\left(\frac{2}{3}\mu\nabla\cdot\vec{v}\right)$$

$$+ \nabla\cdot\left(\mu\frac{\partial\vec{v}}{\partial x}\right) + \nabla\cdot(\mu\nabla x)$$

$$\rho\left[\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right]$$

$$= \rho g_x - \frac{\partial P}{\partial x} - \frac{\partial}{\partial x}\left[\frac{2}{3}\mu\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial y} + \frac{\partial v_x}{\partial z}\right)\right]$$

$$+ \frac{\partial}{\partial x}\left(\mu\frac{\partial v_x}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v_y}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v_z}{\partial x}\right)$$

$$+ \frac{\partial}{\partial x}\left(\mu\frac{\partial v_x}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v_x}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v_x}{\partial z}\right)$$

SIMILARLY IN y & z –

A TOTAL OF 45 TERMS!

9.11 FOR $\vec{v} = \vec{v}_0 + \vec{v}_r$

0 – OF COORDINATE ORIGIN

r – RELATIVE TO COORD. ORIGIN

$$\frac{D\vec{v}}{Dt} = \frac{\partial v_0}{\partial t} + \cancel{\frac{D v_r}{Dt}}^{\,0} = \vec{a}$$

∴ N.S. EQN REDUCES TO

$$\rho\vec{a} = \rho\vec{g} - \nabla P$$

& $\underline{\nabla P = \rho(\vec{g} - \vec{a})}$

9.12 GIVEN THAT

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial \theta}=0$$

a) FOR $v_\theta=0$ $\quad \frac{\partial}{\partial r}(r v_r)=0$

$\therefore$ $r v_r(\theta)=F(\theta)$

OR $v_r=F(\theta)/r$

b) FOR $v_r=0$ $\quad \frac{\partial v_\theta}{\partial \theta}=0$

$$v_\theta=f(r)$$

9.13 N.S. FOR INCOMP, LAM. FLOW

$$\frac{D\vec{v}}{Dt}=\vec{g}-\frac{\nabla P}{\rho}+\nu\nabla^2\vec{v}$$

FOR $\vec{g}$ NEGLIGIBLE

a) VECTOR PROPERTIES ~ $\vec{v}$ & ∇P ARE INDEPENDENT BY THEMSELVES BUT IN SAME RELATIONSHIP MUST LIE IN SAME PLANE.

b) IF VISCOUS FORCES ARE NEGLIGIBLE

$$\frac{D\vec{v}}{Dt}=-\frac{\nabla P}{\rho}$$

$\frac{D\vec{v}}{Dt}$ IS DETERMINED BY $-\frac{\nabla P}{\rho}$ & IS POSITIVE IN DIRECTION OF DECREASING PRESSURE.

c) IN SIMILAR FASHION, ANY FLUID — EITHER MOVING OR STATIC — WILL MOVE OR TEND TO MOVE IN DIRECTION OF DECREASING P.

9.14. FOR 1-D STEADY FLOW:

$$v_x=v_x(x) \qquad v_y=v_z=0$$

$$\rho v_x\frac{\partial v_x}{\partial x}=-\frac{dP}{dx}+\frac{d}{dx}\left[-\frac{2}{3}\left(\mu\frac{dv_x}{dx}\right)+\mu\frac{dv_x}{dx}\right]$$

$$\rho v_x\frac{\partial v_x}{\partial x}=-\frac{dP}{dx}+\frac{4}{3}\left(\mu\frac{dv_x}{dx}\right)$$

$$\frac{d}{dx}(\rho v_x)=0$$

9.15 CONTINUITY: $\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)=0$

MOMENTUM: $\rho\left(\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}\right)=-\frac{\partial P}{\partial x}$

9.16 TAKING z AS POSITIVE DOWN WITH $v_r=v_\theta=0$ & $v_z=f(r)$

EQN. E.6 YIELDS

z direction

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}+v_r\cancel{\frac{\partial v_z}{\partial r}}+\frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}}+v_z\cancel{\frac{\partial v_z}{\partial z}}\right)=-\cancel{\frac{\partial P}{\partial z}}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\cancel{\frac{\partial^2 v_z}{\partial \theta^2}}+\cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

& SINCE $g_z=-g$

$$\frac{g}{\nu}=\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)$$

PROCEED AS WAS DONE IN SOLNS TO PROBS 8.17 & 8.18

9.17 FOR INCOMPRESSIBLE, STEADY FLOW, WITH $v_\theta = v_z = 0$ EQN (E-4) HAS THE FORM

r direction

$$\rho\left(\cancel{\frac{\partial v_r}{\partial t}} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\cancel{\frac{\partial v_r}{\partial \theta}} - \cancel{\frac{v_\theta^2}{r}} + v_z\cancel{\frac{\partial v_r}{\partial z}}\right)$$
$$= -\frac{\partial P}{\partial r} + \rho g_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(\cancel{r v_r})\right) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_r}{\partial \theta^2}} - \frac{2}{r^2}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{\partial^2 v_r}{\partial z^2}}\right]$$

0 DUE TO CONTINUITY

& BECOMES

$$\frac{\partial}{\partial r}\left(P + \rho\frac{v_r^2}{2}\right) = \rho g_r$$

9.18 GOVERNING EQNS. ARE

r direction

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right)$$
$$= -\frac{\partial P}{\partial r} + \rho g_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

θ direction

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \rho g_\theta + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

z direction

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right)$$
$$= -\frac{\partial P}{\partial z} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right]$$

WHEN $v_\theta = f(r)$ & $v_r = v_z = 0$ THE ONLY NON-ZERO TERM ON THE LEFT-HAND SIDE OF ALL COMPONENT EQNS. IS $-v_\theta^2/r$

$$\therefore \frac{D\vec{v}}{Dt} = \frac{d\vec{v}}{dt} = -\frac{v_\theta^2}{r}\vec{e}_r$$

Q.E.D.

9.19 EQN (E-5) IS SIMPLIFIED FOR THIS CASE AS

θ direction

$$\rho\left(\frac{\partial v_\theta}{\partial t} + \cancel{v_r\frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_r v_\theta}{r}} + \cancel{v_z\frac{\partial v_\theta}{\partial z}}\right)$$
$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \rho g_\theta + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right].$$

& IN THE ABSENCE OF GRAVITY WE HAVE

$$\frac{\partial v_\theta}{\partial t} = \nu\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right]$$

9.20 FROM PROB. 9.19 & STEADY FLOW

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r v_\theta)\right] = 0$$

GIVING $\frac{1}{r}\frac{d}{dr}(r v_\theta) = C_1$

INTEGRATING AGAIN

$$r v_\theta = C_1 \ln r + C_2$$

B.C. $v_\theta(R_1) = R_1\Omega_1$

$v_\theta(R_2) = R_2\Omega_2$

$$v_\theta = \frac{1}{r}\left[R_1^2\Omega_1 + \frac{(R_2^2\Omega_2 - R_1^2\Omega_1)\ln r/R_1}{\ln R_2/R_1}\right]$$

CHAPTER 10

10.1 $$\nabla\times\vec{v} = \left(\vec{e}_r\frac{\partial}{\partial r}+\vec{e}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}\right)\times\left(v_r\vec{e}_r+v_\theta\vec{e}_\theta\right)$$

$$= \underbrace{\left(\vec{e}_r\times\vec{e}_r\right)}_{0}\frac{\partial v_r}{\partial r}+v_r\vec{e}_r\times\frac{\partial\vec{e}_r}{\partial r}$$
$$+\underbrace{\left(\vec{e}_r\times\vec{e}_\theta\right)}_{\vec{e}_z}\frac{\partial v_\theta}{\partial r}+v_\theta\vec{e}_r\times\frac{\partial\vec{e}_\theta}{\partial r}$$
$$+\underbrace{\left(\vec{e}_\theta\times\vec{e}_r\right)}_{-\vec{e}_z}\frac{1}{r}\frac{\partial v_r}{\partial\theta}+\frac{v_r}{r}\vec{e}_\theta\times\frac{\partial\vec{e}_r}{\partial\theta}$$
$$+\underbrace{\left(\vec{e}_\theta\times\vec{e}_\theta\right)}_{0}\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_\theta}{r}\vec{e}_\theta\times\frac{\partial\vec{e}_\theta}{\partial\theta}$$

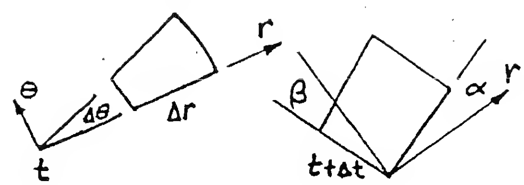
FOR REFERENCE - SEE PROB 9.4

$$\frac{\partial\vec{e}_r}{\partial r}=0,\ \frac{\partial\vec{e}_\theta}{\partial r}=0,\ \frac{\partial\vec{e}_r}{\partial\theta}=\vec{e}_\theta,\ \frac{\partial\vec{e}_\theta}{\partial\theta}=-\vec{e}_r$$

ALL REMAINING (NON-ZERO) TERMS GIVE:

$$\nabla\times\vec{v}=\left[\frac{\partial v_\theta}{\partial r}+\frac{1}{r}\left(v_\theta-\frac{\partial v_r}{\partial\theta}\right)\right]\vec{e}_z$$

10.2



$$\omega_z=\frac{d}{dt}\left(\frac{\alpha+\beta}{2}\right)$$

$$=\lim_{\Delta t\to 0}\frac{1}{2}\left[\frac{\tan^{-1}\dfrac{\left(rv_\theta|_{r+\Delta r}-rv_\theta|_r\right)\Delta t}{r\Delta r}}{\Delta t}+\frac{\tan^{-1}\dfrac{\left(v_r|_{\theta+\Delta\theta}-v_r|_\theta\right)\Delta t}{r\Delta\theta}}{\Delta t}\right]$$

10.2 CONTINUED -

IN THE LIMIT: {NOTE THAT $\tan z\to z$}

$$\omega_z=\lim_{\substack{\Delta x\\ \Delta y\\ \Delta z}\to 0}\left[\frac{rv_\theta|_{r+\Delta r}-rv_\theta|_r}{r\Delta r}-\frac{v_r|_{\theta+\Delta\theta}-v_r|_\theta}{r\Delta\theta}\right]$$

$$=\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)-\frac{1}{r}\frac{\partial v_r}{\partial\theta}$$

$$=\frac{\partial v_\theta}{\partial r}+\frac{1}{r}\left(v_\theta-\frac{\partial v_r}{\partial\theta}\right)$$ Q.E.D.

10.3 $$d\psi=-v_y dx+v_x dy$$
$$=\left(-v_\infty\sin\alpha\right)dx+\left(v_\infty\cos\alpha\right)dy$$
$$\psi=-v_\infty(\sin\alpha)x+v_\infty(\cos\alpha)y+\psi_0$$

10.4 $$\nabla\cdot\vec{v}=\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}=0$$

LET $$rv_r=\frac{\partial\psi(r,\theta)}{\partial\theta}$$

THEN $$\nabla\cdot\vec{v}=\frac{1}{r}\left[\frac{\partial}{\partial r}\frac{\partial\psi}{\partial\theta}+\frac{\partial v_\theta}{\partial\theta}\right]=0$$

$$\frac{\partial}{\partial\theta}\left(\frac{\partial\psi}{\partial r}+v_\theta\right)=0\quad\therefore\ v_\theta=-\frac{\partial\psi}{\partial r}$$

& $$v_r=\frac{1}{r}\frac{\partial\psi}{\partial\theta}\quad\therefore\ v_\theta=-\frac{\partial\psi}{\partial r}$$

Q.E.D.

10.5 $\phi = \frac{5}{3}x^3 - 5xy^2$

SINCE $\vec{v} = \nabla\phi$

CONTINUITY CAN BE EXPRESSED AS $\nabla \cdot \vec{v} = 0$ OR $\nabla^2\phi = 0$

USING $\nabla^2\phi = 0$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

$$10x - 10x = 0$$

$$v_x = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} = 5x^2$$

OR $\psi = 5x^2 y$

$$\sim \frac{\partial\phi}{\partial y} = v_y = -\frac{\partial\psi}{\partial x} \sim \text{CHECK}$$

10.6 IN THE CORE: EULER'S EQN.

$$\frac{D\vec{v}}{Dt} = \vec{g} - \frac{\nabla P}{\rho}$$

$$= -\frac{v^2}{r}\vec{e}_r = \frac{\nabla P}{\rho} = \frac{\partial P}{\partial r}\vec{e}_r$$

$$\frac{dP}{\rho} = -\frac{v^2}{r}dr$$

SINCE VELOCITY VARIATION IS LINEAR

$$v = v_{max}\, r/R$$

$$\int_{P_0}^{P_R} dP = -\frac{\rho v_{max}^2}{R^2}\int_0^R r\,dr$$

$$P_R - P_0 = \frac{\rho v_{max}^2}{2} \qquad (1)$$

OUTSIDE THE CENTRAL CORE - BERNOULLI EQN. APPLIES

10.6 - CONTINUED

$$\frac{P_\infty}{\rho} = \frac{P}{\rho} + \frac{v^2}{2}$$

v VARIES INVERSELY WITH r:

$$v = v_{max}\frac{R}{r}$$

AT $r = R$ $\quad P_\infty - P_r = \rho v_{max}^2/2 \qquad (2)$

ADDING (1) & (2)

$$P_\infty - P_0 = \rho v_{max}^2$$

$$v_{max} = \left[\frac{(38)(32.2)}{0.0766}\right]^{1/2} = \underline{126\ \text{ft/s}} \quad (a)$$

FOR $P = -10$ PSF

$$P_\infty - P = \rho v^2/2$$

$$v = \left[\frac{(10)(32.2)}{0.0766}\right]^{1/2} = 91.7\ \text{ft/s}$$

$$r = \frac{v_{max}}{v}R = \frac{126}{91.7}(100) = 138\ \text{FT}$$

PRESSURE WILL FALL FROM -10 TO -38 PSF IN A DISTANCE OF 138 FT

AT 60 MPH = 88 FT/S

$$\text{TIME} = 138/88 = \underline{1.57\ \text{SECONDS}} \quad (b)$$

PRESSURE AT TORNADO CENTER = -38 PSFG

AT EDGE OF CORE: $= -\frac{\rho v_{max}^2}{2} + P_\infty$

FAR FROM CENTER - $P = P_{ATM}$.

TOTAL $\Delta P = \underline{38\ \text{PSF}} \qquad (c)$

10.7 $v_r = U_\infty \cos\theta \left(1 - a^2/r^2\right)$

ALONG THE STAGNATION STREAMLINE

$\theta = \pi$

$$v_r = -U_\infty\left(1 - a^2/r^2\right) \qquad (a)$$

$$\frac{\partial v_r}{\partial r} = -\frac{2U_\infty a^2}{r^3}$$

$$\frac{\partial v_r}{r}\bigg|_{r=a} = -\frac{2U_\infty}{a} \qquad (b)$$

10.8 FROM CONTINUITY

$$\frac{\partial}{\partial r}(r v_r) = -\frac{\partial v_\theta}{\partial \theta}$$

$$\therefore \frac{\partial v_\theta}{\partial \theta}\bigg|_{\substack{r=a\\ \theta=\pi}} = -\frac{\partial}{\partial r}(r v_r)\bigg|_{\substack{r=a\\ \theta=\pi}}$$

10.9 $P + \frac{\rho v^2}{2} = \text{CONSTANT}$

FOR $P = P_\infty$ $v^2 = v_\infty^2$

$\therefore |v_\infty| = |v_\theta| = 2v_\infty \sin\theta$

$\sin\theta = 0.5$

$\therefore \quad \theta = \pm 30^\circ, \pm 150^\circ$

10.10 (a) $\phi = v_\infty L\left[\left(\frac{x}{L}\right)^3 - 3\frac{xy^2}{L^3}\right]$

$$v = \nabla\phi = v_x \vec{e}_x + v_y \vec{e}_y$$

$$v_x = \frac{\partial \phi}{\partial x} = \frac{3U_\infty}{L^2}\left(x^2 - y^2\right) = \frac{\partial \psi}{\partial y}$$

$$v_y = \frac{\partial \phi}{\partial y} = -\frac{6U_\infty x y}{L^2} = -\frac{\partial \psi}{\partial x}$$

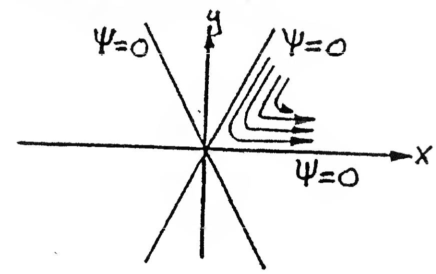
10.10 CONTINUED.

$$\psi = \frac{3U_\infty}{L^2}\left(x^2 y - \frac{y^3}{3}\right) + f(x)$$

$$= \frac{3U_\infty x^2 y}{L^2} + g(y)$$

So $\psi = \frac{U_\infty x^2 y}{L^2}\left(6x^2 - y^2\right)$

FLOW CONFIGURATION IS:



WHEN $\psi = 0$ $\quad y = 0$ OR $\pm\sqrt{6}x$

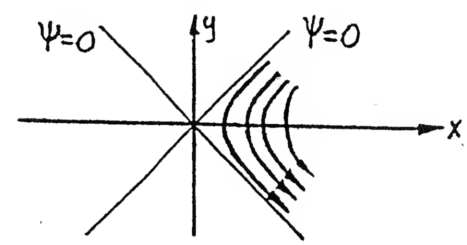
b) $\phi = U_\infty \frac{xy}{L}$

$$v_x = \frac{\partial \phi}{\partial x} = \frac{U_\infty y}{L} = \frac{\partial \psi}{\partial y}$$

$$v_y = \frac{\partial}{\partial y} = \frac{U_\infty y}{L} = -\frac{\partial \psi}{\partial x}$$

$$\psi = \frac{U_\infty}{2L} y^2 + f(x); \quad \psi = -\frac{U_\infty}{2L} x^2 + g(y)$$

$$\psi = \frac{U_\infty}{2L}\left(y^2 - x^2\right)$$



WHEN $\psi = 0$ $\quad y = \pm x$

10.10 CONTINUED

c) $\phi = \frac{U_\infty L}{2}\ln(x^2+y^2)$

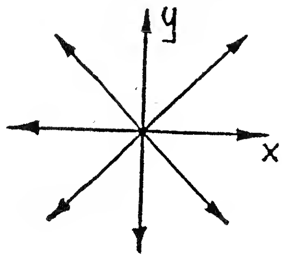
$$V_x = \frac{\partial\phi}{\partial x} = \frac{U_\infty L}{2}\frac{2x}{x^2+y^2} = \frac{\partial\psi}{\partial y}$$

$$V_y = \frac{\partial\phi}{\partial y} = \frac{U_\infty L}{2}\frac{2y}{x^2+y^2} = -\frac{\partial\psi}{\partial x}$$

$$\psi = \frac{U_\infty L}{\cancel{2}}\cancel{2}\tan^{-1}(y/x) + f(x)$$

$$\psi = \frac{U_\infty L}{\cancel{2}}\cancel{2}\tan^{-1}(x/y) + g(y)$$

$$\psi = U_\infty L\left[\tan^{-1}(x/y) - \tan^{-1}(y/x)\right]$$



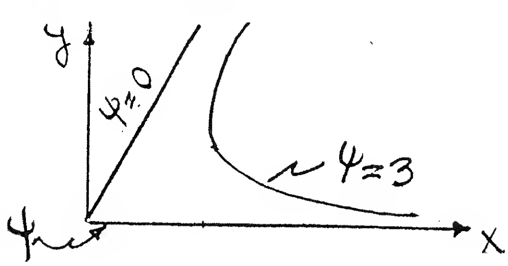
WHEN $\psi = 0$ $y = x$

10.11 $\psi = 2r^3\sin 3\theta$ FOR $\theta = 0, \frac{\pi}{3}$

IN FIGURE — FOR $\psi = 0$ — OR ANY NO. — PICK 3 —

$$r = \left[\frac{3}{2\sin 3\theta}\right]^{1/3}$$

CHOOSE θ — SOLVE FOR r —
PLOT LOOKS LIKE:

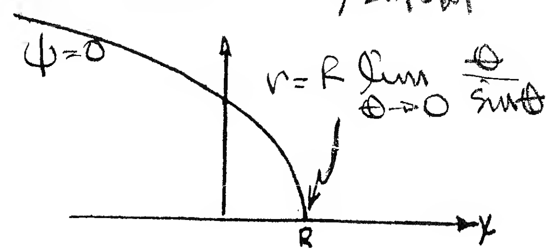


10.12 $\psi = 0 = U_\infty r\sin\theta + \frac{Q}{2\pi}\theta$

SINCE $r > 0$ WHEN $U_\infty > 0$, $\psi = 0$ GIVES $\theta = 0$ ~ THE +X AXIS.

WHEN $U_\infty < 0$ (FLOW IN −X DIRECTION)

$y = R\theta$ — WHERE $R = Q/2\pi|U_\infty|$



10.13 FOR SOURCE AT ORIGIN $\psi = \frac{\dot{m}\theta}{2\pi\rho}$

$\dot{m}$ = SOURCE STRENGTH

FREE STREAM: $\psi = U_\infty y$

ADDING: $\psi = U_\infty y + \dot{m}\theta/2\pi\rho$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = U_\infty\cos\theta + \dot{m}/2\pi\rho r$$

$y = r\sin\theta$

$v_r = 0$ @ $\theta = \pi$

AT $\theta = \pi$ $r = \frac{\dot{m}}{2\pi\rho U_\infty} = \frac{Q}{2\pi U_\infty}$

10.14 $\nabla P = \rho\frac{D\vec{V}}{Dt}$

$$= \rho\left[\frac{\partial\vec{V}}{\partial t} + \nabla\left(\frac{V^2}{2}\right) - V\times(\nabla\times\vec{V})\right]$$

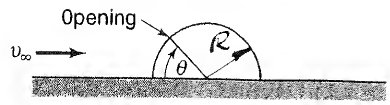
FOR STEADY, IRROTATIONAL FLOW

$$\nabla P = \rho\nabla\left(\frac{V^2}{2}\right)$$

@ STAGNATION POINT — WHERE $V = 0$

$$\nabla P = 0$$

10.15



LIFT FORCE:

$$df_y = df\sin\theta$$

$$= (P_{IN} - P_{OUT})R\sin\theta\, d\theta$$

$$F_y = \int_0^\pi \Delta P\, R\sin\theta\, d\theta$$

FROM BERNOULLI EQN:

$$P + \rho v^2/2 = \text{CONST.}$$

$$P_\infty + \rho\frac{v_\infty^2}{2} = P + \rho\frac{v^2}{2}$$

ON HUT $v = 2v_\infty \sin\theta$

$$\therefore P = P_\infty + \frac{\rho v_\infty^2}{2}\left[1 - 4\sin^2\theta\right]$$

SUBST. INTO EXPRESSION FOR f_y

$$f_y = \int_0^\pi \frac{\rho v_\infty^2}{2}\left[1 - 4\sin^2\theta - 1 + 4\sin^2\theta_0\right]R\sin\theta\, d\theta$$

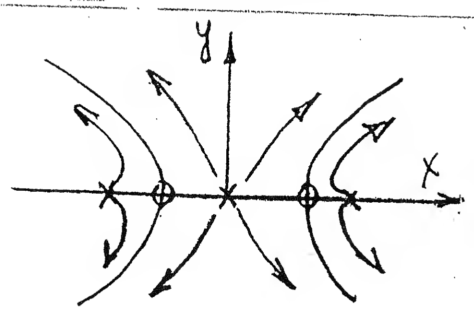
$$= 2\rho v_\infty^2 R\int_0^\pi\left[\sin^3\theta - \sin\theta\sin^2\theta_0\right]d\theta$$

$$= 2\rho v_\infty^2 R\left[4/3 - 2\sin^2\theta_0\right]$$

FOR $f_y = 0$ $\sin^2\theta_0 = 2/3$

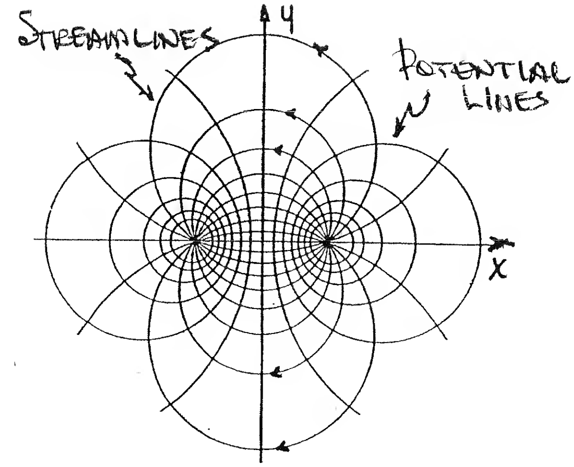
$$\underline{\underline{\theta_0 = 54.7^\circ}}$$

10.16



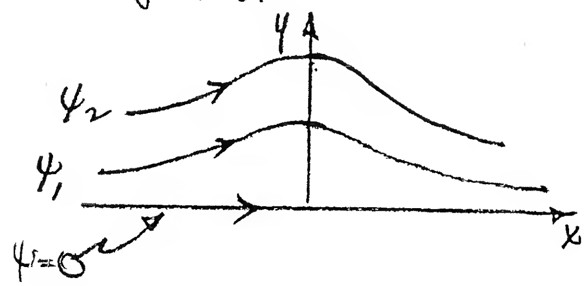
STAGNATION PTS AT CIRCLES

10.17



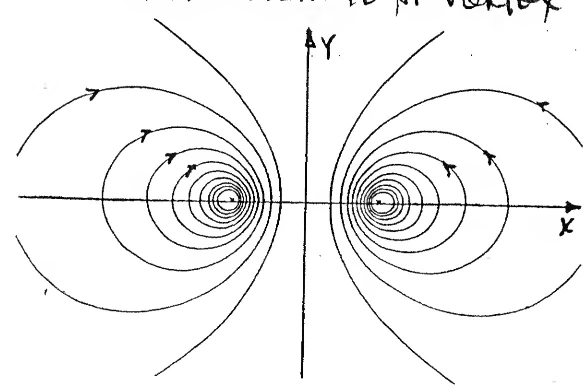
10.18 FOR THIS CASE $-\nabla^2\psi \neq 0$

$$y = \frac{\psi}{1 + 3x^2}$$



10.19 $\psi = -\frac{K}{2\pi}\ln r$, $v_\theta = \frac{K}{2\pi r}$.

WHEN ORIGIN IS AT VORTEX



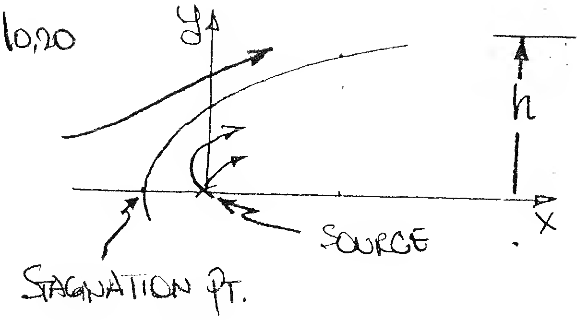
$$v_\theta(-a,0) = \frac{K}{2\pi(2a)} = \frac{K}{4\pi a}$$

$$\vec{v}_\theta(-a,0) = -K/4\pi a\, \vec{e}_y$$

& $\vec{v}(a,0) = -K/4\pi a\, \vec{e}_y$

SINCE $\psi = \frac{K}{2\pi}\ln r$

10.20



$$\psi = U_\infty r \sin\theta + \frac{Q}{2}\frac{\theta}{\pi}$$

a) STAGNATION POINT

$\vec{v} = 0$ REQUIRES $v_r = v_\theta = 0$

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\left(\frac{Q}{2\pi} + U_\infty r\cos\theta\right)$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -U_\infty \sin\theta$$

$$\therefore\ \theta = \pi,\ r\cos\pi = x = -\frac{Q}{2\pi U_\infty}$$

& AT STAGNATION PT

$$x = -\frac{Q}{2\pi U_\infty} = -\frac{1.5}{2\pi(9)}$$

$$= \underline{\underline{-0.0265\ m}}$$

$$y = 0$$

b) BODY HEIGHT

STAGNATION STREAMLINE

$$\psi = U_\infty r\sin\pi + \frac{Q\pi}{2\pi} = \frac{Q}{2}$$

THUS

$$\frac{Q}{2} = U_\infty r\sin\theta + \frac{Q\theta}{2\pi}$$

SO WHEN $\theta = \pi/2$

$$r\sin\theta = y = \frac{1}{U_\infty}\left(\frac{Q}{2} - \frac{Q\pi}{2\pi(2)}\right)$$

$$= \frac{Q}{4U} = \underline{\underline{0.0417\ m}}$$

10.20 CONTINUED

c) FOR X LARGE – ALL FLOW IS AT U_∞

$$\therefore\ Q = U_\infty(2h)$$

$$h = \frac{Q}{2U_\infty} = \frac{1.5}{2(9)} = \underline{\underline{0.0833\ m}}$$

d) MAXIMUM SURFACE VELOCITY

$v^2 = v_r^2 + v_\theta^2$ IS ON S.L. $\psi = Q/2$

v_r & v_θ DETERMINED IN PART (a)

$$v^2 = \frac{Q^2}{4\pi^2 r^2} + \frac{QU_\infty\cos\theta}{\pi r} + U_\infty^2$$

ON SURFACE $\psi = \frac{Q}{2} = U_\infty r\sin\theta + \frac{Q\theta}{2\pi}$

THUS $$\frac{Q^2}{4\pi^2 r^2} = \frac{Q^2 4U_\infty^2\sin^2\theta}{4\pi^2 Q^2(1-\theta/\pi)^2}$$

$$\frac{QU_\infty\cos\theta}{\pi r} = \frac{U_\infty^2 Q\cos\theta\sin\theta}{\pi Q/2(1-\theta/\pi)}$$

RESULTS IN

$$\frac{v^2}{U_\infty^2} = \frac{\sin^2\theta}{\pi^2(1-\theta/\pi)^2} + \frac{2}{\pi}\frac{\sin\theta\cos\theta}{(1-\theta/\pi)} + 1$$

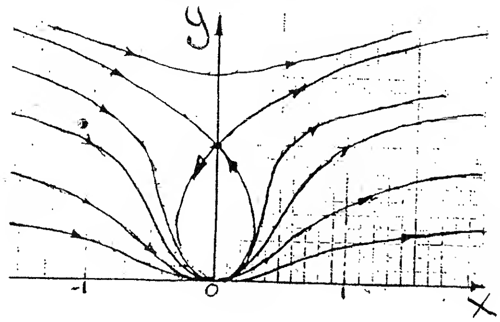
$\frac{v^2}{U_\infty^2}$ IS MAX AT $\theta \cong 63°$

SO $$\underline{\underline{\frac{V_{MAX}}{U_\infty} \cong 1.26}}$$

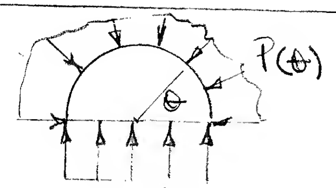
10.21 IN THIS CASE —

$$\psi = U_\infty r \sin\theta \left(1 + a^2/r^2\right)$$

STREAMLINES CAN BE PLOTTED FOR $U_\infty = 1$, $a = 1$
— IN UPPER HALF PLANE THEY APPEAR AS



10.22



P_i

$$\Sigma F_y = 0$$

$$P_i D - 12T - \frac{D}{2}\int_0^\pi P \sin\theta \, d\theta = 0$$

$$P = P_{ATM} + \frac{\rho}{2}\left(U_\infty^2 - U^2\right)$$

$$U = -2U_\infty \sin\theta$$

$$12T = P_i D - P_{ATM} + \frac{\rho}{2} U_\infty^3$$

$$+ 2\rho U_\infty^2 \frac{D}{2}\int_0^\pi \sin^3\theta \, d\theta$$

$$12T = (P_i - P_{atm})D + \frac{5}{6}\rho U_\infty^2 D$$

$\underline{T = 10.1 \text{ kN}}$

PER BOLT

CHAPTER 11

11.1

VARIABLE	DIMENSIONS
D	L
ρ	M/L^3
H	L
g	L/t^2
ω	$1/t$
Q	L^3/t
η	—
P	ML^2/t^3

$i = 8-3 = 5$

CHOOSE CORE AS ρ, D, ω

$\Pi_1 = \eta$ — ALREADY DIMENSIONLESS

$\Pi_2 = \rho^a D^b \omega^c H \sim = H/D$

$\Pi_3 = \rho^d D^\varepsilon \omega^f g \sim = g/D\omega^2$

$\Pi_4 = \rho^g D^h \omega^i Q \sim = Q/D^3\omega$

$\Pi_5 = \rho^j D^k \omega^l P \sim = P/\rho D^5 \omega^3$

11.2

VARIABLE	DIMENSIONS
v	L/t
D	L
ρ	M/L^3
μ	M/Lt
e	L

$i = 5-3 = 2$

CHOOSE CORE AS D, v, ρ

$\Pi_1 = D^a v^b \rho^c \mu \sim = \mu/Dv\rho = \frac{1}{Re}$

$\Pi_2 = D^d v^e \rho^f e \sim = e/D$

$$\underline{f = f(Re, e/D)}$$

11.3

VARIABLE	DIMENSIONS
ΔP	M/Lt^2
ρ	M/L^3
ω	$1/t$
D	L
Q	L^3/t
μ	M/Lt

$i = 6-3 = 3$

CHOOSE CORE AS ρ, D, ω

$\Pi_1 = \rho^a D^b \omega^c \Delta P \sim = \Delta P/\rho D^2 \omega^2$

$\Pi_2 = \rho^d D^\varepsilon \omega^f Q \sim = Q/D^3\omega$

$\Pi_3 = \rho^g D^h \omega^i \mu \sim = \mu/\rho D^2 \omega$

11.4

VARIABLE	DIMENSIONS
C_{MAX}	ML^2/t^2
α	—
β	—
M	M
L	L
ρ	M/L^3
g	L/t^2
R	L

$i = 8-3 = 5$

$\Pi_1 = \alpha$ — ALREADY DIMENSIONLESS

$\Pi_2 = \beta$ " "

CHOOSE CORE AS M, L, g

$\Pi_3 = M^a L^b g^c C_{MAX} \sim = C_{MAX}/MLg$

$\Pi_4 = M^d L^\varepsilon g^f \rho \sim = \rho L^3/M$

$\Pi_5 = M^g L^h g^i R \sim = R/L$

11.5

VARIABLE	DIMENSIONS
k	L/t
D	L^2/t
d	L
ω	$1/t$
ρ	M/L^3
μ	M/Lt

$$i = 6 - 3 = 3$$

CHOOSE CORE AS d, ω, ρ

$$\pi_1 = d^a \omega^b \rho^c k \sim\sim = k/d\omega$$

$$\pi_2 = d^d \omega^e \rho^f D \sim = D/d^2\omega$$

$$\pi_3 = d^g \omega^h \rho^i \mu \sim = \mu/\rho d^2\omega$$

PLOT π_1 VS. π_3

OVER A RANGE IN VALUES OF π_2

11.6

VARIABLE	DIMENSIONS
Q	L^3/t
d	L
ω	$1/t$
μ	M/Lt
σ	M/t^2
ρ	M/L^3

$$i = 6 - 3 = 3$$

CHOOSE CORE AS d, ω, ρ

$$\pi_1 = d^a \omega^b \rho^c Q \sim\sim = Q/\omega d^3$$

$$\pi_2 = d^d \omega^e \rho^f \mu \sim = d^2\omega\rho/\mu$$

$$\pi_3 = d^g \omega^h \rho^i \sigma \sim = \sigma/\rho \omega^2 d^3$$

11.7

VARIABLE	DIMENSIONS
M	M
d	L
ρ	M/L^3
g	L/t^2
σ	M/t^2

$$i = 5 - 3 = 2$$

CHOOSE CORE AS d, ρ, g

$$\pi_1 = d^a \rho^b g^c M \sim\sim = M/d^3\rho$$

$$\pi_2 = d^d \rho^e g^f \sigma \sim = \sigma\rho d^2/g$$

11.8

VARIABLE	DIMENSIONS
n	$1/t$
L	L
d	L
ρ	M/L^3
T	ML/t^2

$$i = 5 - 3 = 2$$

CHOOSE CORE AS n, d, ρ

$$\pi_1 = n^a d^b \rho^c L \sim\sim = L/d$$

$$\pi_2 = n^d d^e \rho^f T \sim = T/n^2 L^4 \rho$$

11.9

VARIABLE	DIMENSIONS
P	ML^2/t^3
d	L
ω	$1/t$
Q	L^3/t
ρ	M/L^3
μ	M/Lt

$$i = 6 - 3 = 3$$

11.9 - CONTINUED

CHOOSE CORE AS d, ρ, Q

$\Pi_1 = d^a \rho^b Q^c \omega$ — $= d^3\omega/Q$

$\Pi_2 = d^d \rho^e Q^f \mu$ — $= d\mu/\rho Q$

$\Pi_3 = d^g \rho^h Q^i P$ — $= d^4 P/\rho Q^3$

$$\underline{\underline{\Pi_3 = f(\Pi_1, \Pi_2)}}$$

11.10

VARIABLE	DIMENSIONS
r	L
t	t
ρ	m/L^3
E	mL^2/t^2

$i = 4 - 3 = 1$

$$\Pi_1 = t^a \rho^b E^c r = \frac{\rho^{1/5} r}{t^{2/5} E^{1/5}}$$

$$\text{OR } \Pi_1 = \frac{\rho r^5}{t^2 E}$$

$$\text{SO: } \underline{\underline{r^5 = C_1 \frac{Et^2}{\rho}}} \qquad (1)$$

SPEED OF WAVE FRONT $= \dfrac{dr}{dt}$

$$\Rightarrow \frac{dr}{dt} = \frac{2}{5} C_2 \frac{t}{r^4}$$

FROM (1) $\quad t = C_3 r^{5/2}$

$$\therefore \frac{dr}{dt} = C^4 / r^{3/2}$$

$\sim \dfrac{dr}{dt}$ DECREASES AS r INCREASES

11.11

VARIABLE	DIMENSIONS
d	L
D	L
v	L/t
ρ	m/L^3
μ	m/Lt
σ	m/t^2

$i = 6 - 3 = 3$

CHOOSE CORE AS D, ρ, v

$\Pi_1 = D^a \rho^b v^c d$ — $= d/D$

$\Pi_2 = D^d \rho^e v^f \mu$ — $= \dfrac{\mu}{D\rho v} = 1/Re$

$\Pi_3 = D^g \rho^h v^i \sigma$ — $= \dfrac{D\sigma}{\rho v^2}$

$$\underline{\underline{\Pi_1 = \Pi_1(\Pi_2, \Pi_3)}}$$

11.12

VARIABLE	DIMENSIONS
ΔP	m/Lt^2
Q	L^3/t
h	L
Ω	$1/t$
μ	m/Lt
L	L
R	L

$i = 7 - 3 = 4$

CHOOSE CORE AS h, Ω, μ

$\Pi_1 = h^a \Omega^b \mu^c L$ — $= L/h$

$\Pi_2 = h^d \Omega^e \mu^f R$ — $= R/h$

$\Pi_3 = h^g \Omega^h \mu^i \Delta P$ — $= \Delta P/\Omega\mu$

$\Pi_4 = h^j \Omega^k \mu^l Q$ — $= Q/h^3\Omega$

11.13 $Re = \frac{LU}{\nu}$ For Air @ 20°C (293K)

$\nu = 1.505 \times 10^{-5}\ m^2/s$

a) Based on L = 5.8 m

$$Re = \frac{(5.8)(22.2)}{1.505\times10^{-5}} = \underline{\underline{8.55 \times 10^6}}$$

b) Based on D = 0.0064 m

$$Re = \frac{(0.0064)(22.2)}{1.505\times10^{-5}} = \underline{\underline{9440}}$$

11.14 $$\rho \frac{D\vec{U}}{Dt} = -\nabla p + \mu \nabla^2 \vec{U} + \rho \vec{g}\left(\frac{T_A}{T_0} - 1\right)$$

To put D.E. into dimensionless form – use text procedure –

L = Reference Length

V_0 = " Velocity

Then $x^* = \frac{x}{L}$, $y^* = \frac{y}{L}$, $t^* = \frac{t U_0}{L}$

$\nabla^* = L\nabla \qquad \nabla^{*2} = L^2 \nabla^2$

$$\rho \frac{D\vec{U}}{Dt} = \rho \frac{D\vec{U}^*}{Dt^*}\left(\frac{\partial U}{\partial U^*}\right)\left(\frac{\partial t^*}{\partial t}\right) = \frac{\rho V_0^2}{L} \frac{DU^*}{Dt^*}$$

Where $\frac{\rho V_0^2}{L}$ is inertial force

We could do all other terms in a like manner but problem statement only asks for ratio of gravity forces to inertial forces

11.14 – Continued –

The gravitational (buoyancy) force is

$$\rho \vec{g}\left(\frac{T_A}{T_0} - 1\right)$$

So ratio asked for is

$$\frac{Lg}{V_0^2}\left[\frac{T_A}{T_0} - 1\right] \qquad \text{Q.E.D.}$$

11.15

Variable	Model	Prototype
D	D	6D
U	U	20 KT
ρ	ρ	ρ
μ	μ	μ
F	10 lbf	F_p
A	D^2	$(6D)^2$

Dynamic similarity requires:

$$Re_M = Re_P$$

$$\left.\frac{DU\rho}{\mu}\right|_m = \left.\frac{DU\rho}{\mu}\right|_p$$

$$U_m = U_p\left[\underbrace{\frac{D_p}{D_m}}_{6}\ \underbrace{\frac{\rho_p}{\rho_m}}_{1}\ \underbrace{\frac{\mu_m}{\mu_p}}_{1}\right] = 6U_p = \underline{\underline{120\ KT}}$$

& also that $Eu_m = Eu_p$

$$\left.\frac{F/A}{\rho U^2}\right|_m = \left.\frac{F/A}{\rho U^2}\right|_p$$

$$F_p = F_m\left[\underbrace{\left(\frac{A_p}{A_m}\right)}_{36}\underbrace{\left(\frac{\rho_p}{\rho_m}\right)}_{1}\underbrace{\left(\frac{U_p}{U_m}\right)^2}_{1/36}\right]$$

$$= \underline{\underline{10\ lb_F}}$$

11.16 SIMILARITY REQUIRES

$$Fr|_m = Fr|_p$$

$$V^2/gL|_m = V^2/gL|_p$$

	MODEL	PROTOTYPE
V	V_m	V_p
L	$L/10$	L

$$\left(\frac{V_m}{V_p}\right)^2 = \frac{L_m}{L_p} = 0.1$$

$$\frac{V_m}{V_p} = 0.316$$

11.17

MODEL	PROTOTYPE
$L = 3m$	$4L$
V_m	16 m/s
ρ_A	ρ_W
ν_A	ν_W
F_m	F_p
A_m	$16A_m$

FOR AIR AT $20°C$ (293 K)

$\nu = 1.505 \times 10^{-5}$ m²/s

AT 6 ATM $\nu = 0.251 \times 10^{-5}$ "

FOR H_2O @ $20°C$ $\nu = 0.995$ " "

$$Re_m = Re_p$$

$$\frac{LV}{\nu}\Big|_m = \frac{LV}{\nu}\Big|_p$$

$$V_m = V_p\left(\frac{\cancel{L_p}}{\cancel{L_m}}\,\frac{\nu_m}{\nu_p}\right) \quad 4$$

11.17 - CONTINUED

$$V_m = 16\left[(4)\frac{0.251 \times 10^{-5}}{0.995 \times 10^{-5}}(6)\right]$$

$$= 96.9 \text{ m/s}$$

$$Eu_m = Eu_p$$

$$\frac{F/A}{\rho V^2}\Big|_m = \frac{F/A}{\rho V^2}\Big|_p$$

$$\frac{F_m}{F_p} = \left(\frac{A_m}{A_p}\right)\left(\frac{\rho_m}{\rho_p}\right)\left(\frac{V_m}{V_p}\right)^2$$

$$= \frac{1}{16}\left(\frac{7.229}{998.2}\right)\left(\frac{96.9}{16}\right)^2$$

AT $20°C$ $\rho_W = 998.2$ kg/m³

AT $20°C$, 6 ATM $\rho_A = 7.229$ "

$$\frac{F_m}{F_p} = 0.0166$$

RESULT IS QUITE TEMPERATURE SENSITIVE

11.18 PROPERTIES -

AIR: $\rho = 5 \times 10^{-3}$ SLUG/FT³ $\nu = 8 \times 10^{-5}$ FT²/s

H_2O: $\rho = 1.94$ " $\nu = 1 \times 10^{-5}$ "

$$Re_m = Re_p$$

$$V_m = V_p\left[\frac{L_p}{L_m}\,\frac{\nu_m}{\nu_p}\right]$$

$$= 60\left[\left(\frac{1}{2}\right)\left(\frac{8 \times 10^{-5}}{1 \times 10^{-5}}\right)\right]$$

$$= 240 \text{ MPH}$$

11.19 VARIABLES DIMENSIONS

VARIABLES	DIMENSIONS
L	L
a	L
v	L/t
τ	t
g	L/t^2

$i = 5 - \underset{\sim}{2} = 3$

{ NOTE – $r = 2$ – NOT THE NO. OF FUNDAMENTAL DIMENSIONS – NO M }

CHOOSE CORE AS L, g

$$\pi_1 = L^a g^b a \sim = \frac{a}{L}$$

$$\pi_2 = L^c g^d v \sim = \frac{v}{(Lg)^{1/2}}$$

$$\pi_3 = L^e g^f \tau \sim = \tau\sqrt{\frac{g}{L}}$$

a) FOR GEOMETRIC SIMILARITY

$$\pi_1|_m = \pi_1|_p$$

$$a_m = a_p \frac{L_m}{L_p}$$

$$= 2\,m\left(\frac{1}{360}\right)$$

$$= 0.0056\,m = \underline{\underline{5.6\,mm}}$$

DYNAMIC SIMILARITY DICTATES –

$$\pi_2|_m = \pi_2|_p$$

$$v_m = v_p \frac{(Lg)_m^{1/2}}{(Lg)_p^{1/2}}$$

$$= 8\,m/s\left(\frac{1}{360}\right)^{1/2}$$

$$= \underline{\underline{0.421\,m/s}}$$

11.19 – CONTINUED

KINEMATIC SIMILARITY DICTATES

$$\pi_3|_m = \pi_3|_p$$

$$\tau_m = \tau_p\left[\sqrt{\frac{g}{L}}\Big|_p \sqrt{\frac{L}{g}}\Big|_m\right]$$

$$= 12\,HR\left(\frac{1}{360}\right)^{1/2} = 0.632\,HR$$

$$= \underline{\underline{37.9\,MIN.}}$$

11.20 FOR EQUAL REYNOLDS NOS.:

$$Re|_m = Re|_p$$

$$\frac{Lv\rho}{\mu}\Big|_m = \frac{Lv\rho}{\mu}\Big|_p$$

$$\rho_m = \rho_p\left(\frac{L_p}{L_m}\right)\left(\frac{v_p}{v_m}\right)\left(\frac{\mu_m}{\mu_p}\right)$$

& FOR IDEAL GAS BEHAVIOR – $\rho = \frac{P}{RT}$

$$\therefore P_m = P_p\left(\frac{T_m}{T_p}\right)\left(\frac{L_p}{L_m}\right)\left(\frac{v_p}{v_m}\right)\left(\frac{\mu_m}{\mu_p}\right)$$

$P_p = 287$ Pa.

$T_p = 250.4\,K$ $T_m = 294\,K$

$v_m \cong 340.3\,m/s$ $v_p = 317.2\,m/s$

$\mu_m = 1.22\times10^{-5}$ LBm/s.FT

$\mu_p = 9.53\times10^{-6}$ "

$$P_m = 287\left(\frac{294}{250.4}\right)\left(\frac{1}{0.4}\right)\left(\frac{317.2}{340.3}\right)\left(\frac{1.22\times10^{-5}}{9.53\times10^{-6}}\right)$$

$$= 1000\,Pa \sim \underline{\underline{1\,KPa}}$$

11.20 – CONTINUED

DIMENSIONLESS TIME SCALE:

$$t^* = \frac{tV}{L}$$

$$\therefore \quad \frac{tV}{L}\bigg|_m = \frac{tV}{L}\bigg|_p$$

$$\frac{t_m}{t_p} = \left(\frac{V_p}{V_m}\right)\left(\frac{L_m}{L_p}\right)$$

$$= \left(\frac{317.2}{340.3}\right)\left(\frac{0.4}{1}\right)$$

$$= \underline{\underline{0.373}}$$

11.21 $FR = \frac{V^2}{gL}$ SPEED RATIO $= \frac{V}{Nd}$

	MODEL	PROTOTYPE
L	0.41	2.45
V	2.58	V

EQUATING FROUDE NUMBERS

$$\frac{V^2}{gL}\bigg|_m = \frac{V^2}{gL}\bigg|_p$$

$$V_p = 2.58\left(\frac{2.45}{0.41}\right)^{1/2}$$

$$= \underline{\underline{6.31\ \text{m/s}}}$$

EQUATING V/Nd

$$\frac{V}{Nd}\bigg|_m = \frac{V}{Nd}\bigg|_p$$

11.21 CONTINUED

$$Nd\big|_p = Nd\big|_m \frac{V_p}{V_m}$$

$$N_p = N_m\left(\frac{d_m}{d_p}\right)\left(\frac{V_p}{V_m}\right)$$

$$= 450\left(\frac{0.41}{2.45}\right)\left(\frac{6.31}{2.58}\right)$$

$$= \underline{\underline{184\ \text{RPM}}}$$

THRUST FORCE INVOLVES EULER NO.

$$Eu\big|_m = Eu\big|_p$$

$$\frac{F/A}{\rho V^2/2}\bigg|_m = \frac{F/A}{\rho V^2/2}\bigg|_p$$

$$F_p = F_m\left(\frac{V_p}{V_m}\right)^2\cancel{\left(\frac{\rho_p}{\rho_m}\right)}^{1}\left(\frac{L_p}{L_m}\right)^2$$

$$= 245\left(\frac{6.31}{2.58}\right)^2\left(\frac{2.45}{0.41}\right)^2$$

$$= \underline{\underline{52.3\ \text{kN}}}$$

TORQUE = FL

– FROM EULER \#

$$\frac{T/L}{\rho V^2/2}\bigg|_m = \frac{T/L}{\rho V^2/2}\bigg|_p$$

$$T_p = T_m\left(\frac{L_p}{L_m}\right)\left(\frac{V_p}{V_m}\right)^2$$

$$= 20\left(\frac{2.45}{0.41}\right)\left(\frac{6.31}{2.58}\right)^2$$

$$= \underline{\underline{715\ \text{N·m}}}$$

CHAPTER 12

12.1 AT TRANSITION $Re_D = 2300$

$$Re = \frac{DV}{\nu} = 2300$$

H_2O @ 20°C: $\nu = 0.995 \times 10^{-6}\ m^2/s$

$$V = \frac{2300(0.995 \times 10^{-6})}{0.038\ m}$$

$$= \underline{\underline{0.060\ m/s \sim 6\ cm/s}}$$

12.2 $F_D = C_D A \rho V^2/2$

FOR 35000 FT – $\rho = 0.0237\ lbm/ft^3$
S.L. $\rho = 0.0766$ "

a) @35,000 FT 500 MPH = 733 FT/S

$$F_D = \frac{0.011(2400)(0.0237)(733)^2}{2(32.2)}$$

$$= \underline{\underline{5220\ LBF}}$$

b) @ SEA LEVEL 200 MPH = 293 FT/S

$$F_D = \frac{0.011(2400)(0.0766)(293)^2}{2(32.2)}$$

$$= \underline{\underline{2700\ LBF}}$$

12.3 AT TRANSITION $Re_x = 2 \times 10^5$

FOR AIR @ 20°C, $\nu = 1.505 \times 10^{-5}\ m^2/s$

$$Re_x = xV/\nu$$

$$x = (2\times10^5)(1.505\times10^{-5})/30\ m/s$$

$$= \underline{\underline{0.100\ m}}$$

12.4 $\frac{v_x}{v_\delta} = C_1 + C_2 y + C_3 y^2 + C_4 y^3$

BOUNDARY CONDITIONS:

(1) $v_x(0) = 0$

(2) $v_x(\delta) = v_\delta$

(3) $\frac{\partial v_x}{\partial y}(\delta) = 0$

ONE MORE B.C. IS NEEDED –

IF $\frac{dP}{dx} = 0$ THE OTHER ONE IS $\frac{\partial^2 v_x}{\partial y^2}(0) = 0$

BUT THIS ISN'T THE CASE CONSIDERED

THE GOVERNING EQN OF MOTION IS

$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{dP_x}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

AT $y = 0$ – $v_x = v_y = 0$

$\frac{dP}{dx}$ CAN BE RELATED TO v_δ BY THE BERNOULLI EQN: $\frac{P}{\rho} + \frac{v_\delta^2}{2} = \text{CONST}$

SO $\frac{dP}{dx} = -\rho v_\delta \frac{dv_\delta}{dx}$

∴ @ $y = 0$ THE EQN OF MOTION GIVES

$$(4) \qquad \frac{\partial^2 v_x}{\partial y^2} = -\frac{1}{\nu} v_\infty \frac{dv_\infty}{dx}$$

THIS IS THE 4th B.C.

FROM (1): $C_1 = 0$ THE REMAINING EXPRESSION FOR $\frac{v_x}{v_\delta}$ WILL BE

$$\frac{v_x}{v_\delta} = C_2 \frac{y}{\delta} + C_3\left(\frac{y}{\delta}\right)^2 + C_4\left(\frac{y}{\delta}\right)^3$$

12.4 CONTINUED -

FROM (2) $1 = C_2 + C_3 + C_4$

(3) $0 = C_2 + 2C_3 + 3C_4$

(4) $-\frac{\delta^2}{\nu}\frac{dv_\delta}{dx} = 2C_3$

SUBSTITUTION YIELDS:

$$\frac{v_x}{v_\delta} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 + \frac{\delta^2}{4\nu}\frac{dv_\delta}{dx}\left[\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^3\right]$$

12.5 GIVEN $v_x = \alpha \sin \beta y$

FOR A LAMINAR B.L. $\frac{dP}{dx} = 0$:

B.C. (1) $v_x(0) = 0$

(2) $v_x(\delta) = v_\infty$

(3) $\frac{dv_x}{dy}(\delta) = 0$

FROM (1) $0 = 0$ — NO HELP

(2) $v_\infty = \alpha \sin \beta\delta$

(3) $0 = \alpha\beta \cos \beta\delta$

GIVING $\beta\delta = \pi/2$ — $\beta = \pi/2\delta$

$\alpha = v_\infty$

SO PROFILE IS $v_x = v_\infty \sin\left(\frac{\pi y}{2\delta}\right)$

VON KARMAN INTEGRAL EQN FOR B.L.

$$\frac{\tau_0}{\rho} = \frac{d}{dx}\int_0^\delta v_x(v_\infty - v_x)\,dy$$

$$\frac{\tau_0}{\rho} = \frac{\mu}{\rho}\frac{dv_x}{dy}\bigg|_0 = \frac{\mu}{\rho} v_\infty \frac{\pi}{2\delta}\cancelto{1}{\cos\frac{\pi y}{2\delta}}$$

12.5 CONTINUED -

EVALUATING $\int_0^\delta v_x(v_\infty - v_x)\,dy$:

$$= v_\infty^2 \int_0^\delta \frac{v_x}{v_\infty}\left(1 - \frac{v_x}{v_\infty}\right)dy$$

$$= v_\infty^2 \int_0^\delta \left[\sin\frac{\pi y}{2\delta} - \sin^2\frac{\pi y}{2\delta}\right]dy$$

$$= v_\infty^2\left[-\frac{2\delta}{\pi}\cos\frac{\pi y}{2\delta} - \frac{y}{2} + \frac{\delta}{2\pi}\sin\frac{\pi y}{\delta}\right]_0^\delta$$

$$= v_\infty^2\left[-2\frac{\delta}{\pi} - \frac{\delta}{2}\right] = v_\infty^2\delta\left[\frac{2}{\pi} - \frac{1}{2}\right]$$

NOW: $\frac{d}{dx}\left[\int_0^\delta \sim\right] = \frac{d}{dx}\left\{v_\infty^2\delta\left[\frac{2}{\pi} - \frac{1}{2}\right]\right\}$

$$= \left(\frac{2}{\pi} - \frac{1}{2}\right)v_\infty^2\frac{d\delta}{dx}$$

EQUATING BOTH PARTS:

$$\nu v_\infty \frac{\pi}{2\delta} = \left(\frac{2}{\pi} - \frac{1}{2}\right)v_\infty^2\frac{d\delta}{dx}$$

$$\delta\, d\delta = \frac{\nu\pi^2}{v_\infty(4-\pi)}\int_0^x dx$$

$$\delta = \left[\frac{\nu x}{v_\infty}\frac{2\pi^2}{4-\pi}\right]^{1/2}$$

$$\delta = 4.81\sqrt{\frac{\nu x}{v_\infty}} = \frac{4.81\,x}{\sqrt{Re_x}}$$

$$C_{fx} = \frac{\tau_0}{\rho v_\infty^2/2} = \frac{\mu}{\rho}\frac{v_\infty\pi/2\delta}{v_\infty^2/2} = \frac{\nu\pi}{v_\infty\delta}$$

PUTTING IN OUR EXPRESSION FOR δ WE HAVE

$$C_{fx} = 0.653\,Re_x^{-1/2}$$

12.5 CONTINUED –

$$C_{fL} = \frac{1}{L}\int_0^L C_{fx}\,dx$$

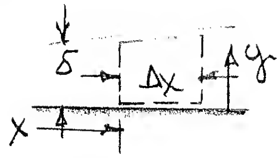
$$= \frac{0.653}{L}\sqrt{\frac{\nu}{v_\infty}}\int_0^L x^{-1/2}\,dx$$

$$= \frac{0.653}{L}\sqrt{\frac{\nu}{v_\infty}}\left(2x^{1/2}\right)_0^L$$

$$= 1.306\,Re_L^{-1/2}$$

Comparison	Approximate	Exact
δ	$4.81x\,Re_x^{-1/2}$	$5.0x\,Re_x^{-1/2}$
C_{fx}	$0.653\,Re_x^{-1/2}$	$0.644\,Re_x^{-1/2}$
C_{fL}	$1.305\,Re_L^{-1/2}$	$1.328\,Re_L^{-1/2}$

12.6



MOMENTUM THEOREM:

$$\sum F_x = \iint_{c.s.}\rho(\vec{v}\cdot\vec{n})\,dA + \cancel{\frac{\partial}{\partial t}}^{\,0}$$

$$\sum F_x = P\delta|_x - P\delta|_{x+\Delta x} + \frac{P|_x + P|_{x+\Delta x}}{2}\left(\delta|_{x+\Delta x} - \delta|_x\right) - \tau_0\,\Delta x$$

$$\iint_{c.s.} v_x\rho(\vec{v}\cdot\vec{n})\,dA = \int_0^\delta \rho v_x^2\,dy\Big|_{x+\Delta x} - \int_0^\delta \rho v_x^2\,dy\Big|_x - v_\infty\left[\int_0^\delta \rho v_x\,dy\Big|_{x+\Delta x} - \int_0^\delta \rho v_x\,dy\Big|_x - v_{y0}\Delta x\right]$$

12.6 – CONTINUED –

REARRANGING, DIVIDING BY Δx, & EVALUATING IN THE LIMIT AS $\Delta x \to 0$:

$$-\delta\frac{dP}{dx} = \tau_0 + v_\infty v_{y0} + \frac{d}{dx}\int_0^\delta \rho v_x^2\,dy - v_\infty\frac{d}{dx}\int_0^\delta \rho v_x\,dy$$

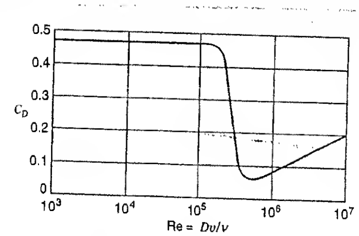
NOTING THAT BERNOULLI'S EQN APPLIES OUTSIDE THE B.L. WE CAN WRITE (SEE CHAPTER)

$$\delta\frac{dP}{dx} = \frac{d}{dx}\left(\delta v_\infty^2\right) - v_\infty\frac{d}{dx}\left(\delta v_\infty\right)$$

& THE FINAL RESULT BECOMES:

$$\frac{\tau_0}{\rho} + \frac{v_\infty v_{y0}}{\rho} = \frac{dv_\infty}{dx}\int_0^\delta (v_\infty - v_x)\,dy + \frac{d}{dx}\int_0^\delta v_x(v_\infty - v_x)\,dy$$

12.7



FOR A SMOOTH SPHERE – FIG. ABOVE

$Re_{cr} \cong 2\times10^5$

FOR AIR @ 20°C $\nu = 1.505\times10^{-5}\ m^2/s$

$$v_{cr} = \frac{\nu}{D}Re_{cr} = \frac{(1.505\times10^{-5})(2\times10^5)}{0.042} = \underline{71.7\ m/s}$$

FOR SUCH A SPHERE (GOLF BALL SIZE) A VELOCITY GREATER THAN THIS WILL REDUCE DRAG & BALL WILL TRAVEL FURTHER

12.8 For Air @ 80°F

$\nu = 0.169 \times 10^{-3}\ ft^2/s$

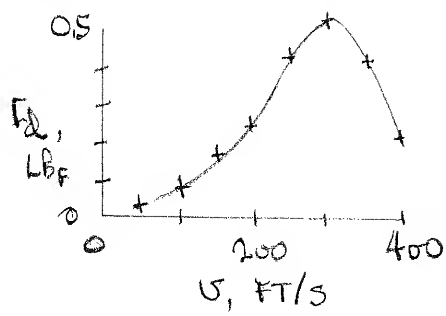
$\rho = 0.0735\ lbm/ft^3$

$$Re = \frac{DU}{\nu} = \frac{(1.65/12)U}{0.169\times10^{-3}}$$

$$F_d = A\rho \frac{U^2}{2} C_D$$

$$= \frac{\pi}{4}\left(\frac{1.65}{12}\right)^2\left(\frac{0.0735}{32.2}\right)\frac{U^2}{2} C_D$$

U, ft/s	C_D	Re	F_D, lbf
50	0.47	4.07×10^4	0.0199
100	0.47	8.14	0.0797
150	0.46	12.2	0.175
200	0.45	16.3	0.305
250	0.41	20.3	0.434
300	0.35	24.4	0.534
350	0.20	28.5	0.415
400	0.08	32.5	0.217



12.9 In the unsteady wake region $1 < Re < 10^3$

@ 20°C $\nu = 1.505\times10^{-5}\ m^2/s$

$Re = DU/\nu$

@ $Re = 1 = \dfrac{0.0127\,U}{1.505\times10^{-5}}$

$\underline{U = 0.00119\ m/s}$

@ $Re = 10^3$ $\underline{U = 1.185\ m/s}$

These are the lower & upper bounds for U

12.10 For Air @ 80°F

$\nu = 0.169\times10^{-3}\ ft^2/s$

$\rho = 0.0735\ lbm/ft^3$

$$Re = \frac{(0.2/12)(88)}{0.169\times10^{-3}} = 8680$$

From Fig 12.2 $C_D \cong 1.2$

$$F_D = C_D A \rho \frac{U^2}{2}$$

$$= 1.2\left(\frac{0.2}{12}\right)(3)\left(\frac{0.0735}{32.2}\right)\left(\frac{88^2}{2}\right)$$

$$\underline{\underline{= 0.530\ lbf}}$$

12.11 $F_D = C_D A \rho \dfrac{U^2}{2}$

Air @ 20°C $\rho = 1.2048\ kg/m^3$

$\nu = 1.505\times10^{-5}\ m^2/s$

$F_D = 0.26(2.33)(1.2048)(30)^2$

$= 657\ N$

Power $= F_D U = (657)(30) = \underline{\underline{19.7\ kW}}$

With a headwind of 6 m/s

$F_D = 0.26(2.33)(1.2048)(36)^2 = \underline{\underline{28.4\ kW}}$

& a tailwind of 6 m/s

$F_D = \sim\sim\ (24)^2 = \underline{\underline{12.6\ kW}}$

For still air	P = 25.9 hp
With head wind	P = 37.4 "
With tailwind	P = 169 "

12.12 $100\ mi/hr = 44.7\ m/s$

$$F_L = C_L A \rho V^2/2$$

$$= \frac{0.21(2.33)(1.2048)(44.7)^2}{2}$$

$$= \underline{589\ N}$$

12.13 IF $C_L = 1$

$$F_L = 589\left(\frac{1}{0.21}\right) = \underline{2.805\ kN}$$

12.14 $F_D = C_D A \rho V^2/2$

IN SAME ENVIRONMENT AT SAME SPEED

$$C_D A|_{CAR} = C_D A|_{PLATE}$$

$$C_D A|_{CAR} = 0.26(2.33)$$

$$C_D A|_{PLATE} = 1.1\ \frac{\pi D^2}{4}$$

$$D = \left[\frac{0.26(2.33)(4)}{(1.1)(\pi)}\right]^{1/2}$$

$$= \underline{0.837\ m}$$

12.15 – CIRCULAR SIGN – $D = 8\ FT$, $V = 120\ mph$ $(176\ FT/s)$

$$F_D = C_D A \rho V^2/2$$

AT $80°F$ – $\rho = 0.0735\ lb_m/FT^3$

$$F_D = \frac{1.1(\pi/4)(8)^2(0.0735)(176)^2}{32.2(2)}$$

$$= \underline{1955\ lb_f}$$

12.16 FOR AIR @ $100°F$ – $\rho = 0.0710\ lb_m/FT^3$
$0°F$ – $\rho = 0.0862$ "

$C_D = 0.28$ $A = 2.4\ m^2 = 25.83\ FT^2$

$$P = F_D V = C_D A \rho V^3/2$$

$$= \frac{0.28(25.83)(0.0710)(102.7)^3}{2(32.2)(550)}$$

$$= \underline{15.7\ hp} \quad \text{– AT } 100°F$$

$$= 15.7\left(\frac{0.0862}{0.0710}\right) = \underline{19.1\ hp}$$
AT $0°F$

12.17 SPHERE – $D = 9.25/\pi = 2.94\ in$
$WT = 5.25$ OUNCES
AT $V = 95$ MPH $(139.3\ FT/s)$

AT $80°F$ $\rho_{AIR} = 0.0735\ lb_m/FT^3$
$\nu_{AIR} = 0.169 \times 10^{-3}\ FT^2/s$

a) $$Re = \frac{DV}{\nu} = \frac{(2.94/12)(139.3)}{0.169 \times 10^{-3}} = \underline{2.02 \times 10^5}$$

$$F_D = C_D A \rho V^2/2$$

b) AT $Re = 2.02 \times 10^5$ $C_D \cong 0.4$

$$F_D = \frac{0.4\left(\frac{\pi}{4}\right)\left(\frac{2.94}{12}\right)^2(0.0735)(139.3)^2}{2(32.2)}$$

$$= \underline{0.418\ lb_f}$$

c) FLOW IS NEAR TRANSITION – BUT STILL IN LAMINAR RANGE

12.18

$Re \cdot 10^{-4}$	7.5	10	15	20	25
C_D	0.48	0.38	0.22	0.12	0.10

AT $80°F$ — $\nu = 0.169 \times 10^{-3}\ FT^2/S$

$\rho = 0.0735\ LBm/FT^3$

FOR $Re = 7.5 \times 10^4 = DU/\nu$

$$U = \frac{(7.5\times10^4)(0.169\times10^{-3})}{1.65/12}$$

$$= 92.18\ FT/S$$

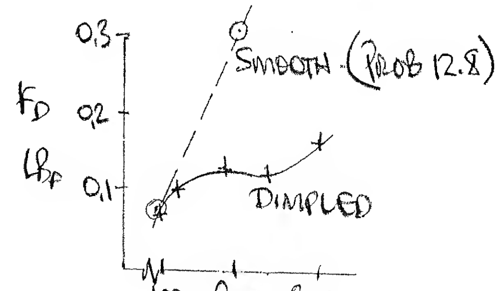
$$F_D = C_D A \rho U^2/2$$

$$= \frac{0.48\left(\frac{\pi}{4}\right)\left(\frac{1.65}{12}\right)^2(0.0735)(92.18)^2}{2(32.2)}$$

$$= 0.069\ LBF$$

DOING THIS CALCULATION FOR ALL GIVEN CONDITIONS WE GENERATE THE FOLLOWING!

$Re \times 10^4$	U	C_D	F_D, LBF
7.5	92.2	0.48	0.069
10	122.9	0.48	0.100
15	184.4	0.47	0.129
20	245.8	0.44	0.125
25	307.3	0.10	0.164



12.19 $W_T = 5.25$ OUNCES $= 0.328\ LB_F$

$$F_L = W_T = C_L A \rho U^2/2$$

$$A = \frac{\pi}{4}\left(\frac{2.94}{12}\right)^2 = 0.04714\ FT^2$$

$$C_L = 0.224$$

FROM PROBLEM STATEMENT

$$C_L \cong 0.24 \frac{R\Omega}{U} - 0.05$$

SO FOR THIS CASE

$$\frac{R\Omega}{U} = \frac{0.224+0.05}{0.24} = 1.142$$

$$U = 110\ mph = 161.3\ FT/S$$

$$\Omega = \frac{1.142(161.3)}{0.385} = 478\ RAD/S$$

$$= 76.1\ REV/S$$

TO TRAVEL 60.5 FT $\quad t = \frac{60.5}{161.3} = 0.375\ S$

NO OF REVOLUTIONS $= 76.1(0.375) = \underline{\underline{28.5}}$

12.20 BLASIUS EQN FOR LAMINAR BOUNDARY LAYER FLOW IS

$$\rho \frac{DU_x}{Dt} = \frac{dP}{dx} + \mu \nabla^2 U_x$$

OR, WRITTEN AS

$$U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\left[\frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2}\right]$$

AT $y=0$ — $U_x = 0$ BUT, IN THIS CASE, $U_y \neq 0$

THE RESULTING FORM IS

$$U_y \frac{\partial U_x}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu \frac{\partial^2 U_x}{\partial y^2}$$

THIS TERM IS NOT PRESENT FOR $U_y(0) = 0$ {EQN (12-33) CASE

12.21 TURBULENCE INTENSITY:

$$I = \frac{\left[\left(\overline{v_x'^2} + \overline{v_y'^2} + \overline{v_z'^2}\right)/3\right]^{1/2}}{v_\infty}$$

KINETIC ENERGY $= \dfrac{v_\infty^2 + \overline{v_x'^2} + \overline{v_y'^2} + \overline{v_z'^2}}{2}$

$$= \frac{v_\infty^2\left(1+3I^2\right)}{2}$$

FOR $I = 0.1$ $\quad K.E.\big|_{TOTAL} = \dfrac{v_\infty^2}{2}(1.03)$

WHILE $\quad K.E._{TURB} = \dfrac{v_\infty^2}{2}(0.03)$

FRACTION DUE TO TURBULENCE

$$= \frac{0.03}{1.03} = \underline{\underline{2.91\%}}$$

12.22 $\dot{V} = 2\text{ gpm} = 0.446\times10^{-2}\ \text{ft}^3/\text{s}$

$$v = \frac{\dot{V}}{A} = \frac{0.446\times10^{-2}}{\frac{\pi}{4}\left(\frac{0.75}{12}\right)^2} = 1.45\ \text{ft/s}$$

FOR H_2O @ 120°F $\quad \nu = 0.62\times10^{-5}\ \text{ft}^2/\text{s}$

@ 45°F $\quad \nu = 1.57\times10^{-5}$ "

@ 120°F

$$Re = \frac{\left(\frac{0.75}{12}\right)(1.45)}{0.62\times10^{-5}} = \underline{14{,}600} \quad (a)$$

@ 45°F

$$Re = \frac{(0.75/12)(1.45)}{1.57\times10^{-5}} = \underline{5770} \quad (b)$$

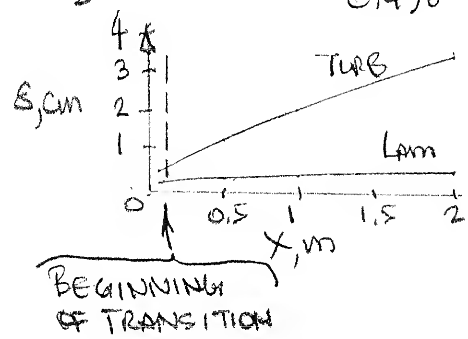
12.23 LAMINAR FLOW: $\dfrac{\delta}{x} = 5\,Re_x^{-1/2}$

TURBULENT " $\quad \dfrac{\delta}{x} = 0.376\,Re_x^{-0.2}$

FOR AIR @ 20°C $\quad \nu = 1.505\times10^{-5}\ \text{m}^2/\text{s}$

$$Re_x = \frac{30\,x}{1.505\times10^{-5}} = 2\times10^6\,x$$

x, m	Re_x	δ_L, cm	δ_t, cm
0	0	0	0
0.1	2×10^5	0.111	0.327
0.5		0.249	1.186
1		0.352	2.063
2		0.498	3.591



12.24 $\dot{V} = 0.006\ \text{m}^3/\text{s}$

$$v = \frac{0.006}{\left(\frac{\pi}{4}\right)(0.15)^2} = 0.34\ \text{m/s}$$

TO CALCULATE y^+ & v^+:

$$\frac{\tau}{\rho} = 0.0225\,\bar{v}_{max}^2\left[\frac{\nu}{v_{x\,max}\,y_{max}}\right]$$

FROM RESULTS OF 1/7 POWER LAW:

$\bar{v} = 0.817\,v_{x\,max} \sim v_{x\,max} = 0.416\ \text{m/s}$

$y_{max} = 0.075\ \text{m}$

AT 20°C $\quad \nu = 0.995\times10^{-6}\ \text{m}^2/\text{s}$

SUBSTITUTING INTO τ/ρ EXPRESSION

$$\sqrt{\tau/\rho} = 0.0171\ \text{m/s}$$

12.24 CONTINUED

- LAMINAR SUBLAYER:

$$y^+ = \frac{\sqrt{\tau/\rho}\, y}{\nu} = 5$$

$$y = \frac{5(0.995\times10^{-6})}{0.0171} = \underline{0.291 \text{ mm}}$$

BUFFER LAYER -

EXTENDS FOR $5 < y^+ < 30$

@ $y^+ = 30$ $y = 1.746$ mm

$\text{THICKNESS}_{B.L.} = \underline{1.455 \text{ mm}}$

TURBULENT CORE EXTENDS FROM $y = 1.455$ mm TO $y = 75$ mm

$\text{THICKNESS}_{T.C.} = \underline{73.55 \text{ mm}}$

12.25 EQN. (12-68)

$$\tau_0/\rho = 0.0225\, v_{max}^2 \left[\frac{\nu}{v_{max}\, y_{max}}\right]^{1/4}$$

$$C_{fx} = \frac{\tau_0/\rho}{v_{avg}^2/2}$$

$$= 0.045\left(\frac{v_m}{v_{av}}\right)^2 [\ \ ]$$

$$[\ \ ]^{-1/4} = \left[\left(\frac{v_m}{v_{av}}\right)\frac{v_{av}R}{\nu}\right]^{-1/4}$$

$$= \left(\frac{v_m}{v_{av}}\right)^{-1/4}\left(2^{1/4}\right) Re_D^{-1/4}$$

GIVING $C_{fx} = 0.0535\left(\frac{v_m}{v_{av}}\right)^{7/4} Re_D^{-1/4}$

NOW - TO FIND $\frac{v_m}{v_{av}}$ FOR PIPE FLOW

12.25 CONTINUED

$$v_{av}(\pi R^2) = \int_A v\, dA$$

$$= 2\pi \int v\, r\, dr$$

$$v_{av} = \frac{2}{R^2}\int_0^R v_{max}\left(1 - \frac{r}{R}\right)^{1/7} r\, dr$$

DOING THE MATH: $\frac{v_m}{v_{av}} = 1.225$

$$\underline{C_{fx} = 0.0763\, Re_D^{-1/4}}$$

12.26 $Re_L = \frac{Lv}{\nu} = \frac{0.5(40)}{0.159\times10^{-3}} = 125{,}800$

$$C_{fL} = \frac{1}{L}\int_0^L C_{fx}\, dx = \frac{1}{L}\int_0^L \frac{0.0576\, x}{Re_x^{1/5}}\, dx$$

$$= 0.072\, Re_L^{-1/5}$$

$$C_{fL} = 0.072\,(125{,}800)^{-1/5} = 6.877\times10^{-3}$$

FOR 2 SIDES & 60°F AIR

$$C_{fL} = 2 C_{fL} A \rho \frac{v^2}{2}$$

$$= \frac{2(6.877\times10^{-3})(1.5)(0.0764)(40)^2}{2(32.2)}$$

$= \underline{0.0392 \text{ LBF}}$ (a)

FOR LAMINAR FLOW

$$C_{fL} = 1.328\, Re_L^{-1/2} = 0.00375$$

$$F_D = 2 C_{fL} A \rho \frac{v^2}{2}$$

$= \underline{0.0213 \text{ LBF}}$ (b)

12.27 $Re = 10^5$

LAMINAR FLOW – $\delta_L = 5\, Re_x^{-1/2}$

TURBULENT " $\delta_t = 0.375\, Re_x^{-0.2}$

FOR $Re = 10^5$ $\dfrac{\delta_t}{\delta_L} = 2.38$

FROM CHAPTER 5: MOMENTUM $\sim \rho v^2$

" " 6: ENERGY $\sim \rho v^3$

FOR $v = v_\infty f\left(\frac{y}{\delta}\right)$

MOMENTUM $= \rho v_\infty^2 f^2\left(\frac{y}{\delta}\right)$

ENERGY $= \dfrac{\rho v_\infty^3}{2} f^3\left(\frac{y}{\delta}\right)$

$$\frac{M}{\rho v_\infty^2} = f^2\left(\frac{y}{\delta}\right)$$

$$\frac{E}{\rho v_\infty^3/2} = f^3\left(\frac{y}{\delta}\right)$$

FOR LAMINAR CASE:

$$\frac{M}{\rho v_\infty^2} = \sin^2\left(\frac{y}{\delta_L}\frac{\pi}{2}\right)$$

$$\frac{E}{\rho v_\infty^3/2} = \sin^3\left(\frac{y}{\delta_L}\frac{\pi}{2}\right)$$

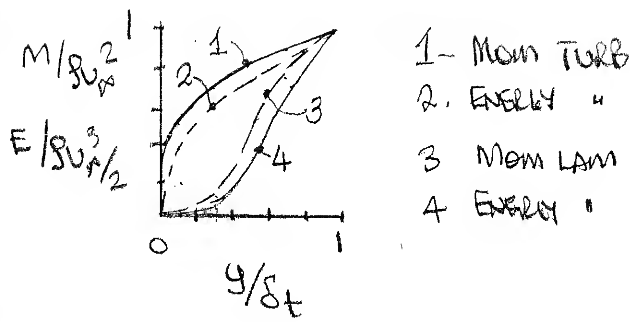
y/δ_L	$\sin\frac{y}{\delta_L}\frac{\pi}{2}$	$\frac{M}{\rho v_\infty^2}$	$\frac{E}{\rho v_\infty^3/2}$
0	0	0	0
0.1	0.156	0.0244	0.0038
0.3	0.455	0.207	0.094
0.5	0.707	0.50	0.355
0.7	0.89	0.795	0.708
0.9	0.99	0.98	0.97
1	1.0	1.0	1.0

12.27 CONTINUED

FOR TURBULENT CASE:

$$\frac{M}{\rho v_\infty^2} = \left(\frac{y}{\delta_t}\right)^{2/7} \qquad \frac{E}{\rho v_\infty^3/2} = \left(\frac{y}{\delta_t}\right)^{3/7}$$

y/δ_t	$\frac{M}{\rho v_\infty^2}$	$\frac{E}{\rho v_\infty^3/2}$
0	0	0
0.1	0.518	0.373
0.3	0.709	0.600
0.5	0.820	0.743
0.7	0.903	0.858
0.9	0.970	0.956
1	1	1



12.28 $F_D = C_{fL} A \rho v^2/2$

$A = (7)(40)(2) = 560\ \text{ft}^2$ {2 SIDES}

$v = 140\ \text{mph} = 205.3\ \text{ft/s}$

@ 500 FT – $\rho = 0.0660\ \text{lbm/ft}^3$

$\mu = 1.165\times10^{-5}\ \text{lbm/ft·s}$

$$Re = \frac{Lv}{\nu} = \frac{7(205.3)}{1.165\times10^{-5}/0.0660} = 8.14\times10^6$$

12.28 CONTINUED -

a) LAMINAR

$$C_{fL} = 1.328\,Re_L^{-1/2} = 0.000465$$

$$F_D = \frac{(0.000465)(560)(0.0660)(205.3)^2}{2(32.2)}$$

$$= \underline{\underline{11.26 \text{ LBF}}}$$

b) TURBULENT

$$C_{fL} = 0.072\,Re_L^{-0.2} = 0.00299$$

$$F_D = \underline{\underline{72.3 \text{ LBF}}}$$

12.29 $\quad Re_x = 10^6$

B.L. THICKNESS —

LAM: $\delta = 5x Re_x^{-1/2}$

TURB: $\delta = 0.376 x Re_x^{-1/5}$

$$\delta_t/\delta_L = \frac{0.376}{5} Re_x^{0.3} = \underline{\underline{4.74}}$$

COEF. OF SKIN FRICTION:

LAM $\quad C_{fx} = 0.664\,Re_x^{-1/2}$

TURB $\quad C_{fx} = 0.0576\,Re_x^{-0.2}$

$$\frac{C_{fx\,t}}{C_{fx\,L}} = \frac{0.0576}{0.664} Re_x^{0.3} = \underline{\underline{5.47}}$$

12.30 FOR TURBULENT B.L. ~ WATER

@ 60°F $\rho = 62.3$ LBM/FT^3

$\nu = 1.22 \times 10^{-5}$ FT^2/S

$v = 20$ FT/S

$$Re_L = \frac{20(20)}{1.22 \times 10^{-5}} = 3.28 \times 10^7$$

12.30 CONTINUED

$$\delta = 0.376 x\,Re_x^{-0.2}$$

$$= 0.376(20)(3.28 \times 10^7)^{-0.2} = \underline{\underline{0.236 \text{ FT}}}$$

$$C_{fL} = 0.072\,Re_L^{-0.2} = 0.00226$$

$$F_D = C_{fL} A \rho v^2/2$$

$$= \frac{(0.00226)(200)(2)(62.3)(20)^2}{2(32.2)}$$

$$= \underline{\underline{350 \text{ LBF}}}$$

IF FLOW IS LAMINAR -

$$C_{fL} = 1.328\,Re_L^{-1/2} = 2.319 \times 10^{-4}$$

$$F_D = \underline{35.91 \text{ LBF}}$$

12.31 EXPANDING $v_x'(x,y)$ IN TAYLOR SERIES:

$$v_x'(x,y) = v_x'(0,0) + x\frac{\partial v'}{\partial x}(0,y) + y\frac{\partial v'}{\partial y}(x,0) + \frac{x^2}{2}\frac{\partial^2 v'}{\partial x^2}(0,y) + \frac{y^2}{2}\frac{\partial^2 v'}{\partial y^2}(x,0) + \frac{\partial^2 v_x'}{\partial x \partial y}(0,0) + \cdots$$

AS $y \to 0 \quad x' \to 0$

THE EXPRESSION FOR v_x' REDUCES TO

$$v_x'(x,y) = a_1 y + a_2 y^2 + a_3 xy + \cdots$$

WHERE $a_1 = \left.\frac{\partial v_x'}{\partial y}\right|_0$ - ETC.

SIMILARLY

$$v_y'(x,y) = b_1 y + b_2 y^2 + b_3 xy + \cdots$$

WHERE $b_1 = \frac{\partial v_y'}{\partial y}$ - etc.

12.31 CONTINUED

CONTINUITY EQN REQUIRES THAT

$$\frac{\partial v_x'}{\partial x} + \frac{\partial v_y'}{\partial y} = 0$$

GIVEN: $a_3 y + b_1 + 2b_2 y + b_3 x = 0$

COEFFICIENTS OF LIKE FNS OF x & y REQUIRE

$$a_3 + 2b_2 = 0$$

$$b_1 = b_3 = 0$$

SO $v_x'(x,y) = a_1 y + a_2 y^2 + a_3 xy$

$$v_y'(x,y) = -a_3 y^2$$

$$v_x' v_y' = -a_3 a_1 y^3 - c_2 c_3 y^4$$

TAKING TIME AVERAGE

$$\overline{v_x' v_y'} = -a_3 a_1 y^3 + \cdots$$

I.E. $\underline{\overline{v_x' v_y'} \simeq y^3}$

WHILE MIXING LENGTH THEORY SAYS $\underline{\overline{v_x' v_y'} \simeq y^2}$

12.32 POWER LAW PROFILE -

$$\frac{v_x}{v_{max}} = \left(\frac{y}{R}\right)^{1/n}$$

$$\frac{\partial v_x}{\partial y} = \frac{v_{max}}{n} \frac{y^{1/n - 1}}{R^{1/n}}$$

AS $y \to 0$ $\underline{\frac{\partial v_x}{\partial y} \to \infty}$

AS $y \to R$ $\frac{\partial v_x}{\partial y} \to \underline{\underline{\frac{v_{max}}{nR}}}$

12.33 $\tau_0 = 0.0225 \rho v_{max}^2 \left(\frac{\nu}{v_{max}\delta}\right)^{1/4}$ (1)

FOR $\frac{v_x}{v_\infty} = \left(\frac{y}{\delta}\right)^{1/n}$

$$\frac{\tau_0}{\rho v_\infty^2} = \frac{d}{dx}\int_0^\delta \frac{v_x}{v_\infty}\left(1 - \frac{v_x}{v_\infty}\right) dy$$

$$= \frac{d\delta}{dx}\int_0^1 \left[\left(\frac{y}{\delta}\right)^{1/n} - \left(\frac{y}{\delta}\right)^{2/n}\right] d\left(\frac{y}{\delta}\right)$$

$$= \left[\frac{1}{1+1/n} - \frac{1}{1+2/n}\right]\frac{d\delta}{dx}$$

EQUATING WITH (1) & DOING ALGEBRA

$$0.0225\left(\frac{\nu}{v_\infty \delta}\right)^{1/4} = \frac{n}{(n+1)(n+2)}\frac{d\delta}{dx}$$

$$0.0225\left(\frac{\nu}{v_\infty}\right)^{1/4}\int_0^x dx = \frac{n}{(n+1)(n+2)}\int_0^\delta \delta^{1/4} d\delta$$

BECOMES

$$\left(\frac{\delta}{x}\right)^{5/4} = \frac{(n+1)(n+2)}{n}(0.0281) Re_x^{-1/4}$$

& FINALLY

$$\underline{\underline{\frac{\delta}{x} = \left[0.0281\,\frac{(n+1)(n+2)}{n}\right]^{0.8} Re_x^{-0.2}}}$$

CHAPTER 13

13.1 OIL - $\nu = 0.08\times10^{-3}\ \text{ft}^2/\text{s}$

$\rho = 57\ \text{lbm/ft}^3$

$\dot{V} = 10\ \text{gal/hr}$

TUBE - DIAM = 0.24 IN., L = 50 FT

$$v = \frac{\dot{V}}{\pi D^2/4} = 1.18\ \text{ft/s}$$

$$Re = \frac{Dv}{\nu} = \frac{(0.24/12)(1.18)}{0.08\times10^{-3}} = 295$$ {LAMINAR}

$$\frac{\Delta P}{\rho} = h_L = 2 f_F \frac{L}{D} v^2 \quad \{f_F = 16/Re\}$$

$$= 2\,\frac{(16)}{295}\,\frac{50}{0.24/12}\,(1.18)^2$$

$$= 377.6\ \text{ft}^2/\text{s}^2$$

$$\Delta P = \frac{57(377.6)}{32.2} = \underline{\underline{668\ \text{lbf/ft}^2}}$$

13.2 OIL - SAME PROPERTIES AS IN PROB 13.1

TUBE - D = 0.1 IN., L = 30 IN.

$\Delta P = 15\ \text{lbf/in}^2$

FOR LAMINAR FLOW - USE H.-P. EQN

$$\Delta P = 32\mu V \Delta x/D^2$$

OR $$\frac{\Delta P}{\rho g} = \frac{32\nu v \Delta x}{g D^2}$$

$$v = \frac{(15)(144)(0.1/12)^2(32.2)}{32(57)(0.08\times10^{-3})(30/12)}$$

$$= 13.24\ \text{ft/s}$$

$$\dot{V} = vA = \underline{\underline{7.22\times10^{-4}\ \text{ft}^3/\text{s}}}$$

$$Re = \frac{(0.1/12)(13.24)}{0.08\times10^{-3}} = 1379$$ {LAMINAR FLOW O.K.}

13.3 $$\Delta P = 2 f_F \frac{L}{D} v^2$$

FOR A SPECIFIED PIPE: $\Delta P \sim f_F v^2$

IF FULLY TURBULENT - $f_F \sim e/D$ ONLY

$\therefore\ \Delta P \sim v^2$

FOR H_2O $\Delta P = 13$ PSI FOR $\dot{m} = 28.3\ \text{lbm/s}$

FOR LOX $\rho = 70\ \text{lbm/ft}^3$ $\dot{m} = 35\ \text{lbm/s}$

$$\frac{\Delta P_{LOX}}{\Delta P_{H_2O}} = \frac{(\dot{m}/\rho A)^2_{LOX}}{(\dot{m}/\rho A)^2_{H_2O}}$$

$$= \left(\frac{35}{70}\right)^2\left(\frac{62.4}{28.3}\right)^2 = 1.21$$

FOR LOX - $\Delta P = 13(1.21) = \underline{\underline{15.8\ \text{PSI}}}$

13.4 ENERGY EQN:

$$-\frac{\delta W_s}{dt} = \dot{m}\left[\frac{P_2-P_1}{\rho} + \frac{v_2^2-v_1^2}{2} + g\Delta y + h_L\right]$$

OIL: $\rho = 810\ \text{kg/m}^3$ $\nu = 4.5\times10^{-6}\ \text{m}^2/\text{s}$

$\dot{V} = 0.56\ \text{m}^3/\text{s}$

LINE: $D = 0.62$ m $y_2 - y_1 = -250$ m

$\Delta P = 101.3 - 300 = -198.7$ kPa

COMMERCIAL STEEL

$$v = \dot{V}/A = \frac{0.56}{\pi/4(0.62)^2} = 1.855\ \text{m/s}$$

$$\frac{P_2-P_1}{\rho} = \frac{+198.7(1000)}{810} = +245.3\ \text{m}^2/\text{s}^2$$

$$\frac{v_2^2-v_1^2}{2} = 0$$

$$g\Delta y = 9.81(-250) = -2452\ \text{m}^2/\text{s}^2$$

13.4 CONTINUED -

$$Re = \frac{DU}{\nu} = \frac{0.62(1.855)}{4.5\times10^{-6}} = 256{,}000$$

FOR THIS Re VALUE & COMMERCIAL STEEL - $\frac{e}{D} \cong 0.00075$

FIG 13.1 - $f_F \cong 0.0045$

$$h_L = 2(0.0045)\left(\frac{280\,000}{0.62}\right)(1.855)^2$$

$$= 13990\ m^2/s^2$$

$$-\frac{\delta W_s}{dt} = (810)(0.56)\left[+245.3 - 2452 + 13990\right]$$

$$= \underline{\underline{5.34\ MW}}$$

13.5 SAME CONDITIONS AS IN PROB 13.4 EXCEPT

2 PIPES IN SERIES -

270 KM OF ORIGINAL PIPE

10 KM OF NEW PIPE WITH D = 0.42 M

FOR THE NEW SYSTEM:

$$-\frac{\delta W_s}{dt} = \dot{m}\left[\frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} + g\Delta y + h_L\right]$$

$$\frac{P_2 - P_1}{\rho} = \{SAME\} = 245.3\ m^2/s$$

$$\frac{\Delta U^2}{2} = \{SAME\} = 0$$

$$g\Delta y = \{SAME\} = -2452\ m^2/s$$

$$h_L = h_{L1} + h_{L2}$$

1 ~ ORIGINAL

2 ~ NEW

13.5 CONTINUED -

$$h_{L1} = \frac{270\,000}{280\,000}(13990) = 13490\ m^2/s^2$$

FOR NEW SECTION:

$$U = \frac{0.56}{\pi/4\,(0.42)^2} = 4.04\ m/s$$

$$Re = \frac{0.42(4.04)}{4.5\times10^{-6}} = 3.773\times10^5$$

$$\frac{e}{D} = 0.00012 \qquad f_F \cong 0.0038$$

$$h_L = 2(0.0038)\frac{10000}{0.42}(4.04)^2$$

$$= 2953\ m^2/s^2$$

$$TOTAL\ h_L = 13490 + 2953 = 16440\ m^2/s^2$$

NEW CASE -

$$-\frac{\delta W_s}{dt} = (810)(0.56)\left[245.3 - 2452 + 16440\right]$$

$$= \underline{\underline{6.46\ MW}}$$

13.6 STEADY FLOW BETWEEN PUMPING STATIONS

$$0 = \frac{P_2 - P_1}{\rho} + \frac{U_2^2 - U_1^2}{2} + g\Delta y + h_L$$

$$\Delta U^2/2 = 0$$

$$\Delta y = 0$$

$$SO\ \frac{\Delta P}{\rho} = h_L = 2 f_F \frac{L}{D} U^2$$

$$Re = \frac{DU}{\nu} = \frac{(0.71)(1.1)}{6.7\times10^{-6}} = 1.166\times10^5$$

$$e/D = 0.000068 \qquad f_F \cong 0.0046$$

$$h_L = 2(0.0046)\left(\frac{320\times10^3}{0.71}\right)(1.1)^2$$

$$= 5017\ m^2/s^2$$

$$\Delta P = h_L/g = \underline{\underline{511\ M\ OF\ OIL}}$$

13.6 CONTINUED -

$$-\frac{\delta W_s}{dt} = \dot{m} g h_L$$

$$= 801\left(\frac{\pi}{4}\right)(0.71)^2(1.1)(9.81)(511)$$

$$\underline{\underline{= 1.749 \text{ MW}}}$$

13.7 ENERGY EQN IN STEADY FLOW.

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g\Delta y + h_L = 0$$

$$-\frac{\Delta P}{\rho} = -\frac{60(144)(32.2)}{62.4} = -4460 \text{ ft}^2/\text{s}$$

$$\frac{v_2^2 - v_1^2}{2} = \frac{v_2^2}{2}$$

$$g\Delta y = 0$$

$$h_L = 2f_F \frac{L}{D} v^2 + \sum K \frac{v^2}{2}$$

$$= v^2\left[2f_F \frac{L}{D} + \frac{1}{2}\sum K\right]$$

$$\sum K = (6)(0.7) + 3.8 + 7.5 = 15.5$$

$$2 f_F \frac{L}{D} = 2(0.007)\frac{160}{0.75/12} = 35.84$$

$$h_L = v^2\left[35.84 + 7.75\right] = 43.6 v^2$$

$$v_2 = \frac{vA}{A_2} = v\left(\frac{D}{D_2}\right)^2 = v\left(\frac{0.75}{0.1}\right)^2 = 56.25 v$$

ENERGY EQN. BECOMES

$$-4460 + \frac{1}{2}(56.25 v)^2 + 43.6 v^2 = 0$$

$$v^2 = \frac{4460}{1625} = 2.74 \text{ ft}^2/\text{s}^2$$

$$v = 1.656 \text{ ft/s}$$

13.7 CONTINUED

NOW TO CHECK f_F:

$$Re = \left(\frac{0.75}{12}\right)(1.656)/(1.22\times10^{-5})$$

$$= 8480$$

$$e/D = \frac{5\times10^{-6}(12)}{0.75} = 0.00008$$

FIG 13.1 - $f_F \cong 0.75$

THIS MAKES A NEGLIGIBLE CHANGE IN THE h_L CALCULATION — ∴

$$v = 1.656 \text{ ft/s}$$

$$\dot{V} = \frac{\pi}{4}\left(\frac{0.75}{12}\right)^2(1.656) = 0.0051 \text{ ft}^3/\text{s}$$

13.8 FOR THIS CASE -

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta y + h_L = 0$$

$$-\frac{\Delta P}{\rho} = \frac{4.55(144)(32.2)}{62.4} = -338.1 \text{ ft}^2/\text{s}^2$$

$$\Delta v^2 = 0$$

$$g\Delta y = 0$$

$$h_L = 2f_F \frac{L}{D} v^2$$

$$v = \frac{118 \text{ ft}^3/\text{m}}{(60)\pi/4(D^2)} = \frac{2.50}{D^2}$$

$$h_L = 2f_F \frac{250}{D}\left(\frac{2.5}{D^2}\right)^2 = \frac{3125}{D^5} f_F$$

GOVERNING EQN. IS

$$-338.1 + \frac{3125}{D^5} f_F = 0$$

$$f_F = 0.1082 D^5$$

OTHER CONSTRAINT IS $f_F(Re)$ ~ FIG 13.1

13.8 CONTINUED -

$$Re = \frac{D v}{\nu} = \frac{\dot{V}}{\pi D^2/4\,\nu}$$

$$= \frac{118}{60\,(\pi) D/4\,(1.22\times10^{-5})}$$

$$\frac{2.052\times10^{5}}{D}$$

TRIAL & ERROR —

ASSUME $f_F = 0.004$

$$D = \left[\frac{0.004}{0.1082}\right]^{1/5} = 0.517 \text{ FT}$$

$$Re = \frac{2.052\times10^{5}}{0.517} = 3.97\times10^{5}$$

FIG 13.1 ... $f_F = 0.00325$

USING THIS VALUE —

$D = 0.496$ FT $Re = 4.137\times10^{5}$

$f_F = 0.0031$

$\Rightarrow D = \underline{0.491 \text{ FT}}$ (5.9 IN)

13.9 ENERGY EQN - STEADY FLOW

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta y + h_L = 0$$

LOCATION 1 — SUMP ($v_1 = 0$)

2 — PUMP INLET

$$\frac{\Delta P}{\rho} = \frac{P_2 - P_{ATM}}{\rho} = \frac{P_{2g}}{\rho}$$

$$v_2 = \frac{\dot{V}}{\pi D^2/4} = \frac{500}{(7.48)(60)\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\right)^2}$$

$$= 5.67 \text{ FT/S}$$

$$\Delta v^2/2 = 16.1 \text{ FT}^2/\text{S}^2$$

13.9 CONTINUED -

$$g\Delta y = 9.81(3) = 29.4 \text{ FT}^2/\text{S}^2$$

$$h_L = 2 f_F \frac{L}{D} v^2$$

$$Re = \frac{(6/12)(5.67)}{1.22\times10^{-5}} = 2.32\times10^{5}$$

$$e/D = 0.003$$

$$f_F \text{ (FIG 13.1)} \cong 0.0066$$

$$h_L = 2(0.0066)\left(\frac{6}{0.5}\right)(5.67)^2 = 5.09 \text{ FT}^2/\text{S}^2$$

SUBSTITUTING INTO ENERGY EQN:

$$-\frac{P_{2g}}{\rho} = 16.1 + 29.4 + 5.09 = 50.6 \text{ FT}^2/\text{S}^2$$

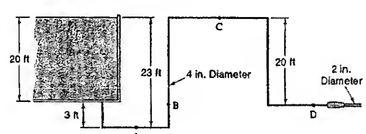
$$-P_{2g} = \frac{62.4(50.6)}{32.2} = \underline{\underline{98.0 \text{ LBF/FT}^2}}$$

$$\underline{P_2 = -0.681 \text{ PSIG}}$$

$$P_2 \text{ ABSOLUTE} = 14.7 - 0.681$$

$$= \underline{\underline{14.02 \text{ PSIA}}}$$

13.10



$L = 35$ m

$= 114.9$ FT

BETWEEN RESERVOIR SURFACE (1) & NOZZLE EXIT (2):

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g\Delta y + h_L = 0$$

$$\Delta P = 0$$

$$v_1^2 = 0$$

$$g\Delta y = -(32.2)(20) = -644 \text{ FT}^2/\text{S}^2$$

IN PIPE: $v_p = v_2 \frac{A_2}{A_P} = v_2\left(\frac{D_2}{D_1}\right)^2 = \frac{v_2}{4}$

$$v_p^2 = v_2^2/16$$

13.10 CONTINUED

$$h_L = 2 f_F \frac{L}{D} v_p^2 + \sum K \frac{v_p^2}{2}$$

$$= v_p^2 \left[2 f_F \frac{114.9}{4/12} + \frac{\sum K}{2} \right]$$

$$\sum K = \underbrace{(5)(0.7)}_{\text{ELBOWS}} + \underbrace{1}_{\text{ENTRANCE}}$$

$$h_L = v_p^2 \left[689.4 f_F + 2.25 \right]$$

ENERGY EQN. IS

$$\frac{v_p^2}{8} - 644 + v_p^2 \left[\quad \right] = 0$$

OR $v_p^2 \left[689.4 f_F + 2.63 \right] = 644$

TRIAL & ERROR —

ASSUME $f_F = 0.005$

$v_p = 10.29$ FT/S

$$Re = \frac{(4/12)(10.29)}{1.22 \times 10^{-5}} = 2.811 \times 10^5$$

$e/D = 0.0005$

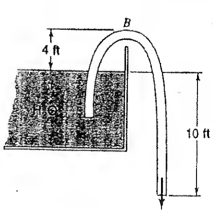
FIG 13.1 — $f_F \cong 0.0045$

WITH $f_F = 0.0045$ $v_p = 10.6$ FT/S

Re CHECKS.

& $$\dot{V} = \frac{\pi}{4}\left(\frac{4}{12}\right)^2 (10.6) = \underline{\underline{0.925 \text{ FT}^3/\text{S}}}$$

13.11



L = 23 FT

D = 1 INCH

BETWEEN RESERVOIR SURFACE (1) & EXIT (2)

13.11 CONTINUED —

$$\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g\Delta y + h_L = 0$$

$\Delta P = 0$

$v_1 = 0$

$g\Delta y = 32.2(-10) = -322$ FT²/S²

$$h_L = 2 f_F \frac{L}{D} v^2 + \sum K \frac{v^2}{2}$$

$$= 2 f_F \frac{23}{1/12} v^2 + v^2 \sum K/2$$

K=1 — ENTRANCE LOSS

$$h_L = v^2 \left[552 f_F + 0.5 \right]$$

ENERGY EQN. IS

$$\frac{v^2}{2} - 322 + v^2 \left[552 f_F + 0.5 \right] = 0$$

$$v^2 \left[552 f_F + 1 \right] = 322$$

TRIAL & ERROR:

ASSUME $f_F = 0.005$

$v = 9.25$ FT/S

$$Re = \frac{(1/12)(9.25)}{1.22 \times 10^{-5}} = 6.32 \times 10^4$$

FIG 13.1 — SMOOTH TUBE — $f_F = 0.0047$

FOR $f_F = 0.0047$ $v = 9.46$ FT/S

$Re = 6.46 \times 10^4$ $f_F = 0.0047$

$$\dot{V} = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 (9.46) = \underline{\underline{0.0516 \text{ FT}^3/\text{S}}}$$

BETWEEN (2) & B

$$\frac{P_B - P_2}{\rho} + \frac{v_B^2 - v_2^2}{2} + g\Delta y + h_L = 0$$

13.11 CONTINUED -

$$\frac{P_B - P_2}{\rho} = \frac{P_{Bg}}{\rho}$$

$$\frac{v_B^2 - v_2^2}{2} = 0$$

$$g\Delta y = 32.2(14) = 450.8 \text{ FT}^2/\text{s}^2$$

$$h_L = 2 f_F \frac{L}{D} v^2 + \sum \cancel{K} \frac{v^2}{2}$$

0

$$= \frac{2(0.0047)(14)(9.46)^2}{1/12}$$

$$= 141.3 \text{ FT}^2/\text{s}^2$$

INTO ENERGY EQN:

$$\frac{P_{Bg}}{\rho} = -450.8 - 141.3 = -592.1 \text{ FT}^2/\text{s}^2$$

$$P_{Bg} = -\frac{(592.1)(62.4)}{32.2} = -1147 \text{ PSF}$$

$$= \underline{\underline{-7.97 \text{ PSI}}}$$

$$P_{B\,ABSOLUTE} = 14.7 - 7.97 = \underline{\underline{6.73 \text{ PSI}}}$$

13.12 RECTANGULAR DUCT - 8" x 8" x 25 FT

$$\dot{V} = 600 \text{ FT}^3/\text{m} \quad \text{STD AIR}$$

$$D_{EQ} = \frac{4(8)(8)}{4/8} = 8 \text{ IN}$$

$$v = \frac{600/60}{8(8)/144} = 22.5 \text{ FT/s}$$

ENERGY EQN. REDUCES TO

$$\frac{\Delta P}{\rho} = 2 f_F \frac{L}{D} v^2$$

$$Re = \frac{(8/12)(22.5)}{1.56 \times 10^{-5}} = 9.59 \times 10^4$$

13.12 CONTINUED -

$$e/D = \frac{0.0005}{8/12} = 0.00075$$

FIG 13.1 - $f_F \cong 0.0054$

$$\frac{\Delta P}{\rho} = 2(0.0054)\left(\frac{25}{8/12}\right)(22.5)^2 = 205 \text{ FT}^2/\text{s}^2$$

$$\Delta P = \frac{205(0.0766)}{32.2} = 0.4876 \text{ PSF}$$

$$= 6.366 \text{ FT AIR} = 76.4 \text{ IN AIR}$$

$$= (76.4)\frac{0.0766}{62.4} = \underline{\underline{0.0938 \text{ IN } H_2O}}$$

13.13 ENERGY EQN.

$$\cancel{\frac{\Delta P}{\rho}} + \cancel{\frac{\Delta v^2}{2}} + g\Delta y + h_L = 0$$

0 0

$$g\Delta y = (32.2)(175) = 5635 \text{ FT}^2/\text{s}^2$$

$$\dot{V} = 3 \times 10^6 \frac{\text{GAL}}{\text{DAY}} = 4.642 \text{ FT}^3/\text{s}$$

$$v = \frac{4.642}{\frac{\pi}{4} D^2} = \frac{5.91}{D^2} \text{ FT/s}$$

$$h_L = 2 f_F \frac{L}{D} v^2 = 2 f_F \frac{10560\, v^2}{D5}$$

$$= 2.112 \times 10^4 \frac{f_F v^2}{D^5}$$

FOR 10-IN PIPE: $v = \frac{5.91}{(10/12)^2} = 8.51 \text{ FT/s}$

$$Re = \frac{(10/12)(8.51)}{1.22 \times 10^{-5}} = 5.81 \times 10^5$$

$e/D = 0.0011 \quad - \quad f_F = 0.0051$

$$h_L = \frac{2.112 \times 10^4 (0.0051)(8.51)^2}{(10/12)^5}$$

$$= \underline{19410 \text{ FT}^2/\text{s}^2}$$

13.13 CONTINUED —

FOR 12" PIPE: $v = 5.91$ FT/S

$Re = 4.84\times10^5$ $e/_D = 0.00085$

$f_F \cong 0.0048$ $h_L = \underline{3540\ FT^2/S^2}$

FOR 14" PIPE: $v = 4.34$ FT/S

$Re = 4.15\times10^5$ $e/_D = 0.00073$

$f_F \cong 0.0047$ $\underline{h_L = 865\ FT^2/S^2}$

$$COST/YR = \left\{\begin{matrix}POWER\\COST\end{matrix}\right\} + \left\{\frac{1}{20}\begin{matrix}INITIAL\\COST\end{matrix}\right\} + 0.06\left\{\begin{matrix}INITIAL\\COST\end{matrix}\right\}$$

$$= \left\{\begin{matrix}POWER\\LOST\end{matrix}\right\} + 0.11\left\{\begin{matrix}INITIAL\\COST\end{matrix}\right\}$$

$$POWER\ COST = \frac{\$0.07}{KWH}(P)$$

$$P = \dot{m}(h_L + g\Delta y)\frac{(1.356)(365)(24)}{(32.2)(1000)}$$

$$= 106.86(h_L + g\Delta y)\ KWH$$

FOR 10-IN PIPE:

$$CPY = 0.11(\$11.40)(2)(5280) + \$0.07(106.86)(19410+5635)$$

$$= \underline{\$200{,}580}$$

FOR 12" PIPE —

$$CPY = 0.11(\$14.70)(2)(5280) + \$0.07(106.8)(3540+5635)$$

$$= \underline{\$85{,}670}$$

FOR 14" PIPE —

$$CPY = 0.11(\$16.80)(2)(5280) + 0.07(106.8)(865+5635)$$

$$= \underline{\$68{,}108} \leftarrow \text{CHEAPEST}$$

13.14 ENERGY EQN. REDUCES TO

$$\frac{\Delta P}{\rho} + 2f_F\frac{L}{D}v^2 = 0$$

$$-\frac{\Delta P}{\rho} = \frac{P_1 - P_{ATM}}{\rho_W} = \frac{P_{1G}}{\rho_W} = \frac{40PSI}{\rho_W}$$

$$= \frac{40(144)(32.2)}{62.4} = 2970\ FT^2/S^2$$

$$2f_F\frac{L}{D}v^2 = 2f_F\frac{50}{0.5/12}v^2 = 2400 f_F v^2$$

~ FOR 1/2-IN. DIAM HOSE ~

INTO ENERGY EQN: $f_F v^2 = 1.2375$

TRIAL & ERROR:

ASSUME $f_F = 0.005$ $v = 15.73$ FT/S

$$Re = \frac{\left(\frac{0.5}{12}\right)(15.73)}{1.22\times10^{-5}} = 5.373\times10^4$$

FIG 13.1 — ASSUME SMOOTH — $f_F = 0.0049$

WITH $f_F = 0.0049$ $v = 15.89$ FT/S

$Re = 5.427\times10^4$ — $f_F = 0.0049$

∴ FOR 1/2-IN. HOSE — $v = 15.89$ FT/S

$$\dot{V} = 15.89\left(\frac{\pi}{4}\right)\left(\frac{0.5}{12}\right)^2 = \underline{\underline{0.0217\ FT^3/S}}$$

FOR 3/4-IN DIAM HOSE:

$h_L = 1600 f_F v^2$ ~ $f_F v^2 = 1.856$

— ASSUME $f_F = 0.004$ — $v = 21.54$ FT/S

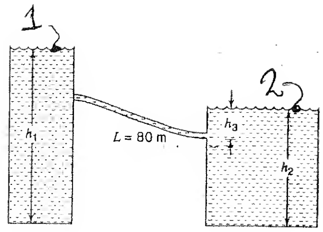
$$Re = \frac{(0.75/12)(21.54)}{1.22\times10^{-5}} = 1.1035\times10^5 \quad f_F = 0.0042$$

WITH $f_F = 0.0042$ $v = 21.02$ FT/S

$Re = 1.077\times10^5$ $f_F = 0.00425$

$v = 20.9$ FT/S $\underline{\dot{V} = 0.0641\ FT^3/S}$

13.15



$h_1 = 60$ m, $h_2 = 30$ m, $h_3 = 8$ m

$L = 80$ m $D = 0.35$ m

ENERGY EQN – BETWEEN 1 & 2

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta y + h_L = 0$$

$$\frac{\Delta P}{\rho} = 0 \quad \frac{\Delta v^2}{2} = 0$$

$$g\Delta y = -(9.81)(30) = -294.3 \text{ m}^2/\text{s}^2$$

$$h_L = 2f_F \frac{L}{D} v^2 + \sum K \frac{v^2}{2}$$

$$= 2(0.004)\left(\frac{80}{0.35}\right)v^2 + 0.5\,v^2$$

$$= 2.329\,v^2$$

INTO ENERGY EQN:

$$2.329\,v^2 = 294.3$$

$$v = 11.24 \text{ m/s}$$

a) $\dot{V} = 11.24\left(\frac{\pi}{4}\right)(0.35)^2 = \underline{\underline{1.082 \text{ m}^3/\text{s}}}$

FOR $e/D = 0.004$

$$h_L = 2f_F \frac{80}{0.35} v^2 = 457 f_F v^2$$

INTO ENERGY EQN:

$457 f_F v^2 = 294.3 \quad f_F v^2 = 0.6438$

TRIAL & ERROR:

ASSUME $f_F = 0.0072$

$v = 9.46$ m/s

13.15 CONTINUED –

$$Re = \frac{(0.35)(9.46)}{0.995 \times 10^{-6}} = 3.328 \times 10^6$$

FULLY TURBULENT – $f_F = 0.0072$

$\therefore v = 9.46$ m/s $\quad \underline{\underline{\dot{V} = 0.910 \text{ m}^3/\text{s}}}$

13.16 ENERGY EQN IS:

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta y + h_L = 0$$

$$\frac{\Delta v^2}{2} = 0$$

$$g\Delta y = (9.81)(-668) = -6553 \text{ m}^2/\text{s}^2$$

$$h_L = 2f_F \frac{L}{D} v^2$$

$\dot{V} = 90$ m^3/s $\quad v = \frac{90}{\frac{\pi}{4}(5)^2} = 4.584$ m/s

$$Re = \frac{5(4.584)}{0.995 \times 10^{-6}} = 2.3 \times 10^7$$

$$e/D \cong \frac{(0.003 \text{ FT})(0.3048)}{5} = 0.00018$$

$$f_F \cong 0.0034$$

$$h_L = 2(0.0034)\frac{(8000)}{5}(4.584)^2 = 228.6 \text{ m}^2/\text{s}^2$$

INTO ENERGY EQN:

$$\frac{\Delta P}{\rho} = 6553 - 228.6 = 6324 \text{ m}^2/\text{s}^2$$

$$\Delta P = 6324\,(1000) = 6324 \times 10^3 \text{ N/m}^2$$

$$\underline{\underline{= 6.324 \text{ MPa}}}$$

13.17 GATE VALVE –

$$\frac{\Delta P}{\rho} = K\frac{v^2}{2}$$

$P_1 = 236$ kPa $P_2 = P_{ATM} = 101.4$ kPa

$\Delta P = 134.6$ kPa $\frac{\Delta P}{\rho} = 134.6\ m^2/s^2$

a) VALVE FULLY OPEN: $K = 0.15$

$$v = \left[\frac{(134.6)2}{0.15}\right]^{1/2} = 42.36 \text{ m/s}$$

$$\dot{V} = (42.36)\left(\frac{\pi}{4}\right)(0.2)^2 = \underline{1.331\ m^3/s}$$

b) VALVE 1/4 CLOSED – $K = 0.85$

$v = 17.8$ m/s $\underline{\underline{\dot{V} = 0.559\ m^3/s}}$

c) VALVE 1/2 CLOSED – $K = 4.4$

$v = 7.82$ m/s $\underline{\underline{\dot{V} = 0.246\ m^3/s}}$

d) VALVE 3/4 CLOSED – $K = 20$

$v = 3.67$ m/s $\underline{\underline{\dot{V} = 0.115\ m^3/s}}$

13.18 $h_L = 2f_F\frac{L}{D}v^2$

$$Re = \frac{Dv}{\nu} = \frac{(0.18)(34)}{0.995\times10^{-6}} = 6.15\times10^6$$

$e/D = 0.0014$ $f_F = 0.0053$

$$h_L = 2(0.0053)\frac{400}{0.18}(34)^2$$

$$= 27230\ m^2/s^2$$

$$= 2776 \text{ m of } H_2O$$

13.19 H_2O @ 15°C $\frac{\Delta P}{\rho} = 0.50$ m

$L = 300$ m $D = 2.20$ m

$\nu = 1.195\times10^{-6}\ m^2/s$

$$h_L = 2f_F\frac{L}{D}v^2$$

13.19 CONTINUED

$$Re = \frac{Dv}{\nu} = \frac{(2.2)(v)}{1.195\times10^{-6}} = 1.841\times10^6 v$$

$$h_L = 9.81(0.5) = 2f_F\frac{300}{2.2}v^2$$

$$f_F v^2 = 0.01799$$

TRIAL & ERROR –

ASSUME TURBULENT FLOW – SMOOTH PIPE

ASSUME $f_F = 0.003$

$v = 2.448$ m/s $Re = 4.508\times10^6$

FIG 13.1 – $f_F = 0.0022$

$v = 2.86$ m/s $Re = 5.26\times10^6$

FIG 13.1 – $f_F \cong 0.0021$

$v = 2.93$ m/s — CLOSE ENOUGH

$$\dot{V} = 2.93\left(\frac{\pi}{4}\right)(2.2)^2 = \underline{\underline{11.13\ m^3/s}}$$

13.20 ENERGY EQUATION:

$$\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta y + h_L = 0$$

$\frac{\Delta P}{\rho} = 0$ (1) = SURFACE OF TANK

(2) = PIPE EXIT

$\frac{\Delta v^2}{2} = \frac{v_2^2}{2}$

$$g\Delta y = 9.81(-16.9) = -165.8\ m^2/s^2$$

$$h_L = 2f_F\frac{L}{D}v^2 = 2f_F\frac{30}{0.6}v^2 = 600f_F v^2$$

INTO ENERGY EQN:

$$v^2\left[600f_F + 0.5\right] = 165.8$$

TRIAL & ERROR –

0.6-m CAST IRON PIPE – $e/D = 0.00045$

13.20 CONTINUED -

$$Re = \frac{0.6\,U}{0.995\times10^{-6}} = 6.03\times10^{5}\,U$$

ASSUME $f_F = 0.003$ $U = 6.36$ m/s

$Re = 3.83\times10^{6}$ $f_F = 0.0041$

THIS IS IN FULLY TURBULENT REGION

$\therefore$ f_F IS 0.0041 & $U = 7.48$ m/s

$$\dot{V} = (7.48)\left(\frac{\pi}{4}\right)(0.6)^2 = \underline{2.116\ m^3/s}$$

13.21 $D = 0.15$ m $L = 100$ m

20°C H_2O - $\nu = 0.995\times10^{-6}\,m^2/s$

$\Delta P = 30$ kPa $\sim \frac{\Delta P}{\rho} = 30\ m^2/s^2$

WROUGHT IRON PIPE $\frac{\varepsilon}{D} = 0.00035$

$$Re = \frac{(0.15)\,U}{0.995\times10^{-6}} = 1.507\times10^{5}\,U$$

ENERGY EQN: $\frac{\Delta P}{\rho} + h_L = 0$

$$2f_F\frac{100}{0.15}U^2 = 30 \qquad f_F U^2 = 0.0225$$

TRIAL & ERROR -

ASSUME $f_F = 0.004$ $U = 2.37$ m/s

$Re = 3.574\times10^{5}$ $f_F \cong 0.0042$

@ $f_F = 0.0042$ $U = 2.31$ m/s

$Re = 3.488\times10^{5}$ $f_F = 0.0042$

$$\dot{V} = (2.31)\left(\frac{\pi}{4}\right)(0.15)^2 = 0.0408\ m^3/s$$

13.22 $\Delta P = 1.3$ m H_2O $L = 10$ m

$D = 0.2$ m $e = 0.0004$ m

ASSUME 20° - $\nu = 0.995\times10^{-6}\,m^2/s$

$$\frac{\Delta P}{\rho} = 1.3(9.81) = 12.75\ m^2/s^2$$

13.22 CONTINUED -

ENERGY EQN - $\frac{\Delta P}{\rho} = 2f_F\frac{L}{D}U^2$

$$12.75 = 2f_F\frac{10}{0.2}U^2 = 100 f_F U^2$$

$$f_F U^2 = 0.1275$$

$$Re = \frac{0.2\,U}{0.995\times10^{-6}} = 2.01\times10^{5}\,U$$

ASSUME SMOOTH PIPE -

IF $f_F = 0.004$ - $U = 5.646$ m/s

$Re = 1.135\times10^{6}$ $f_F \cong 0.002565$

@ $f_F = 0.003$ $U = 6.52$ m/s

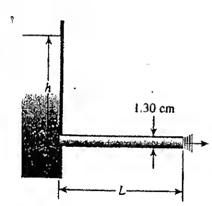
$Re = 1.31\times10^{6}$ $f_F \cong 0.0027$

@ $f_F = 0.0027$ $U = 6.87$ m/s

$Re = 1.381\times10^{6}$ $f_F \cong 0.0027$

$$\dot{m} = (6.87)\left(\frac{\pi}{4}\right)(0.2)^2(1000) = \underline{\underline{0.216\ kg/s}}$$

13.23



$\dot{V} = 5.675\times10^{-4}\ m^3/s$

$L = 20$ m

PIPE IS COM. STEEL

$e/D = 2.446\times10^{-5}$

$$\frac{\Delta P}{\rho} = h_L + \Sigma K U^2/2$$

$$= 2f_F\frac{L}{D}U^2 + 0.5\,U^2 \quad \left\{\begin{array}{c}\text{FOR ENTRANCE}\\ K=1\end{array}\right\}$$

$$U = \frac{5.675\times10^{-4}}{\pi/4\,(0.013)^2} = 4.275\ m/s$$

$$Re = \frac{(0.013)(4.275)}{0.995\times10^{-6}} = 55900$$

FIG 13.1 - $f_F \cong 0.0049$

13.23 CONTINUED

$$\frac{\Delta P}{\rho} = v^2\left[2(0.0049)\left(\frac{20}{0.013}\right) + 0.5\right]$$

$$= 284.7\ m^2/s^2$$

$$h = \frac{284.7}{g} = \underline{\underline{29.02\ m}}$$

13.24 $\dot{V} = 0.25\ m^3/s$

PIPE 1: $v = \dfrac{0.25}{\frac{\pi}{4}(0.16)^2} = 12.43\ m/s$

PIPE 2 $v = \dfrac{0.25}{\frac{\pi}{4}(0.18)^2} = 9.82\ m/s$

PIPE 3 $v = \dfrac{0.25}{\pi/4(0.2)^2} = 7.96\ m/s$

—

PIPE 1 — $\dfrac{\Delta P}{\rho} = 2 f_F \dfrac{L}{D} v^2$

$Re = \dfrac{0.16(12.43)}{0.995 \times 10^{-6}} = 1.998 \times 10^6$

$e/D = 0.0055$ — $f_F = 0.0079$

$$\frac{\Delta P}{\rho g} = \frac{2(0.0079)(900)(12.43)^2}{0.16(9.81)} = \underline{\underline{1400\ m}}$$

PIPE 2 —

$Re = \dfrac{0.18(9.82)}{0.995 \times 10^{-6}} = 1.776 \times 10^6$

$e/D = 0.005$ $f_F = 0.0075$

$$\frac{\Delta P}{\rho g} = \frac{2(0.0075)(1500)(9.82)^2}{0.18(9.81)} = \underline{\underline{1241\ m}}$$

PIPE 3 —

$Re = \dfrac{0.2(7.96)}{0.995 \times 10^{-6}} = 1.6 \times 10^6$

$e/D \cong 0.0045$ $f_F \cong 0.0073$

$$\frac{\Delta P}{\rho g} = \frac{2(0.0073)(800)(7.96)^2}{0.2(9.81)} = \underline{\underline{377\ m}}$$

13.25

Pipe	Length, m	Diameter, cm	Roughness, mm
1	125	8	0.240
2	150	6	0.120
3	100	4	0.200

PIPES IN SERIES — H_2O @ 20°C — $\nu = 0.995 \times 10^{-6}\ m^2/s$

$v_1 = \dot{V} / \frac{\pi D_1^2}{4} = 199\,\dot{V}$

$v_2 = \cdots = 354\,\dot{V}$

$v_3 = \cdots = 796\,\dot{V}$

$\dfrac{\Delta P}{\rho} = h_{L1} + h_{L2} + h_{L3} + 5(9.81)$

$h_{L1} = 2 f_{F1} \dfrac{125}{0.08}(199\dot{V})^2 = 1.238 \times 10^8\ f_{F1}\dot{V}^2$

$h_{L2} = 2 f_{F2} \dfrac{150}{0.06}(354\dot{V})^2 = 6.266 \times 10^8\ f_{F2}\dot{V}^2$

$h_{L3} = 2 f_{F3} \dfrac{100}{0.04}(796\dot{V})^2 = 3.176 \times 10^9\ f_{F3}\dot{V}^2$

Pipe 1 — $e/D = \dfrac{0.24}{80} = 0.003$

ASSUME FULLY TURBULENT — $f_{F1} = 0.0065$

PIPE 2 — $e/D = \dfrac{0.12}{60} = 0.002$

~ SAME ASSUMPTION — $f_{F2} = 0.00585$

PIPE 3 $e/D = \dfrac{0.20}{40} = 0.005$

~ SAME ASSUMPTION $f_{F3} = 0.0077$

$$\Sigma h_L = \dot{V}^2\left[8.047 \times 10^5 + 36.66 \times 10^5 + 244.55 \times 10^5\right]$$

$$= 289.3 \times 10^5\ \dot{V}^2$$

$$\Sigma h_L = \frac{\Delta P}{\rho} + g\Delta y = 180 + 96.27 = 276.3\ m^2/s^2$$

SOLVING — $\dot{V} = 0.00309\ m^3/s$

$v_1 = 0.615\ m/s$ $Re_1 = 4.94 \times 10^4$ $f_{F1} = 0.0071$

$v_2 = 1.094$ " $Re_2 = 6.60 \times 10^4$ $f_{F2} = 0.0065$

$v_3 = 2.460$ " $Re_3 = 9.89 \times 10^4$ $f_{F3} = 0.0077$

13.25 CONTINUED —

USING NEW VALUES FOR f_{F_i} —

$$\sum h_L = \left[\left((8.79 + 40.73 + 245)\times 10^5\right]\dot{V}^2\right.$$

$$= 294.5\times 10^5\,\dot{V}^2 = 276.3$$

$$\underline{\underline{\dot{V} = 0.00306\ m^3/s}}$$

13.26 CONCRETE PIPES IN SERIES

H_2O @ $20°C$ — $\dot{V} = 0.18\ m^3/s$

$h_{L1} + h_{L2} = 18\ m = 176.6\ m^2/s^2$

FOR PIPE 1 — $h_{L1} = 2 f_F \frac{L}{D} v_1^2$

$$v_1 = \frac{0.18}{\left(\frac{\pi}{4}\right)(0.3)^2} = 2.55\ m/s$$

$$Re = \frac{(0.3)(2.55)}{0.995\times 10^{-6}} = 7.678\times 10^5$$

$$e/D = \frac{0.0035}{0.3} = 0.00117$$

FIG 13.1 — $f_F \cong 0.0051$

$$\frac{\Delta P}{\rho} = h_L = 2(0.0051)\left(\frac{312.5}{0.3}\right)(2.55)^2$$

$$= 69.09\ m^2/s^2$$

THIS REQUIRES h_L FOR PIPE 2

TO BE $176.6 - 69.09 = 107.5\ m^2/s^2$

$$107.5 = 2 f_F \frac{312.5}{D} v^2$$

$$\frac{f_F v^2}{D} = 0.172$$

& $v = \frac{\dot{V}}{\pi D^2/4} = \frac{0.18}{\pi D^2/4} = \frac{0.2292}{D^2}$

$\therefore\ \frac{f_F}{D^5} = 3.275$

13.26 CONTINUED —

$$Re = \frac{Dv}{\nu} = \frac{D(0.2292)}{D^2(0.995\times 10^{-6})}$$

$$= \frac{2.304\times 10^5}{D}$$

TRIAL & ERROR —

ASSUME $f_F = 0.006$ — $D = 0.2835\ m$

$e/D = 0.0123$ $Re = 8.127\times 10^5$

FIG 13.1 — $f_F = 0.01$

$D = 0.314\ m$ $e/D = 0.0111$

$Re = 7.338\times 10^5$ $f_F = 0.01$

$\sim$ $\underline{\underline{D = 0.314\ m}}$

13.27 2 PIPES IN PARALLEL:

PIPE 1 — $D = 0.2\ m$ $L = 150\ m$

CAST IRON: $e/D = 0.0013$

PIPE 2 — $D = 0.067\ m$ $L = 150\ m$

STEEL — $e/D = 0.0007$

$\Delta P = 210\ kPa$ $\frac{\Delta P}{\rho} = 210\ m^2/s^2$

PIPE 1: $\frac{\Delta P}{\rho} = 2 f_F \frac{L}{D} v^2$

ASSUME FULLY TURBULENT — $f_F \cong 0.0055$

$210 = 2(0.0055)\frac{150}{0.2} v^2$ — $v = 5.045\ m/s$

$Re = \frac{0.2(5.045)}{0.995\times 10^{-6}} = 1.014\times 10^6$ —

THIS CONFIRMS FULLY TURBULENT &

$\underline{v_1 = 5.045\ m/s}$

PIPE 2: AGAIN ASSUME FULLY TURBULENT

$f_{F_2} = 0.0045$ $\sim$ $v_2 = 3.228\ m/s$

13.27 CONTINUED—

$$Re_2 = \frac{0.067(3.228)}{0.995\times10^{-6}} = 2.173\times10^5$$

FIG 13.1: REVISED $f_F \cong 0.0049$

WITH THIS VALUE $v_2 = 3.094$ m/s

$Re = 2.083\times10^5 \quad f_F \cong 0.0049$

$\therefore$ $\underline{v_2 = 3.049 \text{ m/s}}$

$$\dot{V} = 5.045\left(\frac{\pi}{4}\right)(0.2)^2 + 3.049\left(\frac{\pi}{4}\right)(0.067)^2$$

$$= 0.1585 \text{ ft}^3/\text{s} + 0.0107 \text{ ft}^3/\text{s}$$

$\underline{\underline{\dot{V}_1 = 0.1585 \text{ ft}^3/\text{s}}}$ $\quad$ $\underline{\underline{\dot{V}_2 = 0.0107 \text{ ft}^3/\text{s}}}$

13.28 3 PIPES IN PARALLEL

Pipe	Length, m	Diameter, cm	Roughness, mm
1	100	8	0.240
2	150	6	0.120
3	80	4	0.200

TOTAL $h_L = 24 \text{ m} = 235.4 \text{ m}^2/\text{s}^2$

PIPE 1 — $2f_{F1}\frac{L}{D}v_1^2 = 2f_{F1}\frac{100}{0.08}v^2$

$\sim f_{F1}v_1^2 = 0.0824$

$$Re_1 = \frac{0.08\, v_1}{0.995\times10^{-6}} = 8.04\times10^5\, v_1$$

TRIAL & ERROR—

$e/D = 0.24/100 = 0.0024$

ASSUME FULLY TURBULENT—

$f_F = 0.0063$ — $v_1 = 3.617$ m/s

$$Re = \frac{(0.08)(3.617)}{0.995\times10^{-6}} = 2.91\times10^5$$

$f_F = 0.0062$ —

REVISED VALUE — $\underline{v_1 = 3.65 \text{ m/s}}$

13.28 CONTINUED

PIPE 2 $\quad 235.4 = 2f_{F2}\frac{150}{0.06}v_2^2$

$f_{F2}v_2^2 = 0.0412$

$$Re_2 = \frac{(0.06)v_2}{0.995\times10^{-6}} = 6.03\times10^4\, v_2$$

$e/D|_2 = 0.002$ — ASSUME $f_{F2} = 0.006$

$v_2 = 2.62$ m/s $\quad Re = 1.58\times10^5$

$\sim f_F = 0.0061$

REVISED VALUE FOR v_2: $\underline{v_2 = 2.60 \text{ m/s}}$

PIPE 3: $\quad 235.4 = 2f_{F3}\frac{80}{0.04}v_3^2$

$f_{F3}v_3^2 = 0.059$

$$Re = \frac{(0.04)v_3}{0.995\times10^{-6}} = 4.02\times10^4\, v_3$$

$e/D = 0.005$ — ASSUME $f_{F3} = 0.008$

$v_3 = 2.716$ m/s $\quad Re = 1.092\times10^5$

$f_F = 0.0077$

~ REVISED VALUE: $\underline{v_3 = 2.77 \text{ m/s}}$

TOTAL SYSTEM FLOW RATE:

$$\dot{V} = 3.65\left(\frac{\pi}{4}\right)(0.08)^2 + (2.60)(\pi/4)(0.06)^2 + 2.77\left(\frac{\pi}{4}\right)(0.04)^2$$

$$= \underline{\underline{0.0292 \text{ ft}^3/\text{s}}}$$

CHAPTER 14

14.1 CENTRIFUGAL PUMP:

$\dot{V} = 0.2\ m^3/s \qquad \omega = 850\ rpm$

$r_2 = 0.225\ m \qquad \rho = 1000\ kg/m^3$

$L = 0.05\ m$

$\beta_2 = 24°$

TORQUE – EQN. 14.9

$$M_z = \rho \dot{V} r_2 \left[r_2\omega - \frac{\dot{V}}{2\pi r_2 L} \cot\beta_2 \right]$$

$$\omega = 850\left(\frac{2\pi}{60}\right) = 89.0\ rad/s$$

$$M_z = (1000)(0.2)(0.225) \times \left[(0.225)(89) - \frac{0.2\cot 24}{2\pi(0.225)(0.05)}\right]$$

$$= \underline{615\ N\cdot m} \qquad a)$$

$$\dot{W} = M_z\omega = 615(89)$$

$$= \underline{54.75\ kW} \qquad a)$$

$$\left.\frac{\Delta P}{\rho}\right|_{MAX} = -\frac{\dot{W}}{\dot{m}} = -\frac{\dot{W}}{\rho\dot{V}}$$

$$\Delta P_{MAX} = -\frac{54.75\times10^3\ N\cdot m/s}{0.2\ m^3/s}$$

$$= \underline{-274\ kPa} \qquad b)$$

14.2 CENTRIFUGAL PUMP:

$\rho = 680\ kg/m^3 \qquad r_1 = 0.075\ m$

$L = 0.09\ m \qquad r_2 = 0.14\ m$

$\beta_1 = 25° \qquad \beta_2 = 40°$

$\omega = (1200)\left(\frac{2\pi}{60}\right) = 125.7\ RAD/s$

14.2 CONTINUED

$$\dot{V} = 2\pi r_1^2 L\omega \tan\beta_1$$

$$= 2\pi(0.075)^2(0.09)(125.7)\tan 25°$$

$$= \underline{0.186\ m^3/s} \qquad a)$$

$$\dot{W} = M\omega = \rho\dot{V}r_2\omega\left[r_2\omega - \frac{\dot{V}\cot\beta_2}{2\pi r_2 L}\right]$$

$$= (680)(0.186)(0.14)(125.7)\times \left[(0.14)(125.7) - \frac{0.186\cot 40°}{2\pi(0.14)(0.09)}\right]$$

$$= \underline{32.94\ kW} \qquad b)$$

$$\left.\frac{\Delta P}{\rho g}\right|_{MAX} = \frac{\dot{W}}{\rho g\dot{V}}$$

$$= \frac{32.94\times10^3}{680(9.81)(0.186)}$$

$$= \underline{26.5\ m} \qquad c)$$

14.3 CENTRIFUGAL PUMP –

$r_2 = 0.21\ m \qquad L = 0.05\ m \qquad \beta_2 = 33°$

$\omega = 1200\left(\frac{2\pi}{60}\right) = 125.7\ RAD/s$

$$\frac{\Delta P}{\rho g} = 52\ m\ H_2O$$

$$\dot{W} = \rho\dot{V}r_2\omega\left[r_2\omega - \frac{\dot{V}\cot\beta_2}{2\pi r_2 L}\right]$$

$$= \frac{\dot{m}\Delta P}{\rho} = \dot{V}\Delta P$$

EQUATING:

$$\Delta P = \rho r_2\omega\left[r_2\omega - \frac{\dot{V}\cot\beta_2}{2\pi r_2 L}\right]$$

14.3 CONTINUED

$$\Delta P = 52(1000)(9.81) = 490 \text{ kPa}$$

$$= (1000)(0.21)(125.7) \times \left[(0.21)(125.7) - \frac{\dot{V}\cot 33^\circ}{2\pi(0.21)(0.05)}\right]$$

$$= 26400\left[26.4 - 23.34\dot{V}\right]$$

EQUATING:

$$18.56 = 26.4 - 23.34\dot{V}$$

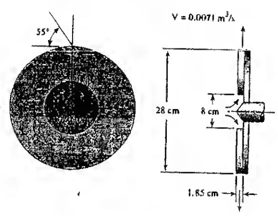
$$\underline{\underline{\dot{V} = 0.336 \text{ m}^3/\text{s}}} \qquad a)$$

$$\dot{W} = \dot{V}\Delta P$$

$$= 0.336(490 \times 10^3)$$

$$= \underline{\underline{164.6 \text{ kW}}} \qquad b)$$

14.4



Pump DEPICTED

$\omega = 1020 \text{ rpm}$

$= 106.8 \text{ rad/s}$

$r_1 = 0.04 \text{ m}$ $\quad \dot{V} = 0.0071 \text{ m}^3/\text{s}$

$r_2 = 0.14 \text{ m}$ $\quad \beta_2 = 55^\circ$

$L = 0.0185 \text{ m}$ $\quad \rho = 1000 \text{ kg/m}^3$

$$\dot{W} = \rho\dot{V}r_2\omega\left[r_2\omega - \frac{\dot{V}\cot\beta_2}{2\pi r_2 L}\right]$$

$$= (1000)(0.0071)(0.14)(106.8) \times \left[(0.14)(106.8) - \frac{0.0071\cot 55^\circ}{2\pi(0.14)(0.0185)}\right]$$

$$= 1555 \text{ W} = \underline{\underline{1.555 \text{ kW}}}$$

14.5 CENTRIFUGAL PUMP -

$\rho = 1000 \text{ kg/m}^3$ $\quad \dot{V} = 0.018 \text{ m}^3/\text{s}$

$\dot{W} = 4.5 \text{ kW}$ $\quad \eta = 63\%$

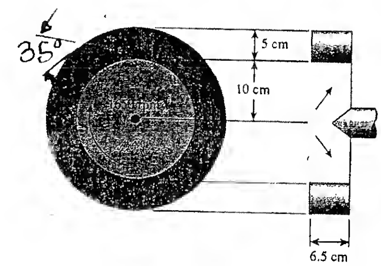
$$\eta = \frac{\dot{m}\Delta P}{\rho\dot{W}}$$

$$\Delta P = \frac{\eta\rho\dot{W}}{\dot{m}} = \eta\frac{\dot{W}}{\dot{V}}$$

$$= \frac{0.63(4500)}{0.018} = 157.5 \text{ kPa}$$

$$\frac{\Delta P}{\rho g} = \frac{157500}{(1000)(9.81)} = \underline{\underline{16.05 \text{ m } H_2O}}$$

14.6



Pump DEPICTED

$\dot{V} = 0.032 \text{ m}^3/\text{s}$

$\rho = 680 \text{ kg/m}^3$

$\omega = 1650 \text{ rpm}$

$= 172.8 \text{ rad/s}$

$$\dot{W} = \rho\dot{V}r_2\omega\left[r_2\omega - \frac{\dot{V}\cot\beta_2}{2\pi r_2 L}\right]$$

$$= (680)(0.032)(0.15)(172.8) \times \left[(0.15)(172.8) - \frac{(0.032)\cot 35^\circ}{2\pi(0.15)(0.065)}\right]$$

$$= 14.2 \text{ kW} = \underline{\underline{19.0 \text{ Hp}}} \qquad a)$$

$$\Delta P = \dot{W}/\dot{V} = \frac{14200}{0.032} = 444 \text{ kPa}$$

$$= \frac{444000}{(680)(9.81)} = \underline{\underline{66.5 \text{ m}}} \qquad b)$$

14.6 CONTINUED

$$\dot{V} = 2\pi r_1^2 L\omega \tan\beta_1$$

$$\tan\beta_1 = \frac{0.032}{2\pi(0.10)^2(0.065)(172.8)}$$

$$= 0.0453$$

$$\underline{\beta_1 = 2.6^\circ} \qquad c)$$

14.7 CENTRIFUGAL PUMP

$\rho = 1000\ kg/m^3$ $\qquad r_1 = 0.12\,m$

$\beta_1 = 32^\circ$ $\qquad r_2 = 0.20\,m$

$\beta_2 = 20^\circ$ $\qquad L = 0.042\,m$

$\omega = 1500\ rpm = 157.1\ rad/s$

$$\dot{V} = 2\pi r_1^2 L\omega \tan\beta_1$$

$$= 2\pi(0.12)^2(0.042)(157.1)\tan 32^\circ$$

$$= \underline{0.373\ m^3/s} \qquad (a)$$

$$\dot{W} = \rho\dot{V}r_2\omega\left[r_2\omega - \frac{\dot{V}\cot\beta_2}{2\pi r_2 L}\right]$$

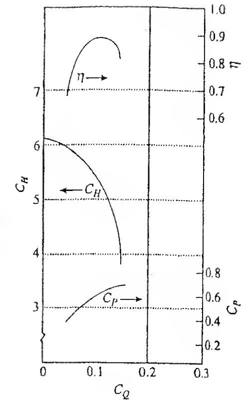
$$= (1000)(0.373)(0.2)(157.1)\times\left[(0.2)(157.1) - \frac{0.373\cot 20^\circ}{2\pi(0.2)(0.042)}\right]$$

$$= 140.7\ kW = \underline{189\ Hp} \qquad (b)$$

$$\Delta p = \frac{\dot{W}}{\dot{V}} = \frac{140.7}{0.373} = 377\ kPa$$

$$\frac{\Delta p}{\rho g} = \frac{377\times10^3}{(1000)(9.81)} = \underline{38.5\ m\ H_2O}$$

14.8 H_2O @ $15^\circ C$



$\rho = 999\ kg/m^3$

$D = 0.45\,m$

$$\omega = 1600\left(\frac{2\pi}{60}\right) = 167.6\ Rad/s$$

@ η MAX –

$C_Q \simeq 0.012$

$C_H \simeq 0.0515$

$C_P \simeq 0.0068$

$\eta \simeq 0.89$

$$C_H = \frac{gh}{n^2D^2} \sim$$

$$h = \frac{(0.0515)(167.6)^2(0.45)^2}{9.81} = \underline{29.9\ m} \quad (a)$$

$$C_Q = \dot{V}/nD^3$$

$$\dot{V} = 0.012(167.6)(0.45)^3 = \underline{0.183\ m^3/s} \quad (b)$$

$$\Delta p = \rho gh = (999)(9.81)(29.9) = \underline{293\ kPa} \quad (c)$$

$$C_P = \frac{\dot{W}}{\rho n^3 D^5}$$

$$\dot{W} = (0.0068)(999)(167.6)^3(0.45)^5$$

$$= 587\ kW$$

$$BHp = \frac{587\times10^3}{0.89} = 659.6\ kW$$

$$= \underline{884\ Hp} \qquad (d)$$

14.9 CENTRIFUGAL PUMP WITH SAME CHARACTERISTICS AS IN PROB 14.8

$\dot{V} = 0.2\,m^3/s$

$$\omega = 1400\left(\frac{2\pi}{60}\right) = 146.6\ Rad/s$$

$\rho = 1000\ kg/m^3$

14.9 CONTINUED –

AT η MAX: $C_Q \cong 0.012$

$C_H \cong 0.0515$

$C_P \cong 0.0068$

$C_Q = \dfrac{\dot{V}}{nD^3}$ $D^3 = \dfrac{0.2}{(146.6)(0.012)}$

$\underline{\underline{D = 0.485 \text{ m}}}$ (a)

$C_H = \dfrac{gh}{n^2D^2}$ $h = \dfrac{(0.0515)(146.6)^2(0.485)^2}{9.81}$

$= 26.5 \text{ m}$

$P_{MAX} = \rho g h = (1000)(9.81)(26.5)$

$\underline{\underline{= 260 \text{ kPa}}}$ (b)

14.10 SAME PUMP FAMILY AS IN PROB 14.8 BUT:

$D = 0.4 \text{ m}$

$\omega = 2200\left(\dfrac{2\pi}{60}\right) = 230.4 \text{ Rad/s}$

$\rho = 999 \text{ kg/m}^3$

@ $\eta_{MAX} \cong 0.89$

$C_Q \cong 0.012$

$C_H \cong 0.0515$

$C_P \cong 0.0068$

$C_H = \dfrac{gh}{n^2d^2}$

$h = \dfrac{(0.0515)(230.4)^2(0.4)^2}{9.81}$

$\underline{\underline{= 44.6 \text{ m } H_2O}}$ a)

14.10 CONTINUED

$C_Q = \dfrac{\dot{V}}{nD^3}$ $\dot{V} = (0.012)(230.4)(0.4)^3$

$\underline{\underline{= 0.177 \text{ m}^3/\text{s}}}$ (b)

$\Delta P = \rho g h = (999)(9.81)(44.6)$

$\underline{\underline{= 437 \text{ kPa}}}$ (c)

$C_P = \dfrac{\dot{W}}{\rho n^3 D^5}$

$\dot{W} = (0.0068)(999)(230.4)^3(0.4)^5$

$= 850.8 \text{ kW}$

$BHp = \dfrac{850.8}{(0.89)(0.746)} = \underline{\underline{1280 \text{ Hp}}}$ (d)

14.11 SAME PUMP FAMILY AS IN PROB 14.8

$D = 0.35 \text{ m}$ $\omega = 2400\left(\dfrac{2\pi}{60}\right) = 251.3 \text{ rad/s}$

$\rho = 999 \text{ kg/m}^3$

$\eta_{MAX} = 0.89$

$C_Q \cong 0.012$

$C_H \cong 0.0515$

$C_P \cong 0.0068$

$C_H = \dfrac{gh}{n^2D^2}$ $h = \dfrac{(0.0515)(251.3)^2(0.35)^2}{9.81}$

$\underline{\underline{= 40.61 \text{ m } H_2O}}$ (a)

$C_Q = \dfrac{\dot{V}}{nD^3}$ $\dot{V} = (0.012)(251.3)(0.35)^3$

$\underline{\underline{= 0.129 \text{ m}^3/\text{s}}}$ (b)

$\Delta P = (999)(9.81)(44.6)$

$\underline{\underline{= 437 \text{ kPa}}}$ (c)

14.11 CONTINUED –

$$C_p = \frac{\dot{W}}{\rho n^3 D^5}$$

$$\dot{W} = (0.0068)(999)(251.3)^3(0.35)^5$$

$$= 566 \text{ kW}$$

$$BHp = \frac{566}{(0.89)(0.746)} = 853 \text{ Hp} \quad (d)$$

14.12 – SAME PUMP FAMILY AS IN PROB 14.8

$$\dot{V} = 0.30 \text{ m}^3/\text{s} \quad n = 1800\left(\frac{2\pi}{60}\right) = 188.5 \text{ r/s}$$

$$@\ \eta_{MAX} = 0.89 \quad C_Q \cong 0.12$$

$$C_H \cong 0.0515$$

$$C_p \cong 0.0068$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad D = \left[\frac{0.30}{(188.5)(0.12)}\right]^{1/3}$$

$$D = \underline{\underline{0.430 \text{ m}}} \quad (a)$$

$$C_H = \frac{gh}{n^2 D^2}$$

$$h = \frac{(0.0515)(188.5)^2(0.43)^2}{9.81}$$

$$= 34.49 \text{ m } H_2O$$

$$\Delta p = \rho g h = (1000)(9.81)(34.49)$$

$$= \underline{\underline{338 \text{ kPa}}} \quad (b)$$

14.13 – SAME PUMP FAMILY AS IN PROB 14.8

$$\dot{V} = 0.201 \text{ m}^3/\text{s} \quad \omega = (1800)(2\pi/60)$$

$$= 188.5 \text{ r/s}$$

14.13 – CONTINUED

$$@\ \eta_{MAX} = 0.89 \quad C_Q \cong 0.12$$

$$C_H \cong 0.0515$$

$$C_p \cong 0.0068$$

$$C_Q = \frac{\dot{V}}{nD^3} \quad D = \left[\frac{0.201}{(188.5)(0.12)}\right]^{1/3}$$

$$= \underline{\underline{0.207 \text{ m}}} \quad (a)$$

$$C_H = \frac{gh}{n^2 D^2} \quad h = \frac{(0.0515)(188.5)^2(0.207)^2}{9.81}$$

$$= 7.99 \text{ m } H_2O$$

$$\Delta p = \rho g h = (1000)(9.81)(7.99)$$

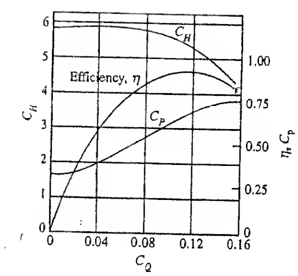
$$= \underline{\underline{78.4 \text{ kPa}}} \quad (b)$$

14.14

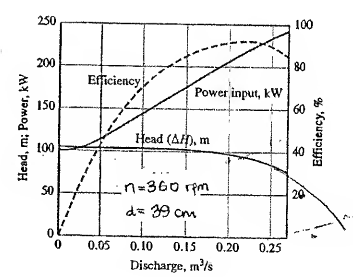
$$@\ \eta_{MAX} \cong 0.88$$

$$C_Q \cong 0.12$$

$$C_H \cong 5.3$$



$$h = 90 \text{ m } H_2O$$



$$C_H = \frac{gh}{n^2 D^2} = 5.3$$

$$\omega^2 = \frac{gh}{C_H D^2}$$

$$= \frac{9.81(90)}{5.3(0.39)^2}$$

$$\omega = 33.1 \text{ rad/s} = \underline{\underline{316 \text{ rpm}}} \quad (a)$$

$$C_Q = 0.12 = \frac{\dot{V}}{nD^3}$$

$$\dot{V} = (0.12)(33.1)(0.39)^3 = \underline{\underline{0.236 \text{ m}^3/\text{s}}}$$

(b)

14.15 SAME PUMP FAMILY AS IN 14.14

NEW PUMP: $n = 400\text{ rpm}$
$= 41.89\text{ rad/s}$

$D_{NEW} = 6\,D_{OLD}$

AT η_{MAX} — $C_Q \cong 0.12 = \dot{V}/nD^3$

$C_H \cong 5.3 = gh/n^2D^2$

$C_P \cong 0.70 = P/\rho n^3 D^5$

$$P_1 = 0.70(1000)(37.70)^3(0.371)^5$$
$$= 263.6\text{ kW}$$

$$P_{NEW} = P_1\left(\frac{\omega_2}{\omega_1}\right)^3\left(\frac{D_2}{D_1}\right)^5$$
$$= 263.6\left(\frac{400}{360}\right)^3(6)^5$$
$$= \underline{2.81\text{ MW}} \qquad \text{(a)}$$

$$h_1 = \frac{5.3(37.70)^2(0.371)^2}{9.81} = 105.7\text{ m}$$

$$h_2 = h_1\left(\frac{\omega_2}{\omega_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$
$$= 105.7\left(\frac{400}{360}\right)^2(6)^2$$
$$= \underline{4.7\text{ km}} \qquad \text{(b)}$$

$$\dot{V}_1 = 0.12\,n_1 D_1^3$$
$$= 0.12(37.70)(0.371)^3$$
$$= 0.231\text{ m}^3/\text{s}$$

$$\dot{V}_2 = \dot{V}_1\left(\frac{n_2}{n_1}\right)\left(\frac{D_2}{D_1}\right)^3$$
$$= 0.231\left(\frac{400}{360}\right)(6)^3$$
$$= \underline{55.4\text{ m}^3/\text{s}} \qquad \text{(c)}$$

14.16 SAME PUMP FAMILY AS IN PROB 14.14

NEW $n = 1000\text{ rpm}$

$C_Q \cong 0.12 = \dot{V}/nD^3 \qquad C_P \cong 0.7 = P/\rho n^3 D^5$

$$\dot{V} = 0.12\left(1000 \times \frac{2\pi}{60}\right)(0.371)^3$$
$$= \underline{0.642\text{ m}^3/\text{s}} \qquad \text{(a)}$$

$$P = 0.7(1000)\left(1000 \times \frac{2\pi}{60}\right)^3(0.371)^5$$
$$= \underline{5.65\text{ MW}} \qquad \text{(b)}$$

14.17 SAME PUMP FAMILY AS IN PROB 14.14

NEW $\omega = 800\text{ rpm} = 83.8\text{ rad/s}$

$h = 410\text{ m}$

$$C_H = \frac{gh}{n^2D^2} = \frac{9.81(410)}{(83.8)^2(0.371)^2} = 4.161$$

AT THIS VALUE OF C_H, $C_Q \cong 0.16$

$C_Q = 0.16 = \dot{V}/nD^3$

$$\dot{V} = 0.16(83.8)(0.371)^3$$
$$= \underline{0.685\text{ m}^3/\text{s}}$$

14.18 SAME PUMP FAMILY AS PROB 14.14

$D_2 = 3D_1 \quad , \quad n_2 = 0.5\,n_1$

@ η_{MAX} $\quad C_Q \cong 0.12 = \dot{V}/nD^3$

$C_H \cong 5.3 = gh/n^2D^2$

$$\frac{\dot{V}_2}{\dot{V}_1} = \left(\frac{n_2}{n_1}\right)\left(\frac{D_2}{D_1}\right)^3 = \frac{1}{2}(3)^2 = 13.5$$

$$\frac{h_2}{h_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2 = \left(\frac{1}{2}\right)^2(3)^2 = 2.25$$

14.18 CONTINUED

$$\dot{V}_1 = 0.12(37.7)(0.371)^3 = 0.231 \text{ m}^3/\text{s}$$

$$\dot{V}_2 = 0.231(13.5) = \underline{\underline{3.12 \text{ m}^3/\text{s}}}$$

$$h_1 = \frac{5.3(37.7)^2(0.371)^2}{9.81} = 105.7 \text{ m}$$

$$h_2 = (105.7)(2.25) = \underline{\underline{238 \text{ m}}}$$

14.19 PUMP PERFORMANCE AS IN PROB 14.14 - $\Delta y = 95$ m

H_2O PUMP - $D = 0.28$ m

$L = 550$ m

$e = 0.457 \times 10^{-4}$ m

$e/D = 0.000163$

ENERGY EQN:

$$-\dot{W}_s = \dot{m}\left[\cancel{\frac{\Delta P}{\rho}} + \cancel{\frac{\Delta V^2}{2}} + g\Delta y + h_L\right]$$

$\qquad 0 \qquad 0$

$$h_L = 2 f_F \frac{L}{D} v^2$$

- ASSUME FULLY TURBULENT

$f_F \cong 0.0033$

$$h_L = 2(0.0033)\left(\frac{550}{0.28}\right)v^2$$

$$= 12.96\, v^2$$

1ST LAW EXPRESSION BECOMES -

$$-\dot{W} = \dot{m}\left[90g + 12.96 v^2\right]$$

SYSTEM HEAD -

$$-\frac{\dot{W}}{\dot{m}g} = h = 90 + 1.32\, v^2 \qquad (1)$$

THIS MUST MATCH PUMP PERFORMANCE -

14.19 CONTINUED -

SYSTEM PERFORMANCE - EQN (1)

$\dot{V}$	h
0.10	93.48
0.15	95.93
0.20	100.54
0.25	106.5

$$\dot{V} = \frac{\pi}{4} D^2 v = 0.0616\, v$$

SYSTEM & PUMP PERFORMANCE INTERSECT AT $\dot{V} \cong 0.21 \text{ m}^3/\text{s}$ — $v = 3.41$

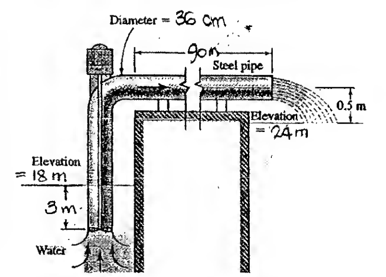
$$Re = \frac{(0.28)(3.41)}{0.995 \times 10^{-6}} = 9.59 \times 10^5$$

$f_F = 0.0035$ ~ CLOSE ENOUGH

SO: WITHIN ACCURACY OF READING PLOTS

$$\underline{\underline{\dot{V} = 0.21 \text{ m}^3/\text{s}}}$$

14.20



FOR STEEL

$e = 0.457 \times 10^{-4}$ m

$e/D = 0.000127$

FOR FULLY-TURBULENT FLOW $f_F \cong 0.0031$

ENERGY EQN: $-\dot{W} = \dot{m}\left[\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g\Delta y + h_L\right]$

BETWEEN RESERVOIR SURFACE (1) & DISCHARGE (2) -

$$\frac{\Delta P}{\rho} = V_1^2 = 0$$

$$g\Delta y = 6.5g$$

$$\Delta V^2/2 = v^2/2$$

14.20 CONTINUED -

$$h_L = 2 f_F \frac{L}{D} v^2 + \sum K \frac{v^2}{2}$$

$$= 2(0.0031)\frac{99.5}{0.36} v^2 + \underbrace{\frac{v^2}{2}}_{\text{ENTRANCE}}$$

$$= 2.21\, v^2$$

ENERGY EQN NOW BECOMES:

$$-\dot{W} = \dot{m}\left[0.5 v^2 + 6.5 g + 2.21 v^2\right]$$

$$-\frac{\dot{W}}{\dot{m} g} = h_{SYST} = 6.5 + 0.276\, v^2 \quad (1)$$

SYSTEM PERFORMANCE - EQN (1)

$\dot{V}$	h_{SYST}
0.20	7.27
0.25	7.71
0.30	8.24
0.35	8.87

PUMP & SYSTEM AREN'T WELL MATCHED - PUMP PERFORMANCE HEAD CURVE MUST BE EXTRAPOLATED

$$\dot{V} \cong \underline{\underline{0.33 \text{ m}^3/\text{s}}}$$

$$-\dot{W} \cong 8.8(1000)(0.33)(9.81)$$

$$= \underline{\underline{28.5 \text{ kW}}}$$

14.21 SAME PUMP FAMILY AS IN PROB 14.14

$$\frac{h_2}{h_1} = \left(\frac{n_2}{n_1}\right)^2\left(\frac{D_2}{D_1}\right)^2$$

$$h_2 = h_1\left(\frac{900}{360}\right)^2 = 6.25\, h_1$$

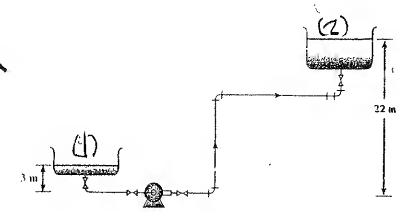
TOTAL MISMATCH -

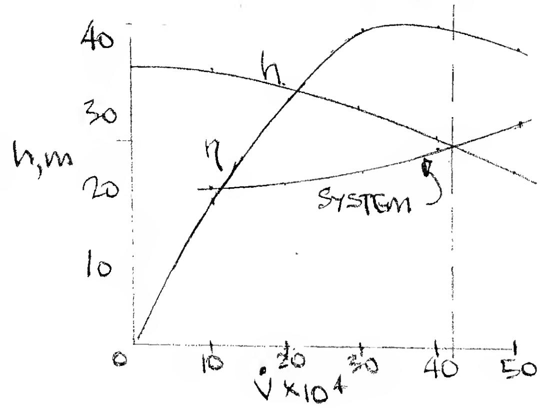
14.22.

PUMP PERFORMANCE →

Capacity, m³/s × 10⁴	Developed head, m	Efficiency, %
0	36.6	0
10	35.9	19.1
20	34.1	32.9
30	31.2	41.6
40	27.5	42.2
50	23.3	39.7

SYSTEM CONFIGURATION →





SYSTEM - INLET - $D = 0.06$ m
$L = 8.5$ m

DISCHARGE - $D = 0.06$ m
$L = 60$ m

STEEL - $e = 0.457 \times 10^{-4}$ m
$e/D = 0.000762$

MINOR LOSSES - 4 VALVES $K = 10$
4 ELBOWS $K = 0.3$
1 CONTRACTION $K = 1.-$

BETWEEN RESERVOIRS - (1) & (2)

$$-\dot{W} = \dot{m}\left[\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g\Delta y + h_L\right]$$

$$\frac{\Delta P}{\rho} = \frac{\Delta v^2}{2} = 0$$

$$g\Delta y = 19 g \ \text{m}^2/\text{s}^2$$

$$h_L = 2 f_F \frac{L}{D} v^2 + \sum K \frac{v^2}{2}$$

14.22 CONTINUED –

ASSUME FLOW IS FULLY TURBULENT –

$f_F \cong 0.0046$

$\sum K = 4(10) + 4(0.3) + 1 = 42.2$

$$h_L = \left[2(0.0046)\frac{68.5}{0.06} + \frac{42.2}{2}\right]v^2$$

$$= 31.6\, v^2$$

ENERGY EQN BECOMES:

$$-\frac{\dot{W}}{\dot{m}g} = \Delta y + \frac{h_L}{g} = h$$

$$= 19 + 3.22\, v^2$$

$\dot{V} \times 10^4$	h, m
20	20.61
30	22.63
40	25.45
50	29.07

INTERSECTION OCCURS AT

$\dot{V} \cong 42 \times 10^{-4}\ m^3/s$

AT $\dot{V} = 42 \times 10^{-4}\ m^3/s$

$v = 1.485\ m/s$

$$Re = \frac{(0.06)(1.485)}{0.995 \times 10^{-6}} = 8.957 \times 10^5$$

USING FIG 13.1 –

CONDITIONS ARE VERY CLOSE TO FULLY TURBULENT FLOW –

INITIAL ASSUMPTION FOR f_F WAS O.K.

$$\underline{\underline{\dot{V} = 0.0042\ m^3/s}}$$

14.23 PUMP – $D = 0.25\ m$

$n = 2000$ rpm

$\dot{V} = 0.065\ m^3/s$

$v_{INLET} = 6.1\ m/s$

H_2O @ 20°C ~ $P_v = 2.34$ kPa

CAVITATION OCCURS AT $P_i = 82.7$ kPa

$$NPSH + \frac{P_v}{\rho g} = \frac{v_i^2}{2g} + \frac{P_i}{\rho g}$$

$$NPSH = \frac{v_i^2}{2g} + \frac{P_i - P_v}{\rho g}$$

$$= \frac{(6.1)^2}{(2)(9.81)} + \frac{(82.7 - 2.34)(10^3)}{1000\,(9.81)}$$

$$\underline{\underline{= 10.09\ m\,H_2O}}$$

14.24 – SAME PUMP AS DESCRIBED IN PROB 14.23 –

NEW TEMP IS 80°C ($P_v = 47.35$ kPa)

$$NPSH = \frac{v_i^2}{2g} + \frac{P_i - P_v}{\rho g}$$

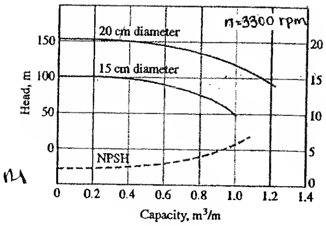
$$= \frac{(6.1)^2}{2(9.81)} + \frac{(82.7 - 47.35)(1000)}{1000\,(9.81)}$$

$$\underline{\underline{= 5.50\ m\ H_2O}}$$

CHANGE FROM 20°C CASE IS

$\Delta = 10.09 - 5.50 = \underline{\underline{4.59\ m}}$

14.25



PUMP –

$D = 0.18\ m$

INLET AT $y = 3.8\ m$ ABOVE SUPPLY RESERVOIR

$\dot{V} = 0.760\ m^3/s$

BETWEEN RESERVOIR SURFACE & PUMP INLET – $h_L = 1.8\ m\ H_2O$

ENERGY EQUATION

$$NPSH = \frac{P_{ATM} - P_V}{\rho g} - y_2 - h_L$$

AT $20°C$ $P_V = 2.34\ kPa$

$$\frac{P_{ATM} - P_V}{\rho g} = \frac{(101.3 - 2.34)(10^3)}{(1000)(9.81)}$$

$$= 10.09\ m$$

$$NPSH = 10.09 - 3.8 - 1.8$$

$$= 4.49\ m\ H_2O$$

FROM PERFORMANCE CURVE –

@ $\dot{V} = 0.760\ m^3/s$

$NPSH \cong 3.9\ m$

CAVITATION SHOULD NOT OCCUR

14.26 $\dot{V} = 220\ m^3/s = 3.487 \times 10^6$ gpm

$h = 420\ m = 1378$ FT

$$N_S = \frac{(400)(3.487 \times 10^6)^{1/2}}{(1378)^{3/4}} = 3302$$

ACCORDING TO FIG 14.11

THIS IS PROBABLY A HIGH CAPACITY CENTRIFUGAL PUMP

14.27 PUMP TO DELIVER 60,000 gpm WITH $h = 300\ m$ @ 2000 rpm.

$$N_S = \frac{(2000)(6 \times 10^4)^{1/2}}{(300/0.3048)^{3/4}} \cong 2790$$

USING FIG 14.11 – PUMP IS PROBABLY A HIGH-CAPACITY CENTRIFUGAL PUMP.

14.28 AXIAL FLOW PUMP – $N_S = 6.0$

$$N_S = \frac{C_Q^{1/2}}{C_H^{3/4}} = \frac{\dot{V}^{1/2}\omega}{h^{3/4} g^{3/4}} \qquad (1)$$

THIS RATIO IS (OBVIOUSLY) DIMENSIONLESS –

BY CONVERTING TO UNITS ON ABSCISSA OF FIG 14.11 –

THE RATIO OF N_S GIVEN BY (1) TO THE VALUE ON FIG 14.11 IS 2733

– SO A VALUE OF 6 FOR EQN (1) IS EQUIVALENT TO $6(2733) = 1.64 \times 10^4$ ON ABSCISSA OF FIG 14.11.

$$\therefore 1.64 \times 10^4 = \frac{n(2400)^{1/2}}{(18)^{3/4}}$$

$$n = 2925\ rpm$$

14.29 Pump @ 520 rpm

$\dot{V} = 3.3\ m^3/s$

$h = 16\ m$

$$\dot{V} = (3.3)\left(\frac{1}{0.3048}\right)^3(7.48)(60)$$

$= 52302$ gpm

$$h = (16)/0.3048 = 42.65\ FT$$

$$N_s = \frac{(520)(5.23\times10^5)^{1/2}}{(42.65)^{3/4}}$$

$= 22532$

FIG 14.11: AXIAL FLOW

14.30 $n = 2400$ rpm

$\dot{V} = 3.2\ m^3/s$

$h = 21\ M$

$$\dot{V} = 3.2\left(\frac{1}{0.3048}\right)^3(7.48)(60)$$

$= 5.072\times10^4$ gpm

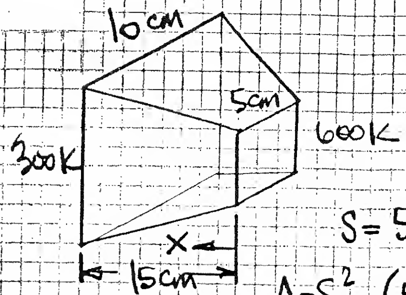
$$h = \frac{21}{0.3048} = 68.9\ FT$$

$$N_s = \frac{(5.072\times10^4)^{1/2}(2400)}{(68.9)^{3/4}}$$

$= 22601$

FIGURE 14.11 AXIAL FLOW

15.1



$$S = 5 + \frac{x}{3} \text{ cm}$$

$$A = S^2 = \left(5+\frac{x}{3}\right)^2 \text{cm}^2$$

$$q = -kA\frac{dT}{dx}$$

$$q\,dx = -kA\,dT$$

$$q\int_0^{15}\frac{dx}{(5+x/3)^2} = k\int_{600}^{300} dT$$

$$q\left[-\frac{3}{5+x/3}\right]_0^{15} = -300\,k$$

$$q\,[0.6-0.3] = 300(0.173\ \text{W/m·K})$$

$$\underline{q = 1.73\ \text{W}}$$

15.2 SAME VALUE AS IN PREVIOUS PROBLEM EXCEPT HEAT FLOWS IN OPPOSITE DIRECTION

$$\underline{q = 1.73\ \text{W}}$$

15.3

$$q\int_0^{15}\frac{dx}{(5+x/3)^2} = -k_0\int_{300}^{600}(1+\beta T)\,dT$$

$$q\left[-\frac{3}{5+x/3}\right]_0^{15} = k_0\Delta T\left[1+\frac{\beta}{2}(T_1+T_2)\right]_{300}^{600}$$

$$q\,[0.3\ \text{cm}^{-1}] = [0.135\ \text{W/m·K}](300\text{K}) \times [1+1.95\times10^{-4}](450)$$

$$\underline{q = 1.50\ \text{W}}$$

15.4

$$R_{BOLT} = \frac{L}{kA} = \frac{0.15}{(40)(\pi/4)(0.0095)^2}$$

$$= 13.16\ \text{K/W}$$

NEGLECTING CHANGE IN CROSS-SECTIONAL AREA OF ASBESTOS:

$$R_{ASB} = \Delta T/q = \frac{300}{1.73} = 173.4\ \text{K/W}$$

$$\frac{1}{R_{EQUIV}} = \frac{1}{13.16} + \frac{1}{173.4} = \frac{1}{12.23}$$

$$q = \frac{\Delta T}{R_{EQUIV}} = \underline{24.5\ \text{W}}$$

15.5

$$q = \frac{kA}{L}\Delta T$$

$$\Delta T = \frac{4000\ \text{W}\ (0.02\ \text{m})}{(0.22\ \text{W/m·K})(2.97\ \text{m}^2)} = 122.4\text{K}$$

$$T_i = 55 + 122.4 = \underline{177.4\ \text{C}}$$

15.6

$$q = \Delta T/\Sigma R$$

$$\Sigma R = \frac{L}{kA} + \frac{1}{hA}$$

$$= \frac{0.02}{(0.22)(2.97)} + \frac{1}{(28.4)(2.97)}$$

$$= 4.246\times10^{-2}\ \text{K/W}$$

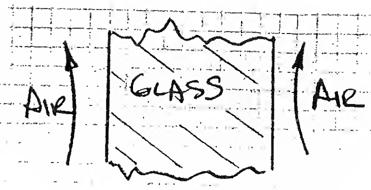
$$\Delta T_{OVERALL} = (4000)(4.246\times10^{-2})$$

$$= 169.9\ \text{K}$$

$$T_{HOT} = 30 + 169.9 = \underline{199.9\ \text{C}}$$

$$T_{SURF} = 30 + \frac{4000}{(28.4)(2.97)} = \underline{77.4\text{C}}$$

15.7



$$q_{MAX} = -k \left.\frac{dT}{dx}\right|_{MAX}$$

$$= (1.35\ W/m{\cdot}K)(15\ K/cm)(100\ cm/m)$$

$$= 2025\ W/m^2 = \Delta T/R$$

$$\Delta T = 2025\,(1/5) = 405\ K$$

$$T_{MIN} = 850 - 405 = \underline{445\ K}$$

15.8 $q_{MAX}\left(\begin{smallmatrix}\text{FROM}\\ \text{PREVIOUS}\\ \text{PROB}\end{smallmatrix}\right) = 2025\ W/m^2$

$$= \frac{\Delta T}{R} + \sigma\left(T_{SURR}^4 - T_A^4\right)$$

$$= \frac{850 - T}{1/5} + 5.676\left[8.5^4 - \left(\frac{T}{100}\right)^4\right]$$

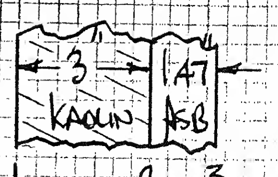
BY TRIAL & ERROR: $\underline{\underline{T = 836\ K}}$

15.9 $\frac{q}{A} = k\frac{\Delta T}{L}$ OR $L = \frac{k\Delta T}{q/A}$

$$L = \frac{(0.10\ BTU/HR\,FT\,F)(1100\ F)}{900\ BTU/HR\text{-}FT^2}$$

$$= \underline{0.122\ FT} = \underline{1.47\ IN.}$$

15.10 ADDING 3 IN. OF KAOLIN:



$T_1 = 250\ F$

$T_3 = 1600\ F$

$$\frac{q}{A} = \frac{\Delta T}{\Sigma R} = \frac{1350}{\frac{1.47/12}{0.10} + \frac{3/12}{0.007}}$$

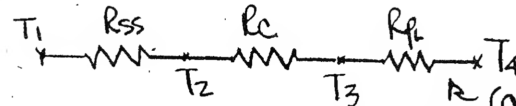
$$= \underline{\underline{281\ BTU/HR\text{-}FT^2}}\ (a)$$

15.10 CONTINUED -

$$\frac{q}{A} = \frac{1600 - T_2}{\frac{1.47/12}{0.10}} = \frac{T_2 - 250}{\frac{3/12}{0.07}}$$

$$\underline{\underline{T_2 = 1254\ F}} \leftarrow (b)$$

15.11



(a)

$$R_{SS} = L/k_{SS} = \frac{1/48}{10} = 0.0028$$

$$R_C = L/k_C = \frac{3/12}{0.025} = 10$$

$$R_{PL} = L/k_{PL} = \frac{1/24}{1.5} = 0.0278$$

(b)

$$q/A = \frac{\Delta T}{\Sigma R} = \frac{170}{10.03} = \underline{\underline{16.95\ BTU/HR\,FT^2}}$$

(c)

$$\frac{250 - T_2}{0.0028} = \frac{T_2 - T_3}{10} = \frac{T_3 - 80}{0.0278} = 16.95$$

$\underline{\underline{T_2 = 249.95\ F}}$ $\underline{\underline{T_3 = 80.47\ F}}$ (d)

15.12 $R_{INS} = \frac{1}{40} = 0.025\ HR\,FT^2F/BTU$

$R_{OUTS} = 1/5 = 0.20$ "

(a)

$$\frac{q}{A} = \frac{\Delta T}{\Sigma R} = \frac{180}{10.225} = 17.6\ BTU/HR\,FT^2$$

$$= \frac{T_3 - 70}{0.0278 + 0.2} \qquad T_3 = 74.0\ F\ (b)$$

CONTROLLING RESISTANCE IS THE CORK BOARD.

15.13

$$q_{TOTAL} = q_{conv} + q_{rad}$$

FOR BLACK BODY RADIATION TO SPACE (NONE INCOMING)

$$q = \frac{\pi}{4}\left(\frac{10}{12}\right)^2\left[5(80) + 0.1714(6.2)^4\right]$$

$$= \underline{356\ \text{Btu/hr}} \qquad (a)$$

BY CONVECTION: PERCENT $= \underline{61.2}$ (b)

RADIATION $= \underline{38.8}$

IF SURROUNDINGS RADIATE @ 540 R

$$q_{conv} = \frac{\pi}{4}\left(\frac{10}{12}\right)^2(5)(80) = 218.2\ \text{Btu/hr}$$

$$q_{conv} = \frac{\pi}{4}\left(\frac{10}{12}\right)^2(0.1714)\left[6.2^4 - 5.4^4\right]$$

$$= 58.6\ \text{Btu/hr}$$

$q = \underline{276.8\ \text{Btu/hr}}$ % CONV $= \underline{78.8}$

" RAD $= \underline{21.2}$

15.14 $800\ \text{W} = 2730\ \text{Btu/hr}$

IF ALL HEAT LEAVES TOP SURFACE

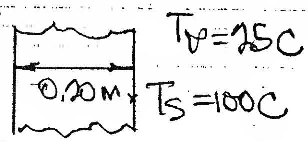
$$q = hA\Delta T + \sigma A\epsilon\left[T^4 - T_s^4\right]$$

$$\frac{2700}{A} = 5(T-40) + 0.1714(1)\left[\left(\frac{T}{100}\right)^4 - 5.4^4\right]$$

BY TRIAL & ERROR: $T = \underline{1086\ R}$

$= \underline{626\ F}$

15.15



$$\frac{q}{A} = \frac{k}{L}\Delta T_W = h(T_s - T_\infty)$$

15.15 CONTINUED -

$$\Delta T_W = \frac{(18\ \text{W/m·K})(75\text{K})(0.20\ \text{m})}{1.3\ \text{W/m·K}}$$

$$= 207\ \text{K}$$

$$T_{INSIDE} = 100 + 207 = \underline{307\ C}$$

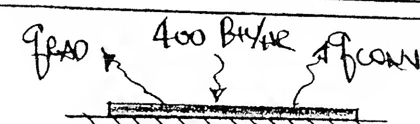
15.16

$$\frac{q}{A} = \frac{k}{L}\Delta T_W = h(T_s - T_\infty) + \sigma\left[T_s^4 - T_\infty^4\right]$$

$$\Delta T_W = \frac{0.2}{1.3}\left[18(75) + 5.676(3.73^4 - 2.98^4)\right]$$

$= 308\ \text{K}$ $\underline{T_{INSIDE} = 408\ C}$

15.17



$$400\ \frac{\text{Btu}}{\text{hr}} = A\left[h(T_s - T_\infty) + \sigma(T_s^4 - T_\infty^4)\right]$$

$$100\ \frac{\text{Btu}}{\text{hr·ft}^2} = 4(T-550) + 0.1714\left[\left(\frac{T_s}{100}\right)^4 - 5.5^4\right]$$

TRIAL & ERROR: $\underline{T = 570\ R = 110\ F}$

15.18

AT TOP:

$$100\ \frac{\text{Btu}}{\text{hr ft}^2} = q_{rad} + q_{conv} + q_{cond}$$

$$100 = 0.1714\left[\left(\frac{T}{100}\right)^4 - 5.5^4\right] + 4(T-550) + \frac{24.8}{1.4/12}(T - T_b) \qquad (1)$$

AT BOTTOM:

$$\frac{24.8}{1.4/12}(T - T_b) = 3(T_b - T_\infty) \qquad (2)$$

FROM (1)

$$8.05\times10^{-4}\left(\frac{T}{100}\right)^4 + 1.019T - T_B = 11.53$$

FROM (2) $T_b = 0.986T + 7.65$

TRIAL & ERROR: $\underline{T = 559\ R = 99\ F}$

15.18 CONTINUED -

WITH RADIATION FROM TOP

WITHOUT " "

EQN (1): $T_B = 1.019T - 11.53$

(2) $T_b = 0.986T + 7.65$

$$\underline{\underline{T = 582\ R = 122\ F}}$$

15.19 $A = 2\left[(0.3)(0.25)+(0.3)(0.5)+(0.25)(0.5)\right]$

$= 0.7\ m^2$

$$q = \frac{kA}{L}\Delta T \qquad L = \frac{kA\,\Delta T}{q}$$

$$L = \frac{(0.30\ W/m\cdot K)(0.7\ m^2)(43\ K)}{400\ W}$$

$$= \underline{\underline{0.0226\ m}} = \underline{\underline{2.26\ cm}}$$

15.20 $A = 0.7\ m^2$

$q = \Delta T / \Sigma R$

$$R_i = 1/hA = \frac{1}{16(0.7)} = 8.93\times10^{-2}\ K/W$$

$$R_{COND} = L/kA = \frac{L}{(0.30)(0.7)} = \frac{L}{0.21}\ "$$

$$R_o = 1/h_oA = \frac{1}{32(0.7)} = 4.46\times10^{-2}\ "$$

$$\Sigma R = 0.1340 + L/0.21 = \frac{43\ K}{400\ W}$$

FOR THESE CONDITIONS - NO INSULATION IS NECESSARY & THE SYSTEM CANNOT TRANSFER 400 W.

15.20 CONTINUED

$$q = \frac{\Delta T}{\Sigma R} = \frac{43\ K}{0.1340} = \underline{\underline{321\ K}}$$

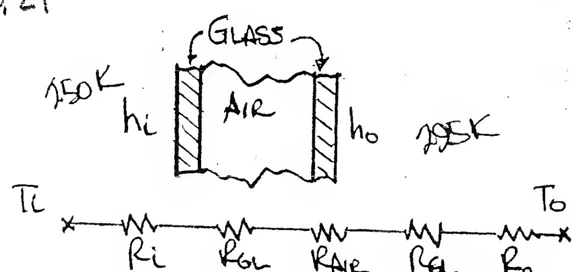
EFFECTIVE WALL TEMP IS

$$q = h_i(0.7)\Delta T_i = h_o(0.7)\Delta T_o$$

$$\Delta T_i = \frac{321}{16(0.7)} = 28.6\ K$$

$$T_{WALL} = \underline{\underline{18.6\ C}}$$

15.21



$$q = \frac{\Delta T}{\Sigma R} \qquad R_i = \frac{1}{(20)(1.83)(3.66)} = 7.465\times10^{-3}$$

$$R_{GL} = \frac{0.0032}{(0.78)(1.83)(3.66)} = 6.125\times10^{-4}$$

$$R_{AIR} = \frac{0.008}{(0.0245)(1.83)(3.66)} = 0.0488$$

$$R_o = \frac{1}{(15)(1.83)(3.66)} = 9.95\times10^{-3}$$

$$\Sigma R = 0.06744$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{45}{0.06744} = \underline{\underline{667\ W}}$$

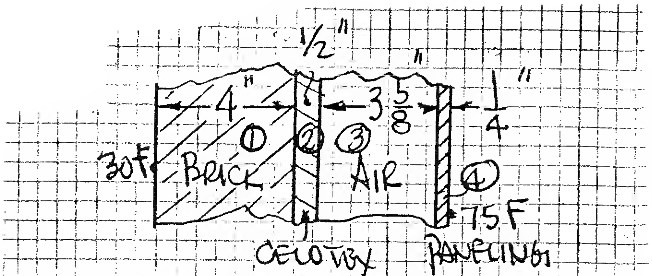
15.22 FOR 1 PANE OF GLASS ONLY

$$\Sigma R = R_i + R_{GL} + R_o$$

$$= 0.0180$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{45}{0.0180} = \underline{\underline{2500\ W}}$$

15.23



$$\frac{q}{A} = -k\frac{dT}{dx} = k\frac{\Delta T}{\Delta x} = \frac{\Delta T}{R}; \quad R = \frac{\Delta x}{k}$$

$$\frac{q}{A} = \frac{75-30 \text{ F}}{\frac{1/3}{0.38} + \frac{1/24}{0.028} + \frac{29/96}{0.015} + \frac{1/48}{0.012}}$$

$$= \frac{45}{0.876 + 1.49 + 20.2 + 0.174}$$

$$= \underline{\underline{1.98 \ \text{Btu/hr-ft}^2}} \leftarrow (a)$$

$$R_{AIR} = \frac{\Delta x}{k} = \frac{1}{1.8} = 0.555$$

$$\Sigma R = 0.876 + 1.49 + 0.555 + 0.174$$

$$= 3.095$$

$$q/A = \frac{45}{3.095} = \underline{\underline{14.54 \ \text{Btu/hr-ft}^2}} \quad (b)$$

$$R_{\text{GLASS WOOL}} = \frac{29/96}{0.025} = 12.1$$

$$\Sigma R = 14.64$$

$$q/A = \frac{45}{14.64} = \underline{\underline{3.07 \ \text{Btu/hr-ft}^2}} \quad (c)$$

15.24

$$q/A = \Delta T / \Sigma R$$

$$R_{\text{INSIDE}} = 1/h_i = 1/4 \ \frac{\text{hr F ft}^2}{\text{Btu}}$$

$$R_{\text{OUTSIDE}} = 1/h_o = 1/2 \quad "$$

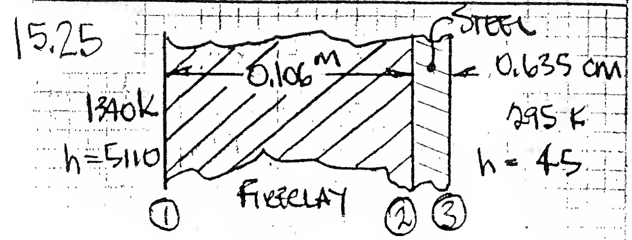
15.24 CONTINUED

Part a) $\Sigma R = 23.4 \quad \frac{q}{A} = 1.92 \ \text{Btu/hr ft}^2$

b) $\Sigma R = 3.74 \quad \frac{q}{A} = 12.04$ "

c) $\Sigma R = 15.28 \quad \frac{q}{A} = 2.94$ "

15.25



$$R_i = 1/A_i h_i = \frac{1}{(5110)(1)} = 1.97 \times 10^{-4}$$

$$R_o = 1/A_o h_o = \frac{1}{(45)(1)} = 0.22$$

$$R_1 = \frac{\Delta x}{kA} = \frac{0.106}{(1.13)(1)} = 0.0938$$

$$R_2 = \frac{0.00635}{42.9} = 1.48 \times 10^{-4}$$

$$\Sigma R = 0.1161$$

$$q = \frac{1340-295}{0.1161} = 9000 \text{ W}$$

$$9000 = \frac{1340 - T_1}{R_i} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2}$$

$$= \frac{T_3 - 295}{R_o}$$

$\underline{\underline{T_1 = 1338 \text{ K} \quad T_2 = 494 \text{ K} \quad T_3 = 493 \text{ K}}}$

15.26

FROM PREVIOUS PROBLEM

$$R_i + R_o + R_1 + R_2 + R_{\text{CELOTEX}}$$

$$= 0.1161 + L/0.069$$

$$= 0.1161 + 14.49 L$$

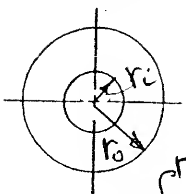
15.26 CONTINUED

$$q = \frac{340-295}{0.022} = 2027\ W$$

$$2027 = \frac{340-295}{0.1161 + 14.49 L}$$

$$\underline{\underline{L = 0.0276\ m}}$$

15.27



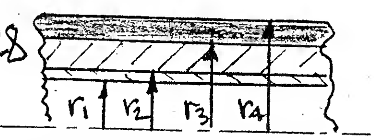
$$\int \frac{q}{A} dr = \int k\, dT$$

$$q \int_{r_i}^{r_o} \frac{dr}{2\pi r L} = -0.08 \int_{1400}^{300} (1 - 0.0003T)\, dT$$

$$q \frac{\ln r_o/r_i}{2\pi L} = 65.52$$

$$q = \frac{2\pi(65.52)}{\ln 4} = \underline{\underline{297\ \frac{Btu}{hr}}}$$

15.28



PER UNIT LENGTH:

$$R_{INS} = 1/2\pi r_i h_i = 0.0238$$

$$R_1 = \frac{\ln r_2/r_1}{2\pi k_1} = 0.00105$$

$$R_2 = \frac{\ln r_3/r_2}{2\pi k_2} = 0.115/k_2$$

$$R_3 = \frac{\ln r_4/r_3}{2\pi k_3} = 0.0659/k_3$$

$$R_4 = \frac{1}{2\pi r_4 h_o} = 0.1296$$

15.28 CONTINUED -

$$\Sigma R = 0.1545 + 0.115/k_2 + 0.0659/k_3$$

CASE 1: k_2 FOR MAGNESIA

$$\Sigma R = 6.134$$

CASE 2: k_2 FOR GLASS WOOL

$$\Sigma R = 7.096$$

GLASS WOOL CASE IS BEST

$$q = \Delta T/\Sigma R = \frac{60}{7.096} = 8.47\ \frac{Btu}{hr\text{-}ft}$$

$$\frac{q}{A} = \frac{8.47}{2\pi(2.95/12)} = \underline{\underline{5.55\ Btu/hr\text{-}ft^2}}$$

15.29 FOR BARE PIPE:

$$q = \pi D_1 h \Delta T = \pi\left(\frac{1.315}{12}\right)(1.5)(310)$$

$$= 160\ Btu/hr\ PER\ FT$$

FOR INSULATED PIPE:

$$80 = \frac{T_s - T_\infty}{\frac{\ln D_2/D_1}{2\pi k} + 1/\pi D_2 h}$$

$$\frac{\pi h}{2\pi k} \ln \frac{D_2}{D_1} + \frac{1}{D_2} = \frac{\pi h}{80} \Delta T$$

$$12.5 \ln D_2/0.1905 + \frac{1}{D_2} = 18.25$$

BY TRIAL & ERROR: $D_2 = 0.382$ FT

$$2t = D_2 - D_1 = 3.265\ IN$$

$$\underline{\underline{t = 1.63\ IN.}}$$

15.30

FOR BARE PIPE, PER FOOT:

$$q = hA\Delta T = \frac{0.575}{(1.315/12)^{1/4}}\pi\left(1.315/12\right)(310)$$

$$= 106.7 \ BTU/HR$$

WITH INSULATION - $q = 53.3$ "

$$q = \frac{\Delta T}{\frac{\ln D_o/D_i}{2\pi k} + \frac{1}{\pi D_o h_o}}$$

$$53.3 = \frac{310}{\frac{\ln D_o/1.315}{2\pi(0.06)} + \frac{(D_o/12)^{1/4}}{0.575\pi D_o/12}}$$

BY TRIAL & ERROR: $D_o = 9.22$ IN

$$\text{INSULATION THICKNESS} = \frac{9.22-1.315}{2} = \underline{\underline{3.95 \text{ IN}}}$$

15.31

$$q_o = \Delta T/\sum R$$

WITHOUT INSULATION:

$$\sum R_{wo} = \frac{1}{2\pi L}\left[\frac{\ln 13.7/12.5}{17.3} + \frac{1}{(12)(0.137)}\right]$$

$$= \frac{0.6136}{2\pi L} \ K/W$$

WITH INSULATION:

$$\sum R_w = \frac{1}{2\pi L}\left[\frac{\ln 13.7/12.5}{17.3} + \frac{1}{12r_o} + \frac{\ln r_o/0.137}{0.13}\right]$$

$$= \frac{1}{2\pi L}\left[5.299\times10^{-3} + \frac{1}{12r_o} + \frac{\ln r_o/13.7}{0.13}\right]$$

$$\text{If } q_w = q_{wo}/4$$

$$\sum R_w = 4\sum R_{wo} = 2.454/2\pi L$$

$$\therefore \ 5.299\times10^{-3} + \frac{1}{12r_o} + \frac{\ln r_o/13.7}{0.13} = 2.454$$

15.31 CONTINUED -

BY TRIAL & ERROR:

$r_o = 0.177$ m

$$\text{INSULATION THICKNESS} = r_o - r_i$$

$$= 0.177 - 0.137 = \underline{\underline{0.04 \text{ m}}}$$

$$= \underline{\underline{4 \text{ cm}}}$$

CHAPTER 16

16.1 IN CYLINDRICAL COORDINATES: (a)

$$\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}=0 \quad \text{OR} \quad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

$$r\frac{dT}{dr}=C_1$$

$$T=C_1\ln r+C_2$$

B.C. $T_i=C_1\ln r_i+C_2$

$T_o=C_1\ln r_o+C_2$

$$C_1=-\frac{T_i-T_o}{\ln r_o/r_i} \qquad C_2=T_i-C_1\ln r_i$$

$$T=T_i-(T_i-T_o)\frac{\ln r/r_i}{\ln r_o/r_i} \qquad (b)$$

$$q=-kA\frac{dT}{dr}=-k(2\pi rL)\frac{dT}{dr}$$

$$=-2\pi kLC_1=\frac{2\pi kL}{\ln r_o/r_i}(T_i-T_o) \qquad (c)$$

16.2 $$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad (a)$$

$$r^2\frac{dT}{dr}=C_1 \qquad T=-\frac{C_1}{r}+C_2$$

B.C. $T_i=-\frac{C_1}{r_i}+C_2$

$T_o=-\frac{C_1}{r_o}+C_2$

$$C_1=\frac{T_i-T_o}{1/r_o-1/r_i} \qquad C_2=T_i+C_1/r_i$$

$$T=T_i-\frac{1/r-1/r_i}{1/r_o-1/r_i}(T_i-T_o) \leftarrow (b)$$

$$q=-k(4\pi r^2)\frac{dT}{dr}=-4\pi kC_1$$

$$=\frac{4\pi k}{1/r_o-1/r_i}(T_i-T_o) \leftarrow (c)$$

16.3 $$\frac{d^2T}{d\theta^2}=0 \qquad (a)$$

$$T=C_1\theta+C_2$$

BC. $T_o=C_2$

$T_\pi=C_1\pi+C_2$

$$C_1=\frac{T_\pi-T_o}{\pi} \qquad C_2=T_o$$

$$T=T_o-\frac{\theta}{\pi}(T_o-T_\pi) \qquad (b)$$

$$dq_\theta=-k\,dA\,dT/dn$$

$$=-k(L\,dr)\frac{dT}{r\,d\theta}=-kL\frac{dT}{d\theta}\frac{dr}{r}$$

$$\int_0^{q_\theta}dq_\theta=-kLC_1\int_{r_i}^{r_o}\frac{dr}{r}$$

$$q_\theta=-kLC_1\ln r_o/r_i$$

$$=\frac{kL}{\pi}\ln r_o/r_i\,(T_o-T_\pi) \qquad (c)$$

16.4 PROBLEM STATEMENT REQUIRES THAT WE DEMONSTRATE

$$\rho\frac{Du}{Dt}+\rho\frac{D}{Dt}(gy)+\vec{v}\cdot\rho\vec{g}=\rho c_v\frac{DT}{Dt}$$

FOR c_v CONSTANT: $\rho\frac{Du}{Dt}=\rho c_v\frac{DT}{Dt}$ ①

$$\rho\frac{D}{Dt}(gy)=\rho\left[\cancelto{0}{\frac{\partial}{\partial t}(gy)}+\cancelto{0}{v_x\frac{\partial}{\partial x}(gy)}+v_y\frac{\partial}{\partial y}(gy)+\cancelto{0}{v_z\frac{\partial}{\partial z}(gy)}\right]$$

$$=\rho g v_y \qquad ②$$

$$\vec{v}\cdot\rho\vec{g}=\vec{v}\cdot\rho g\vec{e}_y=-\rho v_y g \qquad ③$$

SUBSTITUTING ①, ②, & ③ THE DESIRED RESULT IS OBTAINED

16.5 EQN (16-7) WITH CONSTANT k:

$$k\nabla^2 T + \dot{q} + \Lambda = \nabla\cdot P\vec{v} + \rho\frac{D}{Dt}\left(\frac{v^2}{2}\right) + \rho\frac{Du}{Dt} + \rho\frac{D}{Dt}(gy)$$

FOR $\dot{q}=0$ & NO VISCOUS DISSIPATION

$$k\nabla^2 T + \vec{v}\cdot\mu\nabla^2\vec{v} = \nabla\cdot P\vec{v} + \rho\frac{D}{Dt}\left(\frac{v^2}{2}\right) + \rho\frac{Du}{Dt} + \rho\frac{D}{Dt}(gy) \quad (1)$$

FROM NAVIER-STOKES:

$$\rho\frac{D\vec{v}}{Dt} = \rho\vec{g} - \nabla P + \mu\nabla^2 \vec{v} \qquad (9\text{-}19)$$

DOT PRODUCT WITH $\vec{v}$ YIELDS

$$\rho\frac{D}{Dt}\left(\frac{v^2}{2}\right) = \vec{v}\cdot\rho\vec{g} - \vec{v}\cdot\nabla P + \vec{v}\cdot\mu\nabla^2\vec{v}$$

SUBSTITUTING INTO (1) & CANCELLING:

$$k\nabla^2 T = P\nabla\cdot\vec{v} + \vec{v}\cdot\rho\vec{g} + \rho\frac{Du}{Dt} + \rho\frac{D}{Dt}(gy)$$

FOR A POTENTIAL FUNCTION $\phi = gy$

$$\nabla\phi = -\vec{g}$$

THEN $\rho\frac{D}{Dt}(gy) = \rho\frac{D}{Dt}\phi = \rho\left(\frac{\partial\phi}{\partial t} + \vec{v}\cdot\cancel{\nabla\phi}\right)$ (with $\nabla\phi$ replaced by $-g$)

COMBINING WITH THE ENERGY EQN:

$$k\nabla^2 T = P\nabla\cdot\vec{v} + \rho\frac{Du}{Dt}$$

NOW - FROM THERMODYNAMICS:

$$u = u(v,T) \quad du = \left(\frac{\partial u}{\partial v}\right)_T dv + \left(\frac{\partial u}{\partial T}\right)_v dT$$

$$\Rightarrow \rho\frac{Du}{Dt} = \rho\left(\frac{\partial u}{\partial v}\right)_T\frac{Dv}{Dt} + \rho\left(\frac{\partial u}{\partial T}\right)_v\frac{DT}{Dt}$$

GIVING:

$$k\nabla^2 T = P\nabla\cdot\vec{v} + \rho c_v\frac{DT}{Dt} + \rho\frac{Dv}{Dt}\left[-P + T\left(\frac{\partial P}{\partial T}\right)_v\right]$$

$$\rho\frac{Dv}{Dt} = \rho\frac{D}{Dt}\left(\frac{1}{\rho}\right) = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

16.5 CONT. - BY CONTINUITY $-\frac{1}{\rho}\frac{D\rho}{Dt} = \nabla\cdot\vec{v}$

$$\text{SO } k\nabla^2 T = \cancel{P\nabla\cdot\vec{v}} + \rho c_v\frac{DT}{Dt} - \cancel{P\nabla\cdot\vec{v}} + T\left(\frac{\partial P}{\partial T}\right)_v\nabla\cdot\vec{v}$$

FOR INCOMPRESSIBLE FLOW $\nabla\cdot\vec{v} = 0$

$$k\nabla^2 T = \rho c_v\frac{DT}{Dt}$$

Q.E.D.

16.6 @ STEADY STATE $\nabla^2 T + \frac{\dot{q}}{k} = 0$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_0}{k}e^{-\beta x/L} = 0$$

$$\frac{dT}{dx} = \frac{\dot{q}_0}{k}\frac{L}{\beta}e^{-\beta x/L} + C_1$$

$$T = -\frac{\dot{q}_0}{k}\frac{L^2}{\beta^2}e^{-\beta x/L} + C_1 x + C_2$$

B.C. $T(0) = T_0 \quad T_0 = -\frac{\dot{q}_0}{k}\frac{L^2}{\beta^2} + C_2$

$T(L) = T_L \quad T_L = -\frac{\dot{q}_0}{k}\frac{L^2}{\beta^2}e^{-\beta} + C_1 L + C_2$

$$T = T_0 + (T_L - T_0)\frac{x}{L} + \frac{\dot{q}_0 L^2}{k\beta^2}\left[1 - e^{-\beta x/L} - \frac{x}{L}\left(1 - e^{-\beta}\right)\right]$$

16.7 SAME PROBLEM EXCEPT 2ND B.C. IS

$$\frac{\partial T}{\partial x}(L) = 0 \Rightarrow 0 = \frac{\dot{q}_0}{k}\frac{L}{\beta}e^{-\beta} + C_1$$

$$T = T_0 + \frac{\dot{q}_0}{k}\frac{L^2}{\beta^2}\left(1 - \beta\frac{x}{L} - e^{-\beta x/L}\right)$$

16.8. SAME PROBLEM AS 16.6 BUT 2ND B.C. $\frac{dT}{dx}(L) = \xi$ (A CONSTANT)

$$\xi = \frac{\dot{q}_0}{k}\frac{L}{\beta}e^{-\beta} + C_2$$

$$T = T_0 + \xi x - \frac{\dot{q}_0}{k}\frac{L^2}{\beta^2}\left(\frac{\beta x}{L}e^{-\beta} + e^{-\beta x/L}\right)$$

16.9 $T\,dS = dh - dP/\rho = du + P\,dV$

FORMING SUBSTANTIAL DERIVATIVES

$$T\frac{DS}{Dt} = \frac{Du}{Dt} + P\frac{DV}{Dt} = \frac{Du}{Dt} + \frac{P}{\rho^2}\frac{D\rho}{Dt}$$

BY CONTINUITY: $\frac{D\rho}{Dt} = -\rho\nabla\cdot\vec{v}$

SO $\frac{Du}{Dt} = T\frac{Ds}{Dt} - \frac{P}{\rho}\nabla\cdot\vec{v}$

FROM THE ENERGY EQN:

$$\frac{Du}{Dt} = \nabla\cdot k\nabla T + \dot{q} + \phi$$

$$\therefore T\frac{Ds}{Dt} = \nabla\cdot k\nabla T + \dot{q} + \phi + \frac{P}{\rho}\nabla\cdot\vec{v}$$

SINCE ϕ IS ALWAYS >0 ITS EFFECT IS ALWAYS TO INCREASE S

SINCE $\nabla\cdot k\nabla T$ CAN BE EITHER + OR − HT TX CAN EITHER INCREASE OR DECREASE S

16.10 FOR $\frac{v_x}{v_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

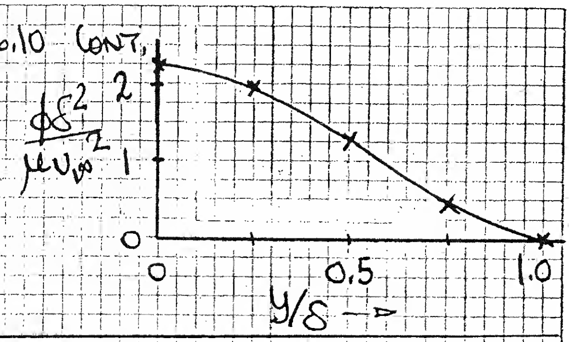
WITH $v_y = v_z = 0$ THE DISSIPATION FUNCTION REDUCES TO

$$\phi = \mu\left[\partial v_x/\partial y\right]^2$$

FOR THIS CASE

$$\frac{\phi\delta^2}{\mu v_\infty^2} = \frac{9}{4}\left[1 - (y/\delta)^2\right]$$

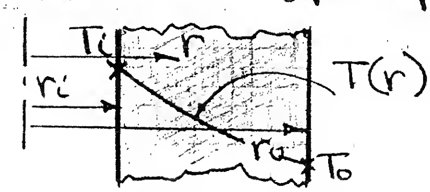
16.10 CONT.



16.11 SPHERICAL COORDINATES

$$\frac{1}{r}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$r^2\frac{dT}{dr} = \text{CONST} \qquad \frac{dT}{dr} = \frac{\text{CONST}}{r}$$



AS r INCREASES $\frac{dT}{dr}$ DECREASES

16.12 FOR THE TRUNCATED CONE:

$A_1 = \pi r_1^2 \quad A_2 = \pi r_2^2 \quad r = r_1 + \frac{r_2 - r_1}{L}x$

$$\Rightarrow A = A_0\left[1 + \left(\frac{r_2}{r_1} - 1\right)\frac{x}{L}\right]^2 = A_0\left(1 + \beta\frac{x}{L}\right)^2$$

$$\beta = \left(r_2/r_1 - 1\right)\frac{1}{L}$$

SINCE $q = -kA\,dT/dx$ WE HAVE

$$q = -kA_0(1 + \beta x^2)\,dT/dx$$

$$q\int_0^L \frac{dx}{1 + \beta x^2} = -kA_0\int_{T_1}^{T_2} dT$$

$$q = kA_0\left[\tan^{-1}(\sqrt{\beta}L)\right](T_1 - T_2)$$

16.12 CONTINUED -

IF, IN ADDITION, $k = k_0 - aT$

WE HAVE

$$q = -(k_0 - aT)(1+\beta x^2)\frac{dT}{dx}$$

$$q\int_0^L \frac{dx}{(1+\beta x^2)} = -\int_{T_1}^{T_2}(k_0 - aT)\,dT$$

$$q = A_0\left[\tan^{-1}(\sqrt{\beta}L)\right]\left[k_0 - a(T_1+T_2)\right] \times (T_1 - T_2)$$

16.13. HT GENERATION IN PLANE WALL

$$\dot{q} = q_{max}\left(1 - \frac{x}{L}\right)$$

FOURIER FIELD EQN, FOR STEADY STATE 1-D CONDUCTION, REDUCES TO

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_{max}}{k}\left(1 - \frac{x}{L}\right) = 0$$

1ST INTEGRATION:

$$\frac{dT}{dx} + \frac{\dot{q}_{max}}{k}\left(x - \frac{x^2}{2L}\right) = C_1$$

SYMMETRY, $\frac{dT}{dx} = 0$ @ $x = 0$ $\therefore C_1 = 0$

SECOND INTEGRATION:

$$\int_{T_c}^{T_0} dT + \frac{\dot{q}_{max}}{k}\int_0^L \left(x - \frac{x^2}{2L}\right)dx = 0$$

$$T_0 - T_c + \frac{\dot{q}_{max}}{k}\left(\frac{x^2}{2} - \frac{x^3}{6L}\right)_0^L = 0$$

$$T_c - T_0 = \frac{\dot{q}_{max}L^2}{3k}$$

16.14 HT GENERATION IN A CYLINDER:

$$\dot{q} = \dot{q}_{max}\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

FOURIER FIELD EQN REDUCES TO

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{q}_{max}}{k}\left[1 - \left(\frac{r}{r_0}\right)^2\right] = 0$$

SEPARATING VARIABLES - 1ST INTEGRATION

$$\int d\left(r\frac{dT}{dr}\right) + \frac{\dot{q}_{max}}{k}\int\left[r - \frac{r^3}{r_0}\right]dr = 0$$

$$r\frac{dT}{dr} + \frac{\dot{q}_{max}}{k}\left(\frac{r^2}{2} - \frac{r^4}{4r_0^2}\right) = C_1$$

SYMMETRY: $\frac{dT}{dr} = 0$ @ $r = 0$ $\therefore C_1 = 0$

SECOND SEPARATION & INTEGRATION

$$\int_{T_c}^{T_0} dT + \frac{\dot{q}_{max}}{k}\int_0^{r_0}\left(\frac{r}{2} - \frac{r^3}{4r_0^2}\right)dr = 0$$

$$T_c - T_0 = \frac{\dot{q}_{max}}{k}\frac{3}{16}r_0^2$$

16.15 HT GENERATION IN A SPHERE -

FOURIER FIELD EQN REDUCES TO:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{q}_{max}}{k}\left[1 - \left(\frac{r}{r_0}\right)^3\right] = 0$$

1ST INTEGRATION YIELDS $\quad$ 0 - SYMMETRY

$$r^2\frac{dT}{dr} + \frac{\dot{q}_{max}}{k}\left(\frac{r^3}{3} - \frac{r^6}{6r_0^3}\right) = \cancel{C_1}$$

SECOND INTEGRATION

$$\int_{T_c}^{T_0} dT + \frac{\dot{q}_{max}}{k}\int_0^{r_0}\left(\frac{r}{3} - \frac{r^4}{6r_0^3}\right)dr = 0$$

$$T_c - T_0 = \frac{\dot{q}_{max}}{k}\frac{2}{15}r_0^2$$

CHAPTER 17

17.1 STEADY-STATE X-DIRECTIONAL CONDUCTION THROUGH A PLANE WALL

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}(T_1 - T_2)$$

FOR $T_1 - T_2 = 75\text{ K}$

$$q = \frac{(30\text{ W/m}\cdot\text{K})(1\text{ m}^2)(75\text{ K})}{0.30\text{ m}}$$

$$= \underline{7500\text{ W/m}^2}$$

$$dT/dx = \frac{\Delta T}{L} = \frac{75\text{ K}}{0.30\text{ m}} = \underline{\underline{-250\text{ K/m}}}$$

FOR $T_1 = 300\text{ K}$ $\quad q = -2000\text{ W/m}^2$

$$dT/dx = -\frac{q}{kA} = \frac{2000\text{ W/m}^2}{(30\text{ W/m}\cdot\text{K})} = \underline{\underline{66.7\ \frac{\text{K}}{\text{m}}}}$$

$$\Delta T = \frac{qL}{kA} = \frac{(-2000)(0.3)}{(30)} = -20\text{ K}$$

$$\underline{\underline{T_2 = 320\text{ K}}}$$

FOR $T_2 = 350\text{ K}$ $\quad dT/dx = -300\text{ K/m}$

$$q = -(30)(-300) = \underline{\underline{9000\text{ W/m}^2}}$$

$$\Delta T = -300\text{ K/m}(0.3\text{ m}) = 90\text{ K}$$

$$\underline{\underline{T_1 = 440\text{ K}}}$$

FOR $T_1 = 250\text{ K}$ $\quad dT/dx = 200\text{ K/m}$

$$q = -(30)(200\text{ K/m}) = \underline{\underline{-6000\text{ W/m}^2}}$$

$$\Delta T = -(200)(0.3) = -60\text{ K} \quad \underline{\underline{T_2 = 310\text{ K}}}$$

17.2

$$q = \frac{k\bar{A}}{r_o - r_i}\Delta T = \frac{k}{r_o - r_i}\,2\pi\frac{(r_o - r_i)}{\ln r_o/r_i}\Delta T$$

$$= \frac{2\pi k}{\ln r_o/r_i}\Delta T \quad (a)$$

$$\%\text{ ERROR} = \frac{A_{LM} - A_{AM}}{A_{LM}} \times 100$$

17.2 CONTINUED

$$= \frac{\dfrac{2\pi(r_o - r_i)}{\ln r_o/r_i} - \pi(r_o + r_i)}{2\pi(r_o - r_i)/\ln r_o/r_i} \times 100$$

$$= \left|1 - \frac{(r_o + r_i)\ln r_o/r_i}{2(r_o - r_i)}\right| \times 100$$

$$= \left|1 - \frac{(r_o/r_i + 1)\ln r_o/r_i}{2(r_o/r_i - 1)}\right| \times 100$$

FOR $\frac{r_o}{r_i} = 1.5$ $\quad$ % ERROR = 1.3 %

3 $\quad$ " $\quad$ = 10.0 "

5 $\quad$ " $\quad$ = 20.7 "

17.3

$$q = \frac{4\pi k r_o r_i}{r_o - r_i}\Delta T \qquad \underline{\underline{\bar{A} = 4\pi r_o r_i}}\ (a)$$

$$A_m = \frac{4\pi(r_o^2 + r_i^2)}{2} = 2\pi(r_o^2 + r_i^2)$$

$$\%\text{ ERROR} = \frac{4\pi r_o r_i - 2\pi(r_o^2 + r_i^2)}{4\pi r_o r_i}$$

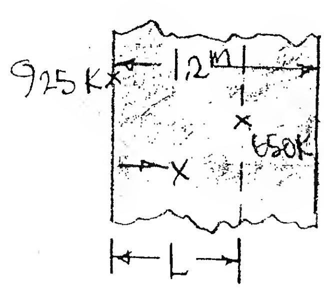
$$= 1 - \frac{1}{2}\left(\frac{r_o}{r_i} - \frac{r_i}{r_o}\right)$$

$r_o/r_i = 1.5$ $\quad$ % ERROR = 8.3 %

3 $\quad$ " $\quad$ = 66.6 " $\quad$ b

5 $\quad$ " $\quad$ = 160 "

17.4



$T_\infty = 300\text{ K}$

17.4 CONTINUED

$$q'' = -k\frac{dT}{dx} = -k_o(1+bT)\frac{dT}{dx}$$

FROM 0 TO 1.2:

$$q''\int_0^{1.2} dx = -k_o\int_{925}^{T}(1+bT)\,dT$$

$$q'' = \frac{k_o}{1.2}\left[T + \frac{b}{2}T^2\right]_{925}^{T} = 23(T-300)$$

SOLVING: $T_{RH\ WALL} = 307.1\ K$

$q'' = 163.3\ W/m^2$

FROM 0 TO L:

$$q''\int_0^L = -k_o\int_{925}^{650}(1+bT)\,dT$$

$$L = \frac{k_o}{q''}\left[T + \frac{b}{2}T^2\right]_{650}^{925}$$

& SOLVING! $\underline{\underline{L = 0.646\ m}}$

17.5 GOVERNING EQN- $\nabla\cdot k\nabla T = 0$

IN ONE DIMENSION: $\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$

FOR CONSTANT k: $\frac{d^2T}{dx^2} = 0$

$\frac{dT}{dx} = C_1$

$T = C_1x + C_2$

$T(0) = T_0 = C_1(0) + C_2 \qquad C_2 = T_0$

$T(L) = T_L = C_1L + C_2 \qquad C_1 = \frac{T_L - T_0}{L}$

FOR VARIABLE k: $\frac{d}{dx}\,k_o(1+\beta T)\frac{dT}{dx} = 0$

$(1+\beta T)\frac{dT}{dx} = C_3$

$T + \frac{\beta T^2}{2} = C_3x + C_4$

$T(0) = T_0 \qquad T_0 + \frac{\beta T_0^2}{2} = C_4$

$T(L) = T_L \qquad T_L + \frac{\beta T_L^2}{2} = C_3L + C_4$

17.5 CONTINUED-

$$C_3 = \frac{T_L}{L}\left(1 + \frac{\beta T_L}{2}\right) - \frac{C_4}{L}$$

$$T^2 + \frac{2}{\beta}T - \frac{2}{\beta}C_3x - \frac{2}{\beta}C_4$$

$$T^2 + BT - C = 0 \qquad B = 2/\beta$$

$$T = -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C} \qquad C = \frac{2}{\beta}(C_3x + C_4)$$

NOW- THE TEMPERATURE DIFFERENCE WE'RE SEEKING IS:

$$\Delta = C_1x + C_2 - \left[-\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - C}\right]$$

MAXIMUM IS WHERE $\frac{d}{dx}\Delta = 0$

$$\frac{d\Delta}{dx} = C_1 \mp \left(\frac{B^2}{4} - C\right)^{-1/2}\left(-\frac{2C_3}{\beta}\right) = 0$$

$$\frac{C_1\beta}{2C_3} \pm \left(\frac{B^2}{4} - C\right)^{-1/2} = 0$$

$$\frac{B^2}{4} - C = \frac{4C_3^2}{\beta^2C_1^2}$$

$$C = \frac{B^2}{4} - \frac{4C_3^2}{\beta^2C_1^2}$$

$$\frac{2}{\beta}C_3x + \frac{2}{\beta}C_4 = \frac{1}{\beta^2} - \frac{4C_3^2}{\beta^2C_1^2}$$

$$\underline{\underline{x = \frac{1}{2\beta C_3} - \frac{2}{\beta C_1^2} - \frac{C_4}{C_3}}}$$

C_1, C_3, & C_4 ARE AS DETERMINED ABOVE

17.6 SAME GENERAL PROCEDURE AS PREVIOUS PROBLEM:

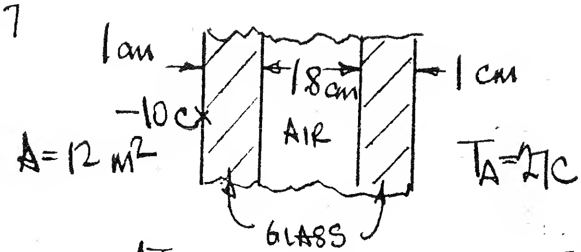
D.E. IS $\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$

FOR CONSTANT k: $\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$

FOR VARIABLE k: $\frac{d}{dr}\left[r(1+\beta T)\frac{dT}{dr}\right] = 0$

-MESSY BUT STRAIGHTFORWARD-

17.7



$q = \frac{\Delta T}{\Sigma R}$ $\quad R_{GL} = \frac{0.01}{(0.78)(12)} = 1.068 \times 10^{-3}$

$R_{AIR} = \frac{0.018}{0.0262(12)} = 5.725 \times 10^{-2}$

$R_{CONV} = \frac{1}{12(12)} = 6.944 \times 10^{-3}$

$\Sigma R = 2(1.068 \times 10^{-3}) + R_{AIR} + R_{CONV}$

$= 0.06633$ K/W

$q = 37/0.06633 = \underline{585 \text{ W}}$

$T_i = 27 - \frac{585}{(12)(12)} = \underline{\underline{22.94 \text{ C}}}$

17.8 BRICK SIZE = 9" × 4.5" × 3"

BRICK #1 $\quad k = 0.44 \frac{\text{Btu}}{\text{HR FT F}}$ $\quad T_{MAX} = 1500$ F

#2 $\quad k = 0.94$ " $\quad T_{MAX} = 2200$ "

MOST ECONOMICAL ARRANGEMENT IS TO USE AS MUCH OF #1 AS POSSIBLE (LOW k). USE #2 NEXT TO HIGH TEMP SUCH THAT ITS COOLER SURFACE HAS $T \leq 1500$ F.

$q'' = \frac{2000 - T}{L/k}$ $\quad L \geq \frac{k(2000 - T_m)}{200}$

$= 2.35$ FT

$= 28.2$ IN.

$L_{ACTUAL} = 28.5$ IN $\quad (9 \times 2 + 4.5 + 2 \times 3)$

$T_{INTERFACE} = T_H - \frac{q''}{k_2/L_2} = 1495$ F

$L_{1\,MIN} = \frac{0.44(1495 - 300)}{200} = 2.63$ FT

$= 31.6$ IN.

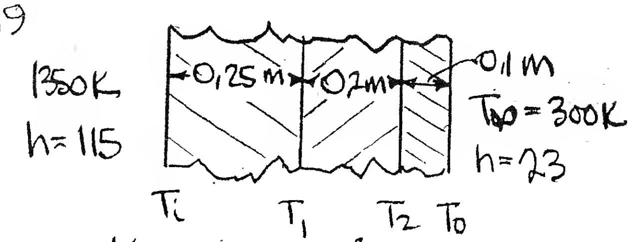
$L_{1\,ACT} = 33$ IN

17.8 CONTINUED -

MOST ECONOMICAL: $\quad \underline{L_1 = 33 \text{ IN}}$

$\underline{L_2 = 28.5"}$

17.9



$R_i = 1/115 = 8.696 \times 10^{-5}$ K/W

$R_1 = 0.25/1.13 = 0.221$ "

$R_2 = 0.20/1.45 = 0.138$ "

$R_3 = 0.10/0.66 = 0.152$ "

$R_o = \frac{1}{23} = 0.0435$ K/W

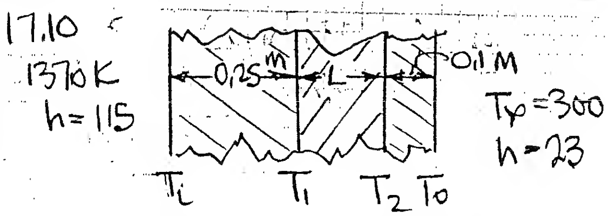
$\Sigma R = 0.563$

$q = \Delta T/\Sigma R = \frac{1070}{0.563} = \underline{1900 \text{ W/m}^2}$

$= \underline{\underline{176.5 \text{ W/FT}^2}}$

$q = 23(T_o - 300)$ $\quad \underline{\underline{T_o = 383 \text{ K}}}$

17.10



$T_o = 325$ K $\quad q = 23(325 - 300)$

$= 575$ W/m²

$R_i = \frac{1}{115} = 8.696 \times 10^{-5}$ K/W

$R_1 = 0.25/1.13 = 0.221$ "

$R_2 = L/1.45$

$R_3 = 0.10/0.166 = 0.152$ "

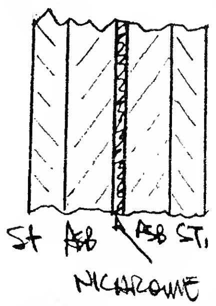
17.10 CONTINUED

$$\Sigma R = 0.416 + L/1.45 = \frac{\Delta T}{q}$$

$$0.416 + L/1.45 = \frac{1045}{575}$$

$$\underline{\underline{L = 2.03 \text{ m}}}$$

17.11



$T_\infty \approx 70$ F (ASSUME)

$L_{st} = 1/8''$ $k = 10 \frac{Btu}{hr\,ft\,F}$

$L_{asb} = 1/8''$ $k = 0.15$ "

$$q = \frac{\Delta T}{\Sigma R} = \frac{1000 - T_\infty}{\frac{0.125/12}{0.15} + \frac{0.125/12}{10} + \frac{1}{3}}$$

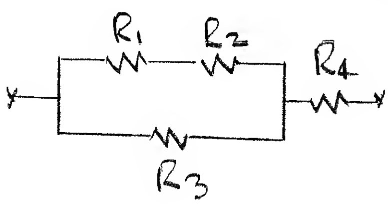
$$= \frac{930}{0.403} = 2305 \text{ Btu/HR FT}^2 \text{ PER SIDE}$$

$$\frac{q}{A} = \frac{2305(2)}{3.413} = \underline{\underline{1351 \text{ W/ft}^2}} \text{ (a)}$$

$$2305 = h\Delta T = 3\Delta T$$

$$\Delta T = 768 \text{ F} \quad T_{SURF} = \underline{\underline{838 \text{ F}}} \text{ (b)}$$

17.12



$$R_1 = \frac{0.125/12}{0.15} = 0.0694$$

$$R_2 = \frac{0.125/12}{10} = 0.00104$$

$$R_3 = \frac{0.25/12}{(22)(2)(\pi/4)(0.75/12)^2}$$

$$= 0.154$$

17.12 CONTINUED –

$$R_{CONDUCTION,EQUIV} = \frac{1}{\frac{1}{R_1+R_2} + \frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{0.07044} + \frac{1}{0.154}} = 0.0483$$

$$\Sigma R_{PER\,SIDE} = 0.0483 + \frac{1}{3} = 0.3817$$

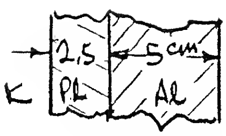
$$\text{NEW HT FLUX} = \frac{930}{0.3817} = 2437 \text{ Btu/HR-FT}^2$$

$$\text{INCREASE} = \frac{2437 - 2305}{2305}$$

$$= \underline{\underline{0.057 = 5.7\ \%}}$$

17.13

$T_\infty = 295$ K



$T_i = 325$ K

$T_\infty = 295$ K

$h_1 = 12$ W/m²K

$h_2 =$ "

$k_1 = 2.42$ W/m·K

$k_2 = 229$

a) APPLIED TO PLASTIC:

$$q = 12(T_1 - 295) + \frac{2.42}{0.025}(T_1 - 325)$$

$$\frac{229}{0.05}(325 - T_2) = 12(T_2 - 295)$$

$$T_2 = 324.9 \quad T_1 = 328.7$$

$$q = 359 + 404 = \underline{\underline{763 \text{ W}}}$$

b) APPLIED TO AL:

$$q = 12(T_2 - 295) + \frac{229}{0.05}(T_2 - 325)$$

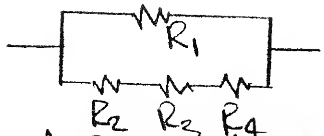
$$\frac{229}{0.05}(T_2 - 325) = \frac{2.42}{0.025}(325 - T_1)$$

$T_1 = 322$K

$T_2 = 325$

$$q = 320 + 361 = \underline{\underline{681 \text{ W}}}$$

17.14



BOLTS IN A SQUARE ARRAY WITH 4 EQUIV. BOLTS/FT^2

$$R_1(\text{BOLTS}) = \frac{L}{kA} = \frac{3.75/12}{(10)(4)(\pi/4)(\frac{1/4}{144})}$$

$= 5.7$ HRF/BTU − STEEL

$= 0.475$ " − ALUM.

$$R_{2SS} = \frac{1/48}{(10)(1)} = 0.0021$$

$$R_{3CB} = \frac{3/12}{(0.25)(1)} = 10$$

$$R_{4PL} = \frac{1/24}{(1.5)(1)} = 0.0278$$

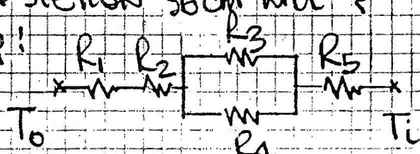
$$\Sigma R = 10.03$$

PER FT^2 OF X-SECTION

$$R_{EQUIV} = \frac{1}{1/5.7 + 1/10.03} = \underline{3.63 \ \frac{HR\text{-}F}{BTU}} \quad (a)$$

$$= \frac{1}{1/0.475 + 1/10.03} = 0.454 \text{ "} \quad (b)$$

17.15 FOR A SECTION 36 cm WIDE & 1 m DEEP!



$$R_1 = \frac{1}{A h_o} = \frac{1}{(0.36)(20)} = 0.139 \ K/W$$

$$R_2 = \frac{L}{kA}\bigg|_{PL} = \frac{0.02}{(0.814)(0.36)} = 0.0683 \text{ "}$$

$$R_3 = \frac{L}{kA}\bigg|_{W} = \frac{0.15}{0.15(0.06)} = 16.67 \text{ "}$$

17.15 CONTINUED -

$$R_4 = \frac{L}{kA}\bigg|_{FG} = \frac{0.15}{0.035(0.30)} = 14.28 \text{ "}$$

$$R_5 = \frac{1}{A_i h_i} = \frac{1}{(0.36)(10)} = 0.278 \text{ "}$$

$$R_{\substack{STUD\\WALL}\,EQUIV} = \frac{1}{1/16.67 + 1/14.28} = 7.691 \text{ "}$$

$$q = \frac{\Delta T}{\Sigma R_{EQUIV}} = \frac{35 \ K}{8.176 \ K/W} = 4.28 \ W$$

$$q_3 = \frac{\Delta T_{SW}}{R_3} \qquad q_4 = \frac{\Delta T_{SW}}{R_4}$$

$$\Delta T_{SW} = \frac{q}{1/R_3 + 1/R_4} = 32.92 \ K$$

$$q_3 = \frac{32.92}{16.67} = \underline{\underline{1.975 \ W}}$$

$$q_4 = \underline{\underline{2.305 \ W}}$$

17.16

$$q_{LOSS} = q_1 + q_2$$

q_1 = HT TO AIR = $hA\Delta T$

$$= (23 \ W/m^2\cdot K)(2.5 \times 0.1 \times 2 \ m^2 + 2.5 \times 0.05 \times 2 \ m^2 + 0.1 \times 0.05 \times 2 \ m^2 - (0.08)^2 \times 2 \ m^2)(T - 300)$$

$$= 17.19 \ (T - 300)$$

q_2 = HT THROUGH PEDESTALS

$$= 2kAm\theta_o\left[1 - 2\,\frac{\theta/\theta_o - e^{-mL}}{e^{mL} - e^{-mL}}\right]$$

$$m = \left[\frac{hP}{kA}\right]^{1/2} = \left[\frac{(23)(0.08)(4)}{(2.6)(0.08)(0.08)}\right]^{1/2} = 21$$

$$e^{mL} = 23.45 \qquad e^{-mL} = 0.0427$$

17.16 CONTINUED -

$$q_2 = 2(2.6)(0.08)^2(21)(T-300) \times \left[1 - 2\frac{-0.0427}{23.45}\right]$$

$$= 0.696(T-300)$$

$$1000 = (17.19 + 0.696)(T-300)$$

$$\underline{\underline{T = 355.9\ K}}$$

17.17

$$q_{LOSS} = q_1 + q_2 + q_3$$

$$q_1 = 17.19(T-300) \quad \text{FROM PREV. PROB.}$$

q_2 = SAME EXPRESSION

$$A = (0.08)(0.08) - \frac{\pi}{4}(0.019)^2$$

$$= 0.00612 \quad \left\{ \begin{matrix} \text{PREVIOUSLY} \\ 0.0064 \end{matrix} \right\}$$

FOR PEDESTAL MAT'L:

$$q_2 \cong 0.7(T-300)$$

q_3 = CONDUCTION THROUGH BOLTS

$$= \frac{kA}{L}\Delta T = 42.9\,\frac{\pi}{4}\,\frac{(0.019)^2}{0.15}\,\Delta T$$

$$= 0.081(T-300)$$

$$1000 = [17.19 + 0.7 + 0.081](T-300)$$

$$\underline{\underline{T = 355.6\ K}}$$

17.18

$$q = \Delta T/\Sigma R$$

$$\Delta T = 292.7 - 70 = 222.7\ F$$

ROOM TEMP (ASSUMED)

FOR 2-IN SCHED 40 ID = 2.067 IN
OD = 2.375 "

$$R_{ST} = \frac{\ln D_o/D_i}{2\pi kL} = 1.475 \times 10^{-5}$$

$$R_{INS} = \frac{\ln D_o/D_i}{2\pi kL} = 0.0529$$

$$R_{OUTS} = 1/hA_o = 0.00537 \quad \text{w/o INSUL}$$

$$= 0.00237 \quad \text{w/ INSUL}$$

$$\Sigma R_{WITH} = 0.0553 \qquad q = 4030\ \text{Btu/hr}$$

$$\Sigma R_{WITHOUT} = 0.00537 \qquad q = 41{,}500\ "$$

$$\Delta q = 37470\ \text{Btu/hr}$$

$$\text{COST} = 37470\left(\frac{\$0.68}{10^5}\right) = \$0.255/\text{HR}$$

$$\text{TIME} = \frac{(60\ \text{FT})(\$0.75/\text{FT})}{\$0.255/\text{HR}} = \underline{\underline{177\ \text{HOURS}}}$$

17.19

$$267.25\,F \;\text{—}\; R_1 \;\text{—}\; R_2 \;\text{—}\; R_3 \;\text{—}\; 80\,F$$

$$R_1 = \frac{1}{\pi D_i h_i L} = \frac{1}{\pi(1.610/12)(1500)(10)} = 0.0015$$

$$R_2 = \frac{\ln D_o/D_i}{2\pi kL} = \frac{\ln 1.078}{2\pi(24)(10)} = 6.19 \times 10^{-5}$$

$$R_3 = \frac{1}{\pi(1.755/12)(2)} = 0.0725$$

$$\Sigma R = 0.072\ \text{HR F/Btu}$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{187.25}{0.072} = 2530\ \frac{\text{Btu}}{\text{HR}} \quad (a)$$

$$267.25\,F \;\text{—}\; R_1{+}R_2 \;\text{—}\; R_3 \;\text{—}\; R_4 \;\text{—}\; 80\,F$$

$$R_1 + R_2 = 0.00156\ \text{HRF/Btu}$$

$$R_3 = \frac{\ln 5.755/1.755}{2\pi(0.041)(10)} = 0.461\ \frac{\text{HRF}}{\text{Btu}}$$

17.19 CONTINUED -

$$R_4 = \frac{1}{\pi(5.755/12)(3)(10)} = 0.0224$$

(b)

$$\sum R = 0.485 \quad q = \frac{\Delta T}{\sum R} = 386 \frac{Btu}{HR}$$

FOR BARE PIPE: $\dot{m}_{STM} = q/h_{fg}$

$$= \frac{2530\ Btu/HR}{933.7\ Btu/LB_m} = 2.71 \frac{LB_m}{HR} \quad (c)$$

17.20

$$\dot{q} \frac{\pi D^2}{4} L = h \pi D L \Delta T$$

$$\frac{I^2 R}{\pi D^2/4 L} \frac{D}{4} = h \Delta T$$

$I^2 R = 10 kW$

$h = 850\ W/m^2 \cdot K$

$L = 0.6 m \quad \Delta T = 1280 K$

$$D = \frac{4(10000\ W)}{(1280K)(\pi)(0.6m)(850\ W/m^2 \cdot K)}$$

$$= 0.0195\ m = 1.95\ cm \quad (a)$$

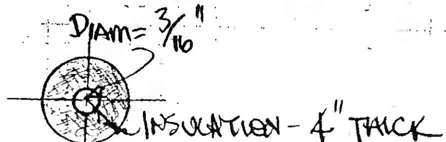
14 GAGE = 0.064 IN, DIAM = 1.626×10^{-3} m

$L = 7.2 m \quad (b)$

FOR $h = 1150\ W/m^2 \cdot K$

$$\left.\begin{array}{l} D = 1.44\ cm \\ L = 5.32\ m \end{array}\right\} (c)$$

17.21



$$\frac{q}{L} = \frac{2\pi k}{\ln r_o/r_i}(T_i - T_o)$$

$$= \frac{2\pi(0.14)}{\ln \frac{8.094}{0.1875}}(120-70)$$

$$= 11.72\ Btu/HR\text{-}FT = 3.43\ W$$

17.21 CONTINUED -

$I^2 R = 3.43\ W$

$R = \rho \frac{L}{A} = 2.95 \times 10^{-4}\ \Omega$

$$I^2 = \frac{3.43}{2.95 \times 10^{-4}} \quad I = 107.9\ Amp$$

17.22

$$\frac{q}{L} = \frac{120-70}{\frac{\ln \frac{8.094}{0.1875}}{2\pi(0.14)} + \frac{1}{\pi\left(\frac{131}{16 \times 12}\right)(4)}}$$

$I = 106.1\ Amp$

$$11.34 = 4\left[\frac{131}{16 \times 12}\right]\pi(T-70)$$

$T = 71.3\ F$

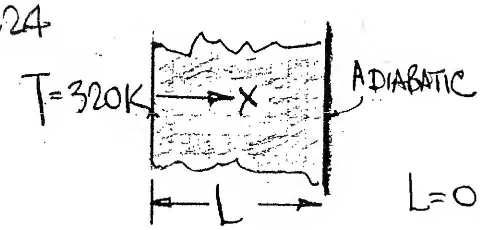
17.23

SAME AS PROB 17.21 EXCEPT WIRE IS ALUMINUM -

$R = 4.85 \times 10^{-4}\ \Omega$

$I = 83.9\ Amp$

17.24



$L = 0.06 m$

$$\dot{q} = q_o\left[1 - x/L\right]$$

POISSON EQN APPLIES: (ENERGY BALANCE)

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{q_o}{k}\left[1 - \frac{x}{L}\right]$$

$$\frac{dT}{dx} = -\frac{q_o}{k}\left[x - \frac{x^2}{2L}\right] + C_1$$

17.24 CONTINUED

$$\frac{dT}{dx}(L)=0 \quad C_1 = \frac{\dot{q}_o}{k}\frac{L}{2}$$

$$\frac{dT}{dx} = \frac{\dot{q}_o}{k}\frac{L}{2}\left[1-2\frac{x}{L}+\frac{x^2}{L^2}\right]$$

$$T = \frac{\dot{q}_o L}{2k}\left[x - \frac{x^2}{L} + \frac{x^3}{3L^2}\right]+C_2$$

$$T(0) = T_0 = C_2$$

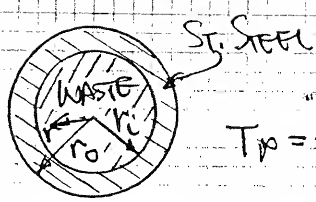
$$T = T_0 + \frac{\dot{q}_o}{k}\frac{L}{2}\left[x - \frac{x^2}{L} + \frac{x^3}{3L^2}\right] \quad (a)$$

(b) $T = T_{MAX}$ WHERE $\frac{dT}{dx}=0$ I.E. @ $x=L$

$$T(L) = T_0 + \frac{180}{0.6}\left(\frac{0.06}{2}\right)\left(\frac{0.06}{3}\right)(1000)$$

$$= 320+180 = \underline{500\ K} \quad (c)$$

17.25



$T(r_i)=T_i$

$T(r_o)=T_o$

$T_\infty = 298 K$

IN WASTE MAT'L: $q = \dot{q}(\pi r_i^2 L)$ (1)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

$$\cancel{r}\frac{dT}{dr}+\frac{\dot{q}r^{\cancel{2}}}{2k} = C_1/r$$

$$\frac{dT}{dx}(0)=0 \Rightarrow C_1=0$$

$$T+\frac{\dot{q}r^2}{4k} = C_2$$

$$T(r_i)=T_i \qquad C_2 = T_i + \frac{\dot{q}r_i^2}{4k}$$

$$T = T_i + \frac{\dot{q}}{4k}(r_i^2-r^2)$$

FOR ST. STEEL:

$$q = \frac{2\pi k L}{\ln r_o/r_i}(T_i-T_o) = 2\pi r_o L h(T_o-T_\infty)$$

EQUATING WITH EQN (1)

$$\frac{2\pi k L}{\ln r_o/r_i}(T_i-T_o) = 2\pi r_o L h(T_o-T_\infty) = \dot{q}\pi r_i^2 L$$

17.25 CONTINUED -

PUTTING IN VALUES - $\underline{T_o = 339.7\ K}$

$$T_i = 339.7+303 = \underline{642.7\ K} \quad (a)$$

@ CENTER OF WASTE MAT'L:

$$T = 642.7 + \frac{\dot{q}}{4k}r_i^2$$

$$= 642.7+625 = \underline{1268\ K}\ (b)$$

17.26

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}r}{k}=0$$

$$\cancel{r}\frac{dT}{dr}+\frac{\dot{q}r^{\cancel{2}}}{2k}=\frac{C_1}{r}$$

$$\frac{dT}{dr}(0)=0 \Rightarrow C_1=0$$

$$T+\frac{\dot{q}r^2}{4k}=C_2$$

$$T(R)=T_R \qquad C_2=T_R+\frac{\dot{q}R^2}{4k}$$

$$T-T_R = \frac{\dot{q}}{4k}(R^2-r^2) \quad (a)$$

$$T_{MAX} = T \text{ @ } r=0$$

$$T_{MAX} = T_R + \frac{\dot{q}R^2}{4k}$$

$$= T_R + \frac{(51.7\times10^6)(0.1077)^2}{4(339)}$$

$$= T_R + 442$$

$$q = \dot{q}V = \dot{q}\frac{\pi D^2 L}{4}$$

$$= \frac{(51.7\times10^6)(\pi)(0.1077)^2(0.1016)}{4}$$

$$= h(\pi)(0.1077)(0.1016)\Delta T$$

$$\Delta T = 307\ K$$

$\underline{T_{SURF} = 332C}$ $\underline{T_{MAX} = 774C}$

(c) (b)

17.27 ASSUME THIN-WALLED INNER VESSEL IS 77 K THROUGHOUT

$$77K \;-\!\!\times\!\!-\!\!\!\sqrt{\ }\!\!\!-\!\!\!\sqrt{\ }\!\!\!-\; 298K \qquad R_i \quad R_o$$

$$R_i = \frac{1/r_i + 1/r_o}{4\pi k} = \frac{1/0.5 - 1/0.55}{4\pi(0.002)} = 7.234$$

$$R_o = \frac{1}{4\pi r_o^2 h} = \frac{1}{4\pi(0.55)^2(18)} = 0.0146$$

$$q = \frac{\Delta T}{\sum R} = \dot{m} h_{fg}$$

$$\dot{m} = \frac{221/7.29}{2\times10^5} = \underline{1.524\times10^{-4}\ kg/s}$$

17.28 FOR $q = \frac{1}{2}$ OF VALUE IN 17.27

$$\sum R = 14.58 = R_{conv} + R_{INS}$$

$$R_{INS} = \frac{1/0.5 - 1/r_o}{4\pi(0.002)} \qquad R_{conv} = \frac{1}{4\pi r_o^2 (18)}$$

$$14.58 = \frac{1}{4\pi}\left[500\left(\frac{1}{0.5} - \frac{1}{r_o}\right) + \frac{1}{18 r_o^2}\right]$$

$r_o = 0.611\ m$ INS, THICKNESS $= 0.0555\ m$

ADDED THICKNESS $= \underline{0.0055\ m}$ OR $\underline{5.5\ mm}$

17.29 PER FOOT OF BARE TUBE:

$$q = hA\Delta T = \left(6\ \frac{Btu}{hr\,ft^2\,F}\right)\left(\frac{\pi}{12}\ ft\right)^2(170\,F)$$

$$= \underline{267\ Btu/hr}$$

FOR LONGITUDINAL FINS:

$$A_f = 12(3/4)(1/12)(2)(1) = 1.5\ ft^2$$

(ENDS NEGLECTED)

$$A_0 = (\pi/12)(1) - 12(3/32)(1/12) = 0.168\ ft^2$$

$$L(h/kt)^{1/2} = \left(\frac{3/4}{12}\right)\left[\frac{6}{24.8(3/64)(1/12)}\right]^{1/2}$$

$$= 0.539 \qquad \eta_F \cong 0.92$$

17.29 CONTINUED

$$q = h(A_o + \eta_F A_F)\Delta T$$

$$= 6(0.168 + 0.92\times1.5)(170) = 1580\ \frac{Btu}{hr}$$

INCREASE $= 1580 - 267 = \underline{1310\ Btu/hr}$

% INCR $= \underline{491\ \%}$

FOR CIRCULAR FINS:

PER FIN: $A_f = 2\frac{\pi}{4}\left[\frac{2.5^2 - 1^2}{144}\right] = 0.0573\ ft^2$

PER FOOT $n = 1.5/0.0573 = 26$ FINS

$$A_0 = \frac{\pi}{12} - \left(\frac{\pi}{12}\times\frac{3}{32}\times\frac{1}{12}\times26\right) = 0.209\ ft^2$$

$$(r_o - r_i)\left[h/kt\right]^{1/2} = \frac{3/4}{12}\left[\frac{6}{(24.8)(3/64)(1/12)}\right]^{1/2}$$

$$= 0.539 \qquad \eta_F \cong 0.88$$

$$q = 6\left[0.209 + (0.88)(26)(0.0573)\right](170)$$

$$= 1560\ Btu/hr$$

INCREASE $= 1560 - 267 = \underline{1290\ \frac{Btu}{hr}}$

$$\%\,INCR = \frac{1290}{267} = \underline{484\ \%}$$

17.30 $h = 60\ Btu/hr\,ft^2\,F$

LONGITUDINAL CASE $L\left(\frac{h}{kt}\right)^{1/2} = 1.70$

$$\eta_F \cong 0.56$$

$$q = 10820\ Btu/hr$$

CIRCULAR CASE - $(r_o - r_i)\left(\frac{h}{kt}\right)^{1/2} = 1.70$

$$\eta_F \cong 0.52$$

$$q = 10180\ Btu/hr$$

W/O FINS $q = 2670\ Btu/hr$

INCREASE:

LONG:	$\underline{8150\ Btu/hr}$	$\underline{305\ \%}$
CIRC:	$\underline{7510\ "}$	$\underline{281\ \%}$

17.31

SOLUTION FOR $\theta = \frac{T-T_\infty}{T_0-T_\infty}$

IS IN FORM $\theta = C_1 e^{mx} + C_2 e^{-mx}$

$$m = \left[\frac{(17\ W/m^2{\cdot}K)(\pi)(0.03\ m)4}{k\pi\,(0.03\ m)^2}\right]^{1/2} = \frac{47.61}{k^{1/2}}$$

LONG FIN APPROXIMATION: $\theta = C_2 e^{-mx}$

$$\theta_1 = 99 = C_2 e^{-mx}$$

$$\theta_2 = 65 = C_2 e^{-m(x_1+0.076)}$$

$$\frac{\theta_2}{\theta_1} = \frac{65}{99} = e^{-m(x_1+0.076-x_1)}$$

$$m = 5.536 = \frac{47.61}{k^{1/2}}$$

$$\underline{\underline{k \cong 74\ W/m^2{\cdot}K}}$$

17.32

$$\eta_f = \text{fn of } L\left[h/kt\right]^{1/2}$$

AIRSIDE: $= \frac{0.75}{12}\left[\frac{(2)(3)}{(0.05/12)(229)}\right]^{1/2}$

$= 0.156 \qquad \eta_f \cong 0.98$

WATERSIDE: $= \frac{0.75}{12}\left[\frac{2(25)}{(0.5/12)(229)}\right]^{1/2}$

$= 0.143 \qquad \eta_f \cong 0.98$

FOR 1 FT² $\quad A_0 = 1 - 150\left(1\times\frac{0.05}{12}\right) = 0.375\ ft^2$

$$A_f = 150(2)\left(\frac{0.75}{12}\right)(1) + 0.625 = 19.35''$$

W/O FINS: $q = 25\,\Delta T_W = 3\,\Delta T_A = \dfrac{\Delta T_{OA}}{\frac{1}{25}+\frac{1}{3}}$

WITH FINS:

WATERSIDE: $q = 25\,\Delta T_W(0.375+18.96) = 483\,\Delta T_W$

AIRSIDE: $q = 3\,\Delta T_A(0.375+18.96) = 58.0\,\Delta T_A$

FINS ADDED TO AIRSIDE ONLY:

17.32 CONTINUED

$$q = \frac{\Delta T_{OA}}{1/25 + 1/58} = 17.47\,\Delta T_{OA}$$

$\%$ GAIN $= \underline{\underline{549\ \%}}$

TO WATERSIDE:

$$q = \frac{\Delta T_{OA}}{1/483 + 1/3} = 2.98\,\Delta T_{OA}$$

GAIN $= \underline{\underline{11.2\ \%}}$

BOTH SIDES:

$$q = \frac{\Delta T_{OA}}{1/483 + 1/58} = 51.78\,\Delta T_{OA}$$

GAIN $= \underline{\underline{1832\ \%}}$

17.33 ONE-DIM. CONDUCTION WITH INTERNAL HT GENERATION

$$\frac{d^2T}{dx^2} = -\frac{\dot q}{k}$$

$$\frac{dT}{dx} = -\frac{\dot q}{k}x + C_1$$

$$T = -\frac{\dot q}{k}\frac{x^2}{2} + C_1 x + C_2$$

$$T(0) = T_0 = C_2$$

$$T(L) = T_L = -\frac{\dot q}{k}\frac{L^2}{2} + C_1 L + T_0$$

$$C_1 = \frac{T_L - T_0}{L} + \frac{\dot q}{k}\frac{L}{2}$$

$$T = \frac{\dot q}{k}\left[\frac{Lx}{2} - \frac{x^2}{2}\right] + \frac{T_L - T_0}{L}x + T_0$$

OR $\quad \underline{T - T_0 = \frac{\dot q L^2}{2k}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] + (T_L - T_0)\left(\frac{x}{L}\right)}$

AT $\frac{L}{2}$: $\quad T - T_0 = \frac{\dot q L^2}{2k}\left(\frac{1}{4}\right) + \frac{T_L - T_0}{2}$

$$\dot q = \frac{I^2 R}{V} = \frac{I^2 \rho \cancel{L}}{A^2 \cancel{L}}$$

$$= \frac{(10)^2(2\times10^{-5})}{\frac{\pi}{4}(0.01)^2} = 25.47\ W/m^3$$

17.33 CONTINUED

MID-PT. TEMP:

$$T_{M.P.} = \frac{25{,}47\,W/m^3\,(0.04\,m)^2}{8\,(2\,W/m\cdot K)} + 50$$

$$\cong 50.00\text{-}C$$

$$q = -kA\frac{dT}{dx} = -kA\left[-\frac{\dot{q}}{k}x + C_1\right]$$

$$= -kA\left[-\frac{\dot{q}}{k}x + \frac{T_L - T_0}{L} + \frac{\dot{q}}{k}\frac{L}{2}\right]$$

$$@x=0 \quad q = -kA\left[\frac{T_L - T_0}{L} + \frac{\dot{q}L}{2k}\right]$$

$$= -\ 0.393\ W$$

$$\text{AT } x=L \quad q = -kA\left[-\frac{\dot{q}L}{2k} + \frac{T_L - T_0}{L}\right]$$

$$= +\,0.393\ W$$

17.34 $$m^2 = \frac{hP}{kA} = \frac{(740)(\pi)(0.019)(4)}{(54)(\pi)(0.019)^2}$$

$$= 2885\ m^{-2}$$

$$\theta = \theta_0 \frac{\cosh mx}{\cosh mL/2}$$

$$\frac{d\theta}{dx} = m\theta_0 \frac{\sinh mx}{\cosh mL/2} \Rightarrow \sinh mx = 0 \quad @x=0$$

$$\theta_{MAX} = \frac{\theta_0}{\cosh mL/2} = \frac{145\,K}{\cosh 12.08}$$

$$T \cong 625\ K$$

17.35 ENERGY BALANCE:

$$\frac{d^2\theta}{dx^2} - m^2\theta = -\frac{W}{kA} \quad \Big\{ W \text{ IN } \frac{Btu}{FT}$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx} + W/hP$$

$$\theta(0)=0 \quad C_1 = \frac{-W/hP\,(1 - e^{-mL})}{e^{mL} + e^{-mL}}$$

$$\theta(L)=0 \quad C_2 = \frac{-W/hP\,(e^{mL} - 1)}{e^{mL} - e^{-mL}}$$

17.35 CONTINUED -

PUTTING IN VALUES:

$$m = 2.28 \quad C_1 = -0.0017\ W/hP$$

$$C_2 = -0.999$$

$$\theta = \theta_{MAX} @ 1.5\,FT$$

$$W_{MAX} = I^2_{MAX}R$$

$$R = \frac{\rho L}{A} = \frac{(1.72\times10^{-6})(3)}{\frac{\pi}{4}(1/48)^2(30.48)}$$

$$= 4.97\times10^{-4}\ \Omega/FT$$

$$\theta_{MAX} = 90\left[-0.00107\,e^{2.28(1.5)} - 0.999\,e^{-0.228(1.5)} + 1\right]\frac{W}{hP}$$

$$\frac{W}{hP} = 96.3 \quad W = 96.3\,(6)(\pi)(0.25/12)$$

$$= 37.8\ Btu/HR\ FT$$

$$= 11.08\ W/FT$$

$$I^2_{MAX} = \frac{11.08}{4.97\times10^{-4}} = 2.23\times10^4\ A^2$$

$$I_{MAX} = 150\ A$$

17.36 $$m^2 = \frac{hP}{kA} = \frac{45(2)}{42\,(0.0159)} = 134.8\ m^{-2}$$

$$m = 11.61\ m^{-1}$$

$$q = kAm\theta_0 \frac{\sinh mL + h/mk \cosh mL}{\cosh mL + h/mk \sinh mL}$$

$$kAm\theta_0 = (42)(0.0159)(11.61)(300)$$

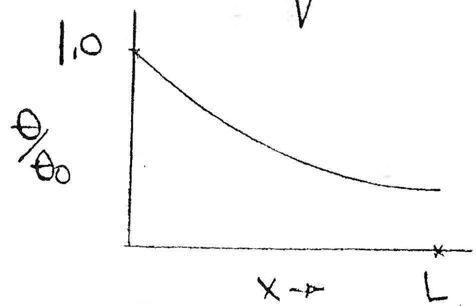
$$= 2330$$

$$\sinh mL = 2.01 \quad \cosh mL = 1.75$$

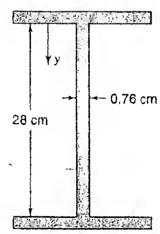
$$h/mk = \frac{45}{11.61(42)} = 0.0923$$

17.36 CONTINUED-

SUBSTITUTING: $\underline{\underline{q = 2.25 \text{ kW}}}$



17.37



$$\frac{\theta}{\theta_0} = \left[\frac{\theta_L}{\theta_0} - e^{-mL}\right]\left[\frac{e^{mx} - e^{-mx}}{e^{mL} - e^{-mL}}\right] + e^{-mx}$$

$\theta_0 = 400 \qquad m = 19.6\,m^{-1}$

$mL = 5.69 \qquad e^{mL} = 242$

$e^{-mL} = 0.0041$

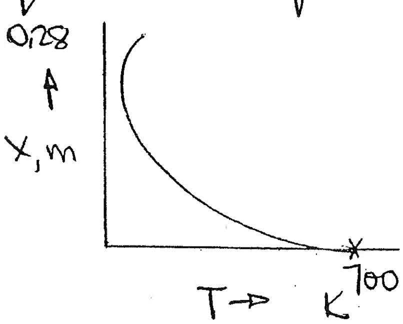
SUBSTITUTING:

$\theta = 0.262\, e^{19.6x} + 399.7\, e^{-19.6x}$

$q = -kA \frac{dT}{dx} \qquad \frac{dT}{dx}(0) = -1829$

$\frac{dT}{dx}(L) = 1305$

$q_0 = \underline{\underline{-2.32 \text{ kW}}} \quad q_L = \underline{\underline{387 \text{ W}}}$



17.38

FOR ALUMINUM I-BEAM:

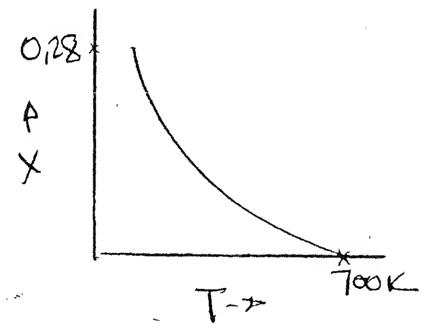
SAME PROCEDURE AS PREVIOUS PROB.

$m = 8.08 \qquad e^{mL} = 9.61$

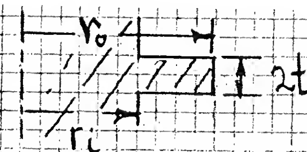
$e^{-mL} = 0.104$

$\theta = 3e^{8.08x} + 688\, e^{-8.08x}$

$q_0 = \underline{\underline{551 \text{ kW}}} \quad q_L = \underline{\underline{170 \text{ W}}}$



17.39



$r_i = \frac{0.3}{2} = 0.15\text{ m} \quad r_0 = 0.15 + 0.02 = 0.17\text{ m}$

$t = 0.003/2 = 0.0015\text{ m}$

$\text{AREA} = 2\pi(r_0^2 - r_i^2) + A_{END}$

$= 2\pi(0.17^2 - 0.15^2) + 2\pi(0.17)(0.003)$

$= 0.0434\ m^2$

$$(r_0 - r_i)\sqrt{h/kt} = 0.02\left[\frac{12}{(46.4)(0.0015)}\right]^{1/2}$$

$= 0.263$

$r_0/r_i = 1.13$

FROM FIG 17.11 $\quad \eta_F \cong 0.96$

$q = A_F \eta_F h(T_0 - T_\infty)$

$= (0.0434)(0.96)(12)(270)$

$= \underline{135 \text{ W}} \quad \underline{\text{PER FIN}}$

17.39 CONTINUED -

FOR A 30% 3 kW ENGINE

$$Q_{IN} = \frac{3\text{ kW}}{0.3} = 10\text{ kW}$$

$$Q_{OUT} = Q_{IN} - W = 7\text{ kW}$$

AMOUNT Tx FROM FINS $= 0.5(7)$
$= 3.5\text{ kW}$

NO. OF FINS REQ'D:

$$n = \frac{3500\text{ W}}{135\text{ W/FIN}} = 25.92$$

26 FINS REQUIRED

17.40

a) L = 20 mm b)
t = 6 mm

FOR BOTH CASES: $T_b = 120C$
$T_\infty = 20C$

$h = 60\text{ W/m}^2\text{·K}$ $k_{S.S.} = 15.3\text{ W/m·K}$

FOR CASE a) STRAIGHT FIN

$$q = \eta_F h A_F \theta_b$$

USING TEXT - FIG 17.11

$$\left(L+\frac{t}{2}\right)^{3/2}\left[\frac{h}{kt\left(L+t/2\right)}\right]^{1/2}$$

$$= (0.02+0.003)^{3/2}\left[\frac{60}{(15.3)(0.006)(0.023)}\right]^{1/2}$$

$= 0.588$ $\eta_F \cong 0.80$

PER m OF WIDTH: (NEGLECT ENDS)

$$q_F = 0.80(60)(100)(2)(0.020)$$

$$= \underline{192\text{ W/m}}$$

17.40 CONTINUED -

FOR CASE b) - TRIANGULAR

$$L^{3/2}\left[\frac{h}{ktL}\right]^{1/2}$$

$$= 0.02^{3/2}\left[\frac{60}{(15.3)(0.003)(0.02)}\right]^{1/2}$$

$= 0.723$ $\eta_F \cong 0.81$

$$q = \eta_F A_F h \theta_b$$

$$= (0.81)(2)(0.02)(60)(100)$$

$$= \underline{194.4\text{ W/m}}$$

17.41 W/OUT FINS:

$$q_0 = h\Delta T\pi\left(\frac{2}{12}\right) = 712\text{ Btu/HR}$$

FOR LONGITUDINAL FINS:

$$L\left(\frac{h}{kt}\right)^{1/2} = \frac{1}{12}\left[\frac{8}{10\left(\frac{1/32}{12}\right)}\right] = 1.46$$

$\eta_F \cong 0.6$

$$A_0 = \pi D_0 - 16(2t) = 0.440\text{ ft}^2$$

$$A_F = 16(2)(1/12) + 16(1/16)(1/12) \cong 2.67\text{ ft}^2$$

$$q = h\Delta T\left[A_0 + \eta_F A_F\right]$$

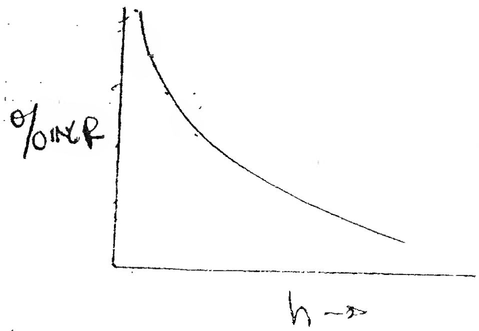
$$= 8(170)\left[0.440 + 0.6(2.67)\right] \cong 2780\ \frac{\text{Btu}}{\text{HR}}$$

INCREASE $= \underline{2068\ \frac{\text{Btu}}{\text{HR}}}$ %INCR $= \underline{290\%}$ a)

FOR VARYING VALUES OF h:

h	$L\sqrt{h/kt}$	η_F	%INCR
2	0.731	0.84	412
5	1.156	0.71	346
8	1.462	0.60	290
15	2.00	0.48	229
50	3.65	0.27	122
100	5.17	0.19	81

17.41 CONTINUED



I.E. FINS ARE MOST EFFECTIVE WHEN h IS SMALL

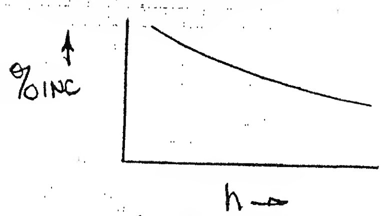
17.42 SAME AS 17.41 EXCEPT MAT'L IS ALUMINUM

W/O FINS $q = 712$ BTU/HR

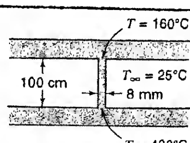
$L\left[h/kt\right]^{1/2} = 0.121 \qquad \eta_F \hat{=} 0.99$

(a) $q \cong \underline{4300 \text{ BTU/HR}} \qquad \% \text{INCR} = \underline{503\%}$

h	2	5	15	50	100
%INCR	488	471	448	370	303



17.43



FOR KNOWN END TEMPS:

$$\frac{\theta}{\theta_0} = \frac{T-T_\infty}{T_0-T_\infty} = \left[\frac{\theta_L}{\theta_0} - e^{-mL}\right]\left[\frac{e^{mx}-e^{-mx}}{e^{mL}-e^{-mL}}\right] + e^{-mx}$$

$\theta_0 = 160-25 = 135 \qquad \dfrac{\theta_L}{\theta_0} = 2.78$

$\theta_L = 400-25 = 375$

$$m = \left[hP/kA\right]^{1/2} = \left[\frac{300(2)}{(0.008)(229)}\right]^{1/2} = 18.1\ m^{-1}$$

$e^{mL} = 6.11 \qquad e^{-mL} = 0.164$

17.43 CONTINUED -

SUBSTITUTING:

$$\frac{\theta}{\theta_0} = 0.440\,e^{18.1x} + 0.560\,e^{-18.1x}$$

$$\frac{d\theta}{dx} = \theta_0\left[7.964\,e^{18.1x} - 10.14\,e^{-18.1x}\right]$$

$@x=0 \qquad \dfrac{d\theta}{dx} = -2.176\,\theta_0$

$@x=L \qquad \dfrac{d\theta}{dx} = 47.0\,\theta_0$

$\dfrac{q}{L} = -kt\dfrac{d\theta}{dx} \qquad \dfrac{q}{L}(0) = \underline{-294 \text{ W/m}}$

$\dfrac{q}{L}(L) = \underline{6,345 \text{ kW/m}}$

17.44 USING TABLE 17.1

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{1+\rho^2-\epsilon^2}{2\rho}\right)}$$

$\rho = 0.5 \quad \epsilon = 1/6$

$$S = \frac{2\pi}{\cosh^{-1}\left(\frac{1+1/4-1/36}{1}\right)} = \frac{2\pi}{\cosh^{-1} 11/9}$$

$= 9.6$

$$\frac{q}{L} = kS\Delta T = (0.023)(9.6)(300)$$

$= \underline{66.3 \text{ BTU/HR-FT}}$

17.45 TABLE 17.1

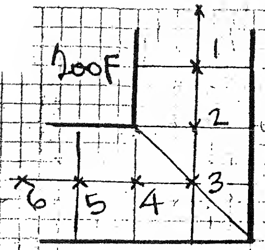
$$S = \frac{2\pi}{\cosh^{-1}(\rho/r)} = \frac{2\pi}{\cosh(2.5)} = 1.311$$

$$\frac{q}{L} = kS\Delta T$$

$= (0.341)(1.311)(60)$

$= \underline{26.83 \text{ W/m}}$

17.46



$200 + 0 + 2T_2 - 4T_1 = 0$

$200 + 0 + T_1 - T_3 - 4T_2 = 0$

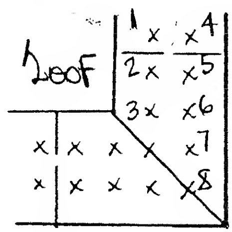
$0 + 0 + 2T_2 - 4T_3 = 0$

$T_1 = 91.7\text{F} \quad T_2 = 83.3\text{F} \quad T_4 = 41.7\text{F}$

$$\frac{q}{L} = 8k\left[\frac{200-91.7}{2} + 200 - 83.3\right]$$

$$= \underline{\underline{205 \text{ BTU/HR-FT}}}$$

17.47



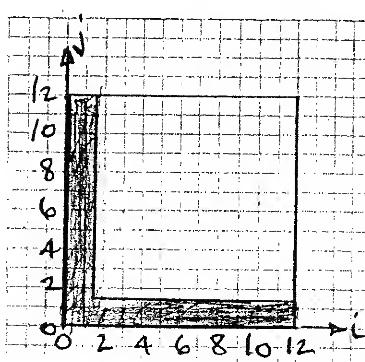
BY ITERATION:

NODE	T, F
1	123
2	112
3	74
4	58
5	51.5
6	36
7	18

$$\frac{q}{L} = 8k\left[(200-123) + (200-112)\right]$$

$$= \underline{\underline{198 \text{ BTU/HR-FT}}}$$

17.48



17.48 CONTINUED-

NUMERICAL SOLUTION USING A 12×12 MESH YIELDS THE FOLLOWING:

$\frac{q}{L} \cong \underline{\underline{1400 \text{ BTU/HR-FT}^2}}$ PER FT.

$T_{MIN} \cong \underline{\underline{91.8\text{ F}}}$ AT $i,j = 12,12$

17.49

$$S = \frac{2\pi}{\ln r_o/r_i - 0.271} = \frac{2\pi}{\ln \sqrt{2}/0.5 - 0.271} = 8.17$$

$q = kS L \Delta T$

$= (0.037)(8.17)(300)(50)$

$= 2180 \text{ BTU/HR}$

$$\text{STEAM CONDENSED} = \frac{q}{h_{fg}} = \frac{2180}{826} = \underline{\underline{2.65 \text{ LBm/HR}}}$$

17.50

$$S = \frac{2\pi}{\cosh^{-1}(\delta/r)} = \frac{2\pi}{\cosh^{-1}\frac{1.2}{0.324}} = 3.17$$

$q = kS\Delta T$

$= (0.66 \text{ W/m·K})(3.17)(90\text{K})(1.45\text{m})$

$= \underline{\underline{273 \text{ W}}}$

CHAPTER 18

18.1 $Bi = \frac{hV/S}{k} = \frac{3}{10}\ \frac{3/488}{0.5} = 0.0369$

$\therefore$ CAN USE LUMPED PARAMETERS

BY 1^{ST} LAW: $\dot{q}V - hA\theta = \rho c_p V \frac{d\theta}{dt}$

WHERE $\theta = T - T_\infty$

$$\frac{d\theta}{dt} = \frac{\dot{q}V}{\rho c_p V} - \frac{hA\theta}{\rho c_p V} = a - b\theta$$

SOLN: $t = \frac{1}{b} \ln \frac{a}{a - b\theta} = \frac{1}{b} \ln \frac{1}{1 - \frac{b}{a}\theta}$

$$a = \frac{\dot{q}}{\rho c_p} = \frac{(500)(3.413)}{3(0.11)} = 5170\ \text{F/HR}$$

$$b = \frac{hA}{\rho c_p V} = \frac{3(0.05)}{(0.11)(3)} = 4.54\ \text{HR}^{-1}$$

$$\Rightarrow t = \frac{1}{4.54} \ln \frac{1}{1 - 4.54/5170\,(160)}$$

$= \underline{0.0333\ \text{HOURS}} = \underline{2.0\ \text{MIN.}}$

18.2 $V = \frac{\pi D^2}{4} L = \frac{\pi}{4}(0.0001)^2(0.005)$

$= 3.93 \times 10^{-11}\ m^3$

$A = 2\frac{\pi D^2}{4} + \pi D L$

$= \frac{\pi}{2}(0.0001)^2 + \pi(0.0001)(0.005)$

$= 1.587 \times 10^{-6}\ m^2$

$$\frac{hV}{kA} = \frac{10\ (3.93\times10^{-11})}{20\ (1.587\times10^{-6})} \cong 1.24 \times 10^{-5}$$

CLEARLY A LUMPED PARAMETER CASE

ENERGY BALANCE:

$$\dot{q} - hA(T - T_\infty) = \rho c_p V \frac{dT}{dt}$$

LET $\theta = T - T_\infty$:

$$\frac{d\theta}{dt} = \frac{\dot{q}}{\rho c_p V} - \frac{hA}{\rho c_p V}\theta$$

$= A - B\theta$

18.2 CONTINUED –

$$\int_0^\theta \frac{d\theta}{A - B\theta} = \int_0^t dt$$

$$-\frac{1}{B} \ln \frac{A - B\theta}{A} = t$$

$$\Rightarrow t = \frac{1}{B} \ln \frac{A}{A - B\theta}$$

$$A = \frac{\dot{q}}{\rho c_p V} = \frac{9(0.2)}{k/\alpha\,(V)} = \frac{1.8}{(20/5\times10^{-5})(3.93\times10^{-11})}$$

$= 1.145 \times 10^5$

$$B = \frac{hA}{\rho c_p V} = \frac{10(1.587\times10^{-6})}{(20/5\times10^{-5})(3.93\times10^{-11})} = 1.010$$

$$t = \frac{1}{1.01} \ln \frac{1.145\times10^5}{1.145\times10^5 - (1.01)(870)}$$

$= \underline{7.63 \times 10^{-3}\ S} = \underline{7.63\ MS}$

18.3 ALUMINUM WIRE:

$D = 0.794$ mm $\quad R = 0.0572\ \Omega/m$

$k = 229$ W/m·K $\quad \rho = 2701$ kg/m³

$c_p = 938$ J/kg·K $\quad \alpha = 9.16\times10^{-5}$ m²/s

$$Bi = \frac{hV}{kS} = \frac{h}{k}\ \frac{\pi D^2 L}{4\ \pi D L}$$

$$= \frac{(500)(0.794\times10^{-3})}{(229)(4)} = 4.33\times10^{-4}$$

– LUMPED PARAMETER SOLN IS OK.

STEADY STATE CASE – PER M

$I^2 R = hA\Delta T$

$$\Delta T = \frac{(100)^2(0.0572)\ W}{(550\ W/m^2{\cdot}K)\pi(0.794\times10^{-3})m^2}$$

$= 416.9$ K

$T_{MAX} = 25 + 416.9 = \underline{441.9\ C}$

18.3 CONTINUED –

TRANSIENT CASE –

$$\rho c_p V \frac{dT}{dt} = I^2R - hS(T-T_\infty)$$

$$\frac{d\theta}{dt} = \frac{I^2R}{\rho c_p V} - \frac{hS}{\rho c_p V}\theta = A - B\theta$$

$$A = \frac{I^2R}{\rho c_p V} = \frac{(100)^2(0.0572)}{(2701)(938)\frac{\pi}{4}(0.794\times10^{-3})^2}$$

$$= 456 \text{ K/s}$$

$$B = \frac{hS}{\rho c_p V} = \frac{(550)(\pi)(0.794\times10^{-3})}{(2701)(938)\frac{\pi}{4}(0.794\times10^{-3})^2}$$

$$= 1.094 \text{ s}^{-1}$$

$$\frac{d\theta}{dt} = A - B\theta$$

$$\int_0^\theta \frac{d\theta}{\theta - B\theta} = \int_0^t dt$$

$$-\frac{1}{B}\ln\frac{A-B\theta}{A} = t$$

$$t = \frac{1}{B}\ln\frac{A}{A-B\theta}$$

$$= \frac{1}{1.094}\ln\frac{456}{456-1.094(411.9)}$$

$$= \underline{\underline{4.06 \text{ S}}}$$

18.4 $Bi = \frac{hV}{kS} = \frac{6}{0.151}\,\frac{(0.6)(0.3)(0.45)}{[0.6(0.45)(2) + 0.6(0.3)(2) + 0.3(0.45)(1)]}$

$= 2.75$

A DISTRIBUTED PARAMETER PROB.

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{320-297}{420-297} = 0.187 = Y_A Y_B Y_C$$

$m_x = \frac{0.151}{6(0.15)} = 0.168$ $\quad X_x = \frac{\alpha t}{0.15^2} = 2.75\times10^{-6}t$

$m_y = 0.119$ $\quad X_y = 1.22\times10^{-6}t$

$m_z = 0.042$ $\quad X_z = 1.72\times10^{-6}t$

18.4 CONTINUED –

BY TRIAL & ERROR

$t \cong \underline{\underline{80 \text{ HOURS}}}$

18.5 $Bi = \frac{hV}{kS} = \frac{16}{23}\left(\frac{6/12}{6}\right) = 0.058$

LUMPED PARAMETER!

$$Fo = \frac{\alpha t}{(V/S)^2} = \frac{23}{460(0.10)}\frac{\text{FT}^2}{\text{HR}}\,\frac{t}{\left(\frac{6/12}{6}\right)^2}$$

$= 72t$

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{600}{2000} = e^{-(0.058)(72t)}$$

$t = \underline{0.288 \text{ HR}} = \underline{17.3 \text{ MIN}}$

18.6 LUMPED PARAMETER SOLN APPLIES IF $Bi = \frac{hV}{kS} < 0.1$ OR $h < 0.1\,kS/V$

$$\frac{kS}{V} = \frac{k\pi D^2}{\pi D^3/6} = 0.47(6) = 2.82$$

SO h MUST BE $< 0.1(2.82) = 0.282 \text{ W/m}^2\text{K}$

BUT $h = 15 \Rightarrow$ USE DISTRIBUTED PARAM. SOLN.

$$\frac{\alpha t}{r_0^2} = \frac{(0.47)\,t}{(940)(3800)(0.05)^2} = 5.26\times10^{-5}t$$

$$\frac{T_s-T_\infty}{T_0-T_\infty} = 0.5 \qquad \frac{k}{hr_0} = \frac{0.47}{15(0.05)} = 0.627$$

$X \cong 0.17 = 5.26\times10^{-5}t$

$t = \underline{\underline{3230 \text{ S}}} = \underline{\underline{53.9 \text{ MIN}}}$

18.7 $Bi \cong 0.005$

USE LUMPED PARAMETER SOLN.

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-h/\rho c_p \frac{S}{V}t}$$

$$t = \frac{\rho c_p}{h}\frac{V}{S}\ln\frac{T_0-T_\infty}{T-T_\infty}$$

$$= \frac{51}{4}\frac{V}{A\times12}(0.62) = 0.658\frac{V}{A}$$

18.7 CONTINUED –

V/A IN INCHES

$$\frac{V}{A} = \frac{\pi D^2 L/4}{\pi D L + 2\pi D^2/4} = \frac{D}{4+2D/L} = \frac{3}{4+\frac{6}{L}}$$

CASE	L(IN)	V/A	t (MIN)
a	3	0.5	19.7
b	6	0.6	23.6
c	12	0.67	26.3
d	24	0.706	27.9
e	60	0.732	28.9

18.8

$$\frac{T_c - T_\infty}{T_0 - T_\infty} = \frac{500-1000}{70-1000} = 0.538$$

$$\frac{V}{A} = \frac{D}{4+2D/L} = 1/17$$

$$Bi = \frac{hV/S}{k} = \frac{4/17}{k}$$

a) Cu – $Bi \le 0.1$ – LUMPED

$$t = \frac{\rho c_p V}{hA} \ln \frac{1}{0.538} = \underline{\underline{27.9 \text{ MIN.}}}$$

b) Al $\quad Bi \le 0.1$

$$t = 0.345 \text{ HR} = \underline{\underline{20.7 \text{ MIN}}}$$

c) Zn $\quad Bi < 0.1$

$$t = 0.381 \text{ HR} = \underline{\underline{22.9 \text{ MIN}}}$$

d) STEEL $\quad Bi < 0.1$

$$t = 0.502 \text{ HR} = \underline{\underline{30.2 \text{ MIN}}}$$

18.9 WATER IS WELL-STIRRED

$\therefore$ LUMPED ~ $T = T(t)$ ONLY

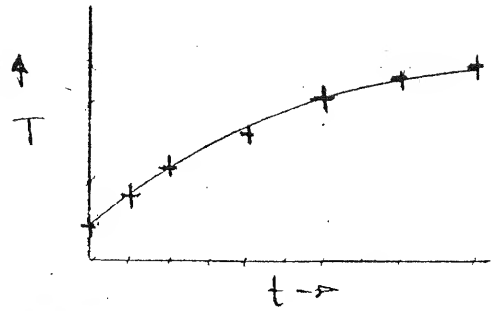
$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{hAt}{\rho c_p V}\right)$$

$$= \exp\left[-\frac{40(\pi)(1.5)(2)t}{62.4(1)(\pi)(1.5^2/4)(2)}\right]$$

$$T = 300 - 260\, e^{-1.71t}$$

t, HR	0	0.1	0.2	0.4	0.6	0.8	1.0
T F	40	81	115	169	210	237	253

18.9 CONTINUED



18.10

$$Bi = \frac{hV/S}{k} = \frac{h}{k}\,\frac{\pi D^2 L/4}{\pi D L + 2\pi D^2/4}$$

$$= \frac{hD}{4k(L+D/2)} = \frac{85\,(0.6)(0.6)}{(2.29)(4)(0.9)}$$

$$= 0.0371$$

~ LUMPED PARAMETER CASE

TEMP MAY BE CONSIDERED UNIFORM AT ANY TIME

$$Fo = \frac{\alpha t}{(V/S)^2} = \frac{(9.16\times10^{-5})(3600)}{(0.10)^2}$$

$$= 32.98$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-Bi\,Fo} = e^{-(0.0371)(32.98)}$$

$$= 0.294$$

$$T = 345 + 0.294(130)$$

$$= \underline{\underline{383.2 \text{ K}}}$$

18.11

$$Bi = \frac{hV/S}{k} = \frac{15\left[\frac{\pi D^2}{4} L / \pi D L\right]}{12.4}$$

$$= 0.151$$

MUST USE DISTRIBUTED PARAMETER SOLN, FIG F.8

$$\frac{T_s - T}{T_s - T_0} = \frac{2300-1500}{2300-200} = 0.381$$

$$\frac{\alpha t}{x_1^2} = \frac{0.15}{(0.25)^2} t = 2.4t$$

18.11 CONTINUED -

$$\eta = \frac{x}{x_1} = 0 \qquad m = \frac{k}{hx_1} = 3.31$$

$$X \cong 1.7 \Rightarrow t = \frac{1.7}{2.4} = 0.708 \text{ HR}$$

$$\text{VELOCITY} = \frac{20}{0.708} = \underline{28.2 \frac{FT}{HR}} = \underline{0.47 \frac{FT}{M}}$$

18.12 $\frac{T_c - T_\infty}{T_0 - T_\infty} = \frac{410-435}{295-435} = 0.179$

$$\frac{\alpha t}{x_1^2} = \frac{(6.19\times10^{-8})t}{(0.015)^2} = 2.75\times10^{-4}t$$

$m \cong 0$ FROM CHART $X \cong 0.8$

$$t = \frac{0.8}{2.75\times10^{-4}} = \underline{2912 S} = \underline{48.5 \text{ MIN}}$$

18.13 $Bi = \frac{hV}{kS} = \frac{40 \;\pi D^3/6}{19.3\;\pi D^2} = 0.00575$

LUMPED PARAMETER!

$$Fo = \frac{\alpha t}{(V/S)^2} = \frac{0.8(15/3600)}{[0.2/2(6)]^2} = 432$$

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-Bi\,Fo} = 0.0834$$

$$T = \underline{115.9 \text{ F}}$$

18.14 $x_1 = 0.15$ m $\quad x = 0.05$ m

$$x/x_1 = 1/3$$

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{100-380}{25-380} = 0.789$$

$$m = k/hx_1 = \frac{0.20}{140(0.15)} = 0.00952$$

$$\alpha t/x_1^2 = \frac{1.1\times10^{-7}t}{(0.15)^2} = 4.89\times10^{-6}t$$

18.14 CONTINUED -

{USING CHART FOR CYL}

@ $\frac{x}{x_1} = 0.2 \quad \frac{\alpha t}{x_1^2} = 0.10$

@ $0.4 \quad$ " $= 0.07$

INTERPOLATING - @ 0.33 $\frac{\alpha t}{x_1^2} \cong 0.08$

$$t \cong \frac{0.08}{4.89\times10^{-6}} \cong 16360 \text{ S}$$

$$= \underline{273 \text{ MIN}} = \underline{4.54 \text{ HR}}$$

18.15 $Bi = \frac{hV/S}{k} = \frac{22.8\,\pi \frac{D^2}{4} L}{0.19\;\pi D L} = 3.9$

~ DISTRIBUTED PARAMETER SOLN!

$$\frac{\alpha t}{x_1^2} = \frac{0.19\,t}{(580)(1050)(0.065)^2} = 7.38\times10^{-5}t$$

$$\frac{k}{hx_1} = \frac{0.19}{22.8(0.065)} = 0.128$$

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{530-810}{295-810} = 0.544 \quad n=0$$

FROM CHART $X \cong 0.23 = 7.38\times10^{-5}t$

$$t = \underline{3114 \text{ S}} = \underline{51.9 \text{ MIN}}$$

18.16 USING CHART SOLN!

$$\frac{T_s - T}{T_s - T_0} = \frac{100-250}{100-400} = 0.5$$

$Y_A Y_B Y_C = 0.5 \qquad n_A = n_B = n_C = 0$

$m_A = m_B = m_C = 0 \qquad X_A = 442t$

$X_B = 70.7t$

$Y_C \approx 1 \qquad X_C = 0.884t$

BY TRIAL & ERROR! $t \cong \underline{8.4\times10^{-4} \text{ HR}}$

$$\cong \underline{3.05 \text{ S}}$$

18.17

$$Bi = \frac{hV}{kS} = \frac{230}{0.151}\;\frac{(0.6)(0.3)(0.45)}{[\text{SEE PROB } 18.7]}$$

$$= 105 \quad \left\{\begin{matrix}\text{DISTRIBUTED}\\ \text{PARAMETER}\end{matrix}\right\}$$

$$\frac{T_S - T_\infty}{T_0 - T_\infty} = 0.187 = Y_A Y_B Y_C$$

$$m_x = \frac{0.151}{230(0.15)} = 4.377 \times 10^{-3}$$

$$m_y = 2.92 \times 10^{-3}$$

$$m_z = 1.09 \times 10^{-3}$$

$$X_x = \frac{(6.19 \times 10^{-8})t}{(0.15)^2} = 2.75 \times 10^{-6} t$$

$$X_y = 1.22 \times 10^{-6} t$$

$$X_z = 1.09 \times 10^{-6} t$$

TRIAL & ERROR: $t \cong \underline{62 \text{ HOURS}}$

18.18

$$Bi = \frac{hV}{kS} = \frac{(90\,W/m^2{\cdot}K)(\pi D^2 L/4)}{(0.5\,W/m{\cdot}K)(\pi D L)}$$

$$= 0.9 \quad \left\{\begin{matrix}\text{DISTRIBUTED}\\ \text{PARAMETER}\end{matrix}\right\}$$

$$\frac{T_c - T_\infty}{T_0 - T_\infty} = \frac{80-100}{5-100} = 0.211$$

$$m = \frac{k}{hx_1} = \frac{0.5}{90(0.01)} = 0.556$$

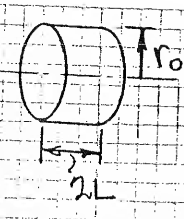
$$n = 0 \qquad X = \frac{\alpha t}{x_1^2} = \frac{0.5\,t}{880(3350)(0.01)^2}$$

$$= 1.70 \times 10^{-3} t$$

FROM CHART $X \cong 0.76 = 1.70 \times 10^{-3} t$

$$t = \underline{447\ S} = \underline{7.45\ \text{MIN.}}$$

18.19



$$2L = 2r_0$$

$$V = \pi r_0^2 (2L)$$

$$= \frac{2.25}{991} = 2.27 \times 10^{-3}\,m^3$$

$$L = r_0 = \frac{2.27 \times 10^{-3}}{2\pi} = 0.0712\,m$$

FOR A FINITE CYLINDER:

$$\frac{T(0,0,t) - T_\infty}{T_i - T_\infty} = R(0,t)X(0,t)$$

$$\frac{T(0,0,t) - T_\infty}{T_i - T_\infty} = \frac{95-190}{5-190} = 0.514$$

FOR BOTH r & x DIRECTIONS:

$$\frac{hr_0}{k} = \frac{hL}{k} = \frac{15(0.0712)}{0.675} = 1.582$$

$$\frac{\alpha t}{r_0^2} = \frac{\alpha t}{L^2} = \frac{0.167 \times 10^{-6} t}{(0.0712)^2} = 3.29 \times 10^{-5} t$$

TRIAL & ERROR {USING CHARTS} $\quad \frac{\alpha t}{L^2} = \frac{\alpha t}{r_0^2} \cong 0.34$

$$t = 0.34/3.29 \times 10^{-5} = 10330\,S$$

$$= \underline{172\ \text{MIN}} = \underline{2.87\ \text{HR}}$$

18.20 SAME CYL AS IN PROB 18.15 BUT H VARIES—

AS $H/D \to \infty$ $\quad t = 3114\,S = 51.9$ MIN.

WITH ENDS CONSIDERED: $D = \text{CONST} = 13\,\text{CM}$

$$\left.\frac{\alpha t}{x_1^2}\right|_{CYL} = 7.38 \times 10^{-5} t \qquad \left.\frac{k}{hx_1}\right|_{CYL} = 0.128$$

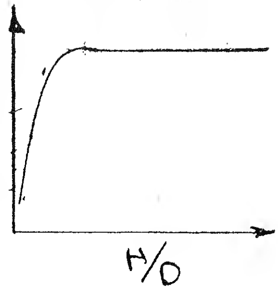
$$Y = 0.544 = Y_{CYL} Y_{PL}$$

FOR PLANE: $\frac{\alpha t}{x_1^2} = \frac{1.25 \times 10^{-5} t}{H^2}$

$$k/hx_1 = 0.0167/H$$

18.20 CONTINUED -

H	H/D	t,s
∞	∞	3114
1.3	10	3114
0.65	5	~3000
0.26	2	~1600
0.13	1	~600



18.21 USE SEMI-INFINITE WALL SOLN.

$$\frac{T_\infty - T}{T_\infty - T_0} = \text{erf}\,\eta + \exp\left(\beta + \frac{\beta^2}{4\eta^2}\right)\left[1-\text{erf}\left(\frac{\beta}{2\eta}+\eta\right)\right]$$

$$\eta = x/2\sqrt{\alpha t} \quad \beta = \frac{hx}{k} \quad \frac{\beta}{2\eta} = \frac{h\sqrt{\alpha t}}{k}$$

@ SURFACE ~ $x=0$

$$\frac{T_\infty - T}{T_\infty - T_0} = \exp\left(\frac{\beta^2}{4\eta^2}\right)\left(1-\text{erf}\,\frac{\beta}{2\eta}\right)$$

$$\frac{h^2\alpha t}{k^2} = \frac{(200)^2(0.35)t}{(1.73)^2} = 4678\,t$$

t IN HOURS

$$\frac{h\sqrt{\alpha t}}{k} = 68.4\,t^{1/2}$$

$$0.1 = e^{68.4\,t^{1/2}}(1-\text{erf}\,4678t)$$

APPROXIMATION: USE 1ST TERM IN SERIES EXPANSION:

$$\frac{T_\infty - T_s}{T_\infty - T_0} = 0.1 = \frac{2\eta}{\sqrt{\pi}\,\beta} = \frac{1}{\sqrt{\pi}}\frac{k}{h\sqrt{\alpha t}}$$

$$t = 6.80\times10^{-3}\ \text{HR} = \underline{24.5\ \text{S}}$$

AT THIS TIME $\eta = 0.427$ $\beta = 4.81$

SOLVING FOR T: $\underline{\underline{T = 1413\ \text{F}}}$

18.22 USE SEMI-INFINITE SOLN.

$$\frac{T-T_s}{T_0-T_s} = \text{erf}\,\frac{y}{2\sqrt{\alpha t}} = \frac{2}{\sqrt{\pi}}\int_0^{y/2\sqrt{\alpha t}} e^{-\beta^2}\,d\beta$$

GIVEN: $\alpha = 0.0456\ \text{FT}^2/\text{HR}$ $T_s = 0\ \text{F}$

$\frac{\partial T}{\partial y}(0) = 0.02\ \text{F/FT}$ $T_0 = 7000\ \text{F}$

$$\frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\frac{2}{\pi}\int_0^{y/2\sqrt{\alpha t}} e^{-\beta^2}\,d\beta\right](T_0-T_s)$$

$$= \frac{2}{\sqrt{\pi}}\left[e^{-y^2/4\alpha t}\,\frac{1}{2\sqrt{\alpha t}}\right](T_0-T_s)$$

FOR $y=0$ $\quad \frac{\partial T}{\partial y}(0) = \frac{T_0-T_s}{\sqrt{\pi\alpha t}}$

$$t = \frac{(T_0-T_s)^2}{\pi\alpha\left(\partial T/\partial y|_0\right)^2} = \frac{(7000)^2}{\pi(0.0456)(4\times10^{-4})}$$

$$\underline{\underline{= 9.75\times10^7\ \text{YEARS}}}$$

18.23 USE SEMI-INF WALL SOLN.

$$\frac{T-T_0}{T_\infty-T_0} = \text{erfc}\,\frac{x}{2\sqrt{\alpha t}} - \left[\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\right] \times \left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

@ $x=0$ THIS REDUCES TO

$$\frac{T-T_0}{T_\infty-T_0} = 1 - e^{z^2}(1-\text{erf}\,z)$$

WHERE $z = \frac{h\sqrt{\alpha t}}{k}$

$$\alpha = \frac{k}{\rho c_p} = \frac{0.17}{(545)(2385)} = 1.3\times10^{-7}\ \text{m}^2/\text{s}$$

$$\frac{T-T_0}{T_\infty-T_0} = \frac{400-21}{900-21} = 0.431$$

$$z = \frac{h\sqrt{\alpha t}}{k} = \frac{30}{0.17}\sqrt{1.3\times10^{-7}}\,t^{1/2}$$

$$= 0.0636\,t^{1/2}$$

$$z^2 = 0.00405\,t$$

18.23 CONTINUED -

SUBSTITUTING:

$$0.431 = 1 - e^{z^2}(1-\text{erf}\, z)$$

$$e^{z^2}(1-\text{erf}\, z) = 0.569$$

BY TRIAL & ERROR: $z \cong 0.6$

$$t = \frac{z^2}{0.00405} = \underline{\underline{88.9\ s}}$$

$$= \underline{\underline{1.48\ \text{MIN}}}$$

18.24 $$\frac{T-T_s}{T_0-T_s} = \frac{2}{\sqrt{\pi}}\int_0^{x/2\sqrt{\alpha t}} e^{-\beta^2}d\beta$$

$$\frac{\partial T}{\partial t} = (T_0-T_s)\frac{2}{\sqrt{\pi}}\, e^{-x^2/4\alpha t}\left[-\frac{x}{4\sqrt{\alpha}\, t^{3/2}}\right]$$

$$= -\frac{(T_0-T_s)}{t\sqrt{\pi}}\,\frac{x}{2\sqrt{\alpha t}}\, e^{-x^2/4\alpha t}$$

$$\frac{\partial T}{\partial t} = -\frac{(T_0-T_s)}{t\sqrt{\pi}}\, z\, e^{-z^2} \qquad \left\{z = \frac{x}{2\sqrt{\alpha t}}\right\}$$

SO $\left|\frac{\partial T}{\partial t}\right|$ IS MAX WHEN $z e^{-z^2}$ IS MAX

$$\frac{d}{dz}\left(z e^{-z^2}\right) = e^{-z^2} - 2z^2 e^{-z^2} = 0$$

$$z = 1/\sqrt{2} \quad \text{OR} \quad \infty$$

$$\Rightarrow \frac{x}{2\sqrt{\alpha t}} = \frac{1}{\sqrt{2}} \quad \text{OR} \quad x = \sqrt{2\alpha t}$$

$$x = \left[2(0.0456)(9.8\times10^{-7})(24)(365)\right]^{1/2}$$

$$= \underline{\underline{2.8\times10^5\ \text{FT}}} = \underline{\underline{53\ \text{MILES}}}$$

18.25 THIS IS A SEMI-INFINITE CASE:

$$\frac{T_s-T}{T_s-T_0} = \text{erf}\,\frac{x}{2\sqrt{\alpha t}}$$

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.25\ m}{2\left[(5.16\times10^{-7}\ m^2/s)(18000\ s)\right]^{1/2}}$$

$$= 1.297 \qquad \text{erf}\ 1.297 \cong 0.934$$

$$\frac{1100-T}{1100-280} = 0.934 \qquad \underline{\underline{T = 334\ K}}$$

18.26 SOLN IS AMENABLE TO EITHER NUMERICAL OR ANALYTICAL APPROACH

$$\frac{T_s-T_\infty}{T_0-T_\infty} = \exp\left(\frac{h^2\alpha t}{k^2}\right)\left[\text{erfc}\,\frac{h\sqrt{\alpha t}}{k}\right]$$

$$\Delta T_s = T_s - T_\infty \qquad h = 0.44\,(T-T_\infty)^{1/3}$$

$$\Delta T_s = 900\,\exp\left(0.00732\,\Delta T_s^{2/3}\, t\right)\times$$

$$\times\left[\text{erfc}\left(0.0856\,\Delta T_s^{1/3}\, t^{1/2}\right)\right]$$

TRIAL & ERROR - AT EACH t:

t -	1 HR	6 HR	24 HR
T, F	580	396	275

18.27

$$\frac{T-T_0}{T_\infty-T_0} = \text{erfc}\,\frac{x}{2\sqrt{\alpha t}} - \left[\exp\left(\frac{hx}{k}\right) + \frac{h^2\alpha t}{k^2}\right]\times$$

$$\times\left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$\frac{T-T_0}{T_\infty-T_0} = \frac{400-25}{800-25} = 0.484$$

18.27 CONTINUED-

@ SURFACE ($x=0$)

$\frac{x}{2\sqrt{\alpha t}}=0 \quad \operatorname{erf}(0)=0 \quad \operatorname{erfc}(0)=1$

$\frac{h\sqrt{\alpha t}}{k}=\frac{20}{0.21}(1.07\times10^{-7})^{1/2}t^{1/2}$

$=0.0311\,t^{1/2}=z$

GOVERNING EXPRESSION BECOMES:

$0.484=1-e^{z^2}(1-\operatorname{erf} z)$

TRIAL & ERROR: $z\cong 0.73=0.0311\,t^{1/2}$

$t=\underline{551\text{ s}}=\underline{9.18\text{ MIN.}}$

18.28

FOR GLASS:

$\alpha=\frac{k}{\rho c_p}=\frac{0.45}{(170)(0.2)}=0.0132\ \text{ft}^2/\text{HR}$

USING SEMI-INFINITE WALL EXPRESSION:

$\frac{T-T_0}{T_s-T_0}=\frac{32-30}{65-30}=0.0572=\operatorname{erfc}\frac{x}{2\sqrt{\alpha t}}$

$\frac{x}{2\sqrt{\alpha t}}=1.38$

$\underline{\underline{t=3.9\text{ s}}}$

18.29 CHARTS APPLY BUT ARE DIFFICULT TO READ -

CHECK VALIDITY OF INFINITE WALL SOLN

$\frac{L}{2\sqrt{\alpha t}}=\frac{1\text{ FT}}{2(0.0231t)^{1/2}}>2$

WORKS FOR $t>27$ HOURS

$\frac{x}{2\sqrt{\alpha t}}=\frac{1}{2(0.0231t)^{1/2}}=\frac{3.29}{t^{1/2}}$

$\frac{T_s-T}{T_s-T_0}=\operatorname{erf}\frac{3.29}{t^{1/2}} \qquad \underline{\underline{t\cong 5.2\text{ HR}}}$

18.30 APPLICABLE EXPRESSION IS

$\frac{T_\infty-T}{T_\infty-T_0}=\operatorname{erf}A+\exp\left(\frac{hx}{k}+B^2\right)\left[1-\operatorname{erf}(A+B)\right]$

$A=\frac{x}{2\sqrt{\alpha t}}=\frac{0.05}{2(0.444\times10^{-5}t)^{1/2}}$

$=11.92\,t^{-1/2}$

$\frac{hx}{k}=\frac{22(0.05)}{17.3}=0.0636$

$B=\frac{h\sqrt{\alpha t}}{k}=\frac{22}{17.3}(0.444\times10^{-5}t)^{1/2}$

$=0.00267\,t^{1/2}$

$B^2=7.11\times10^{-6}\,t$

TRIAL & ERROR: $t\cong 49000\text{ s}$

$\cong\underline{\underline{13.6\text{ HOURS}}}$

18.31

$\frac{T_\infty-T}{T_\infty-T_0}=\operatorname{erf}A+\exp\left(\frac{hx}{k}+B^2\right)\left[1-\operatorname{erf}(A+B)\right]$

@ $x=0,\ t=180\text{ s}$

$A=\frac{x}{2\sqrt{\alpha t}}=0 \qquad \frac{hx}{k}=0$

$B=\frac{h\sqrt{\alpha t}}{k}=\frac{110}{17.3}\sqrt{180\,\alpha}=0.180$

$\frac{20-T}{20-300}\cong 0.96$

$T=20+268=\underline{\underline{288\text{ C}}}$

AT $x=50$ mm

$A=0.89$

$\frac{hx}{k}=0.318$

$B=0.180 \qquad B^2=0.0324$

$\frac{20-T}{20-300}=0.974 \qquad \underline{\underline{T=293\text{ C}}}$

18.32. EQN (18-33)

$$\frac{q_x}{A} = \frac{d}{dt}\int_0^\delta \rho c_p T\,dx - \rho c_p T_0 \frac{d\delta}{dt}$$

FOR $\frac{T - T_0}{T_S - T_0} = \phi\left(\frac{x}{\delta}\right)$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = (T_S - T_0)\,\frac{1}{\delta}\,\left.\frac{\partial \phi}{\partial x}\right|_{x=0}$$

$$\frac{\partial \phi}{\partial x}(0) = K \qquad \text{(A CONSTANT)}$$

$$\frac{q_x}{A} = -k\frac{\partial T}{\partial x}(0) = F(t)$$

$$\therefore\ T_S - T_0 = \frac{F(t)\,\delta}{k\,K}$$

$$\frac{F(t)}{\rho c_p} = \frac{d}{dt}\int_0^\delta T\,dx - T_0\frac{d\delta}{dt}$$

$$\frac{d}{dt}\int_0^\delta T\,dx = T_0\frac{d\delta}{dt} + \frac{d}{dt}(T_S - T_0)\int_0^\delta \phi\,dx$$

$$= T_0\frac{d\delta}{dt} + \frac{d}{dt}\left[(T_S - T_0)\,B\,\delta\right]$$

$\{B$ A CONSTANT$\}$

$$\Rightarrow \frac{F(t)}{\rho c_p} = \frac{d}{dt}\left[(T_S - T_0)\,B\,\delta\right]$$

$$= \frac{d}{dt}\left[\frac{B\,\delta^2 F(t)}{k\,K}\right]$$

$$\frac{k}{\rho c_p}\,\frac{K}{B}\,F(t) = \frac{d}{dt}\left[\delta^2 F(t)\right]$$

$$\delta^2 F(t) = \frac{K}{B}\,\alpha\int_0^t F(t)\,dt$$

$$\delta = (\text{CONSTANT})\sqrt{\alpha}\left[\frac{\int_0^t F(t)\,dt}{F(t)}\right]^{1/2}$$

18.33 NUMERICAL SOLN REQ'D

INITIAL TEMP PROFILE –

$T = 35 + 0.5X \qquad T$ IN °F, X IN FT

ALGORITHMS –

FOR ALL NODES EXCEPT SURFACE:

$$T_i^{t+1} = \frac{T_{i+1} + T_{i-1}}{2}$$

FOR SURFACE NODE:

$$T_0^{t+1} = T_1^t - \frac{h\Delta x}{k}\,T_0^t$$

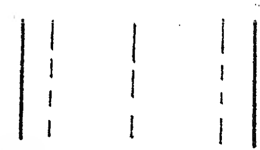
$$\sim \frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2} \qquad \frac{2h\Delta t}{\rho c_p \Delta x} = \frac{h\Delta x}{k} \qquad T_\infty = 0$$

RESULT – USING SPREADSHEET OR PROGRAM

TIME $\doteq$ 1800 HOURS

18.34 NUMERICAL SOLN REQ'D

ALGORITHMS –



NODES → 0 1 i $n-1$ n

NODE 1: $T_1^{t+1} = \frac{T_0 + T_2^n}{2}$

$$T_{n-1}^{t+1} = \frac{T_n + T_{n-2}^t}{2}$$

$$T_i^{t+1} = \frac{T_{i+1} + T_{i-1}}{2}$$

FOR $\frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2}$; IF $\Delta x = 0.25$ FT

$\Delta t = 1.95$ HR

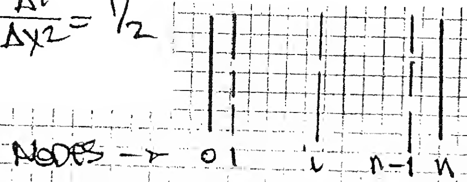
NO OF INCREMENTS $\cong 7.4$

TIME $= 7.4(1.95) \cong$ 144 HR

18.35. $T = 520 + 330 \sin \frac{\pi x}{L}$

$\alpha = \frac{k}{\rho c_p} = \frac{0.66}{(1670)(8.38)} = 4.72 \times 10^{-5}\ m^2/s$

$\frac{\alpha \Delta t}{\Delta x^2} = 1/2$



SAME ALGORITHMS AS PROB (18-34)

FOR $\frac{\alpha \Delta t}{\Delta x^2} = \frac{1}{2}$ IF $\Delta x = 0.225\ m$

$\Delta t = 536\ s$

NO OF INCREMENTS $\cong 2.4$

TIME $\cong 2.4(536) = 1286\ s$

$= \underline{21.4\ MIN}$

AT THIS TIME: $T_{surf} \cong \underline{360\ K}$

CHAPTER 19

19.1 For a plane wall:

VARIABLES	DIMENSIONS
T	T
T_0	T
T_∞	T
x	L
L	L
α	L^2/t
k	Q/LtT
t	t
h	Q/L^2tT

$i = n - r = 5$

If temps are grouped as

$T - T_\infty$, $T_0 - T_\infty$ $\quad i = n - r = 4$

$\pi_1 = \Delta T^a L^b k^c \alpha^d (T - T_\infty)$

$\pi_2 = (\qquad)(x)$

$\pi_3 = (\qquad)(t)$

$\pi_4 = (\qquad)(h)$

$$\pi_1 = \frac{T - T_\infty}{T_0 - T_\infty} \quad \pi_2 = \frac{x}{L} \quad \pi_3 = \frac{\alpha t}{L^2} \quad \pi_4 = \frac{hL}{k}$$

19.2

	AIR	H_2O	BENZ	Hg	GLYC
ν	1.9×10^{-5}	0.474×10^{-5}	0.473×10^{-5}	1.06×10^{-6}	0.128×10^{-2}
c_p	1.008×10^{-3}	1.0	0.45	0.033	0.598
k	0.0293	0.383	0.0162	5.03	0.165
Re	2.3×10^{5}	1.02×10^{7}	1.02×10^{7}	4.57×10^{7}	37,800
Pr	0.699	2.72	5.21	0.021	13.1
Nu	348	15.4	77.3	1.17	35.7
St	2.16×10^{-3}	5.55×10^{-7}	1.45×10^{-6}	1.22×10^{-6}	7.21×10^{-5}

19.3 ~ PLOTS ~

19.4 Air @ 310 K: $Pr = 0.705$

$k = 2.7\times10^{-2}\ \mathrm{W/m\cdot K}$

$\beta g/\nu^2 = 1.161\times10^{8}\ /\mathrm{m^3\cdot K}$

$$Gr = \frac{\beta g}{\nu^2} x^3 \Delta T = (1.161\times10^8)(110)x^3$$

$$\frac{\delta}{x} = 3.94\, Pr^{-1/2}(Pr + 0.954)^{1/4} Gr_x^{-1/4}$$

$x =$	15 cm	30 cm	1.5 m
$Gr_x =$	4.31×10^{7}	3.45×10^{8}	4.31×10^{10}
$\delta =$	0.985 cm	1.17 cm	1.75 cm
$Nu_x =$	30.5	51.2	171.3
$h_x =$	$5.48\ \mathrm{W/m^2\cdot K}$	4.61 -	3.08 -

19.5

$$h_x = \frac{k}{x}\, 0.332\, Re_x^{1/2} Pr^{1/3}$$

$$= 0.055 \frac{Re_x^{1/2}}{x} Pr^{1/3}$$

$T = 30\ \mathrm{F}$ $\quad T_f = 55\ \mathrm{F}$ $\quad h_x = 0.4 \dfrac{Re_x^{1/2}}{x}$

x	$Re_x^{1/2}/x$	h_x
0	∞	∞
0.5	10.92	8.74
1	15.46	6.19
1.5	18.92	5.05
2	21.85	4.37

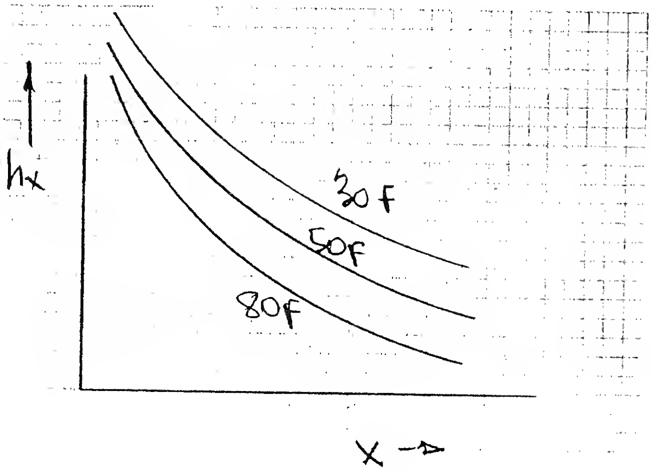
$T = 50\ \mathrm{F}$ $\quad T_f = 75\ \mathrm{F}$ $\quad h_x = 0.256 \dfrac{Re_x^{1/2}}{x}$

x	$Re_x^{1/2}/x$	h_x
0	∞	∞
0.5	22.2	11.37
1	31.45	8.06
1.5	38.5	6.57
2	44.5	5.70

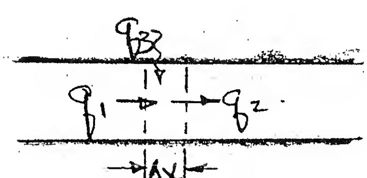
19.5 CONTINUED –

$T = 80F \qquad T_f = 105 \qquad h_x = 0.119\,\frac{Re_x^{1/2}}{x}$

x	$Re_x^{1/2}/x$	h_x
0	∞	∞
0.5	67.5	16.06
1	95.2	11.32
1.5	117	9.29
2	135	8.03



19.6



AS PER DEVELOPMENT IN TEXT:

$$q_2 - q_1 - q_3 = 0$$

$$\rho v c_p z T\big|_{x+\Delta x} - \rho v c_p z T\big|_x - \frac{q}{A}(2\Delta x) = 0$$

$$\frac{\rho v c_p z}{2}\,\frac{T|_{x+\Delta x} - T|_x}{\Delta x} - \frac{q}{A} = 0$$

19.6 CONTINUED –

IN LIMIT AS $\Delta x \to 0$

$$\frac{\rho v c_p z}{2}\frac{dT}{dx} - \frac{q}{A} = 0$$

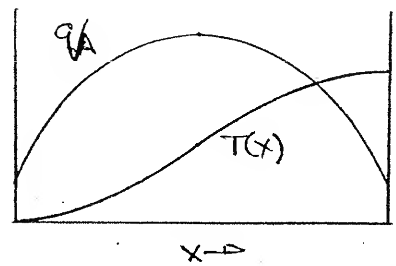
$$\int_{T_0}^{T} dT = \frac{2}{\rho v c_p z}\int_0^x \left(\alpha + \beta \sin\frac{\pi x}{L}\right)dx$$

$$T - T_0 = \frac{2}{\rho v c_p z}\left[\alpha x + \frac{\beta L}{\pi}\left(1 - \cos\frac{\pi x}{L}\right)\right]$$

$$\frac{2}{\rho v c_p z} = \frac{1}{30}\ \frac{\text{HR FT F}}{\text{BTU}}$$

$$\beta L/\pi = 1910\ \text{BTU/HR-FT}$$

x	q/A	$T-120$
0	250	0
1	1310	27
2	1750	80.3
3	1310	134
4	250	161



19.7

FOR A SINGLE 4-FT-LONG PLATE:

$$q = W\int_0^L \left(\alpha + \beta \sin\frac{\pi x}{L}\right)dx$$

$$= W\left(\alpha x + \frac{\beta L}{\pi}\cos\frac{\pi x}{L}\right)_0^L$$

$$= WL\left(\alpha + 2\beta/\pi\right)$$

$$= 16\ \text{FT}^2\left(250 + \frac{2(1500)}{\pi}\right)$$

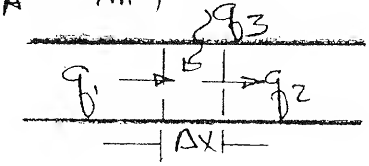
$$= 19300\ \text{BTU/HR}$$

FOR STACK OF PLATES:

$$q = 19300\,\frac{(640)}{16} = \underline{\underline{772{,}000\ \text{BTU/HR}}}$$

19.8 $\frac{q}{A} = a + b\sin\frac{\pi y}{y} = 900 + 2500\sin\frac{\pi x}{1.22}$

$\frac{q}{A}$ IN W/m^2, x IN m.



ENERGY BALANCE: $q_2 - q_1 - q_3 = 0$

STANDARD PROCEDURE

RESULTING EXPRESSION IS

$$\frac{dT}{dx} = \frac{2\,q/A}{S V c_p D(1)} = C\frac{q}{A} \quad \left\{\begin{array}{l} C \text{ A} \\ \text{CONSTANT}\end{array}\right.$$

$$\int_{T_E}^{T} dT = C\int_0^x \left(a + b\sin\frac{\pi y}{L}\right)dx$$

$$T - T_E = C\left[ax + \frac{Lb}{\pi}\left(1 - \cos\frac{\pi x}{L}\right)\right]$$

$$C = \frac{2}{SVc_pD(1)} = \frac{2}{(7.5)(1034)(0.003)(1)}$$

$$= 0.086 \text{ m}\cdot\text{kg}/W$$

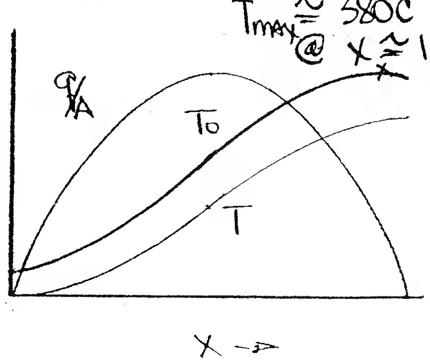
$$T - T_E = 77.4\,x + 83.5\left(1 - \cos\frac{\pi x}{1.22}\right)$$

FOR $h = 56\ W/m^2\cdot K$ $\quad \frac{q}{A} = h(T_0 - T)$

$$T_0 = T + \frac{q/A}{56}$$

x	q/A	T	T_0
0	900	100	116
0.4	3040	171	226
0.8	3110	285	340
1.2	1030	360	378
1.22	900	361	377

$T_{max} \cong 380C$ @ $x \cong 1.15$ M



19.9 $q = \int_0^L h_x \Delta T\,dx = \int_0^L Nu_x \frac{k}{x}\Delta T\,dx$

$$= k\Delta T^{5/4}(0.508)\,Pr^{1/2}(Pr + 0.954)^{-1/4} \times \left(\frac{\beta g}{\nu^2}\right)^{1/4}\left(\frac{4}{3}L^{3/4}\right)$$

$= \underline{\underline{995\ W}}$ PER m OF WIDTH

19.10 FOR $v = a + by + cy^2$

B.C. $v(0) = 0$

$v(\delta) = v_\infty$ $\quad \frac{v}{v_\infty} = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$

$\frac{dv}{dy}(\delta) = 0$

FOR $T - T_s = \alpha + \beta y + \gamma y^2$

B.C. $(T - T_s)|_0 = 0$

$(T - T_s)_{\delta_t} = T_\infty - T_s$ $\quad \frac{T - T_s}{T_\infty - T_s} = 2\frac{y}{\delta_t} - \left(\frac{y}{\delta_t}\right)^2$

$\frac{\partial}{\partial y}(T - T_s)\Big|_{\delta_t} = 0$

INTO MOMENTUM EQN – TO GET

$$\delta^2 = 30\frac{\nu x}{v_\infty} \qquad (1)$$

INTO ENERGY EQN.:

$$\delta\xi\, d\left(\delta\xi^2 - \frac{1}{5}\delta\xi^3\right) = 12\frac{\alpha}{v_\infty}dx \qquad (2)$$

WHERE $\xi = \delta_t/\delta$

SOLN GIVES $\xi \cong Pr^{-1/3}$

$\therefore\ \delta_t = Pr^{-1/3}\delta$

SINCE $\frac{q}{A} = -k\frac{dT}{dy}(0) = h(T_s - T_\infty)$

$$\frac{h}{k} = \frac{2}{\delta_t} = \frac{2Pr^{1/3}}{\delta} = 0.365\,Pr^{1/3}\left(\frac{v_\infty}{\nu x}\right)^{1/2}$$

OR; $Nu_x = \frac{hx}{k} = 0.365\,Pr^{1/3}\,Re_x^{1/2}$

19.11 $\frac{q}{A} = \alpha + \beta \sin\frac{\pi x}{L}$

$= \pi D \int_0^L \left(\alpha + \beta \sin\frac{\pi x}{L}\right) dx$

$= \pi D \left[\alpha L + 2\frac{\beta L}{\pi}\right]$

$= \pi\left(\frac{15}{12}\right)\left[250 + \frac{3000}{\pi}\right] = \underline{\underline{4730 \ \frac{Btu}{hr}}}$

$T_{EXIT} = T_i + \frac{q}{\rho A v c_p} = 60 + \frac{4730}{60(1)(0.062)(3600)}$

$= \underline{\underline{60.3\ F}}$

$T_w = 60.3 + \frac{250}{976} \approx \underline{\underline{60.6 F}}$

19.12 $T_0 = 300 - 240 e^{-720\,St}$

$q = \pi D L\ \rho c_p v (T_s - T_b)\, St$

$= 471\ (\rho c_p v)\, St$

FOR AIR @ 180F $\rho = 0.0622$

$c_p = 0.241$

$q = 471(0.0622)(0.241)(15\times 3600)\,St$

$= 3.82\times 10^5\ St$

$\nu = 0.228\times 10^{-3}$

$Re = \frac{DV}{\nu} = 5480$

$f = 8.8\times 10^{-3}$

REYNOLDS! $St = 0.00444$

$\underline{\underline{T_0 = 290 F}}$ $\underline{\underline{q = 937\ Btu/hr}}$

COLBURN!

$St = 0.00444\, Pr^{-2/3} = 0.00467$

$\underline{\underline{T_0 = 298 F}}$ $\underline{\underline{q = 959\ Btu/hr}}$

19.13 N_2 AT 100F 200F 150F

	100F	200F	150F
ρ_∞	0.069	0.0583	
ν_∞	1.77×10^{-3}	0.236×10^{-3}	0.209×10^{-3}
k	0.0154	0.0174	0.0164
Pr	0.71	0.71	0.71

$Re = \frac{xV}{\nu} = \frac{4 FT(10 FT/s)}{0.209\times10^{-3} FT^2/s} = 1.91\times10^5$

a) $\delta = \frac{5x}{Re_x^{1/2}} = \frac{5(4)}{(1.91\times10^5)^{1/2}} = 0.0457\ FT$

$= 0.549$ IN

b) $\delta_t = \delta / Pr^{1/3} = \delta/(0.71)^{1/3} = 0.615$ IN

c) $C_{fx} = 0.664\, Re_x^{-1/2} = 0.00152$

d) $C_{fL} = 1.328\, Re_L^{-1/2} = 0.00304$

e) $h_x = 0.332\frac{k}{x} Re_x^{1/2} Pr^{1/3} = 0.531\ \frac{Btu}{hr\,ft^2\,F}$

f) $h = 0.664\frac{k}{L} Re_L^{1/2} Pr^{1/3} = 1.06$ "

g) $f_d = A\, C_f \frac{\rho V^2}{2} = \frac{2(0.00304)(0.063)(10)^2}{2(32.2)}$

$= 5.95\times10^{-4}\ lb_f$

h) $q = hA\Delta T = 1.06(2)(100)$

$= 212\ Btu/hr$

19.14

FOR AIR AT $T_f = 325 K$:

$\rho = 1.087\ kg/m^3$ $\nu = 1.807\times10^{-5}\ m^2/s$

$c_p = 1.008\ kJ/kg\cdot K$ $Pr = 0.702$

$k = 2.816\ W/m\cdot K$

(a) $C_{fL} = 1.328\, Re_L^{-1/2}$

$Re = \frac{LV}{\nu} = \frac{(1m)(2.8\ m/s)}{1.807\times10^{-5}\ m^2/s} = 1.55\times10^5$

$C_{fL} = (1.328)(1.55\times10^5)^{-1/2} = \underline{\underline{0.00337}}$

19.14 CONTINUED-

(b) $F_D = C_{f_L} A \frac{\rho v^2}{2}$

$= \frac{(0.00337)(0.25)(1)(1.087)(2.8)^2}{2}$

$= \underline{\underline{3.59 \times 10^{-3}\ N}}$

(c) $q = hA\Delta T$

USING COLBURN ANALOGY:

$St_L = \frac{C_{f_L}}{2} Pr^{-2/3}$

$= \frac{(0.00337)}{2}(0.702)^{-2/3}$

$= 2.133 \times 10^{-3}$

$= h/\rho c_p V$

$h = (2.133\times10^{-3})(1.087)(1008)(2.8)$

$= 6.54\ W/m^2 \cdot K$

$q = 6.54(1)(0.25)(55)$

$= \underline{\underline{90.0\ W}}$

19.15

MOMENTUM THEOREM ~ X DIR.

$\sum F_x = \iint_{C.S.} v_x \rho(\vec{v}\cdot\vec{n})dA + \frac{\partial}{\partial t}\iiint_{C.V.} v_x \rho dV$

AT LOW VELOCITY: $\rho \simeq$ CONST, $v_\infty = 0$

& STEADY STATE

LHS: $\sum F_x = \begin{pmatrix}\text{BUOYANT}\\ \text{FORCE}\end{pmatrix} - \begin{pmatrix}\text{VISCOUS}\\ \text{FORCE}\end{pmatrix}$

BUOYANT FORCE (B.F.): $g\Delta\rho$ $\left\{\begin{matrix}\text{PER UNIT}\\ \text{VOLUME}\end{matrix}\right\}$

$\beta \equiv \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \cong \frac{1}{\rho}\frac{\Delta\rho}{\Delta T}$

$\therefore \Delta\rho \cong -\rho\beta\Delta T$

$B.F. = \Delta x \beta g \rho \int_0^{\delta_t}(T - T_\infty)dy$

19.15 CONTINUED-

VISCOUS FORCE (V.F.) $= \Delta x \mu \frac{\partial v_x}{\partial y}(0)$

RHS $\iint_{C.S.} v_x \rho(\vec{v}\cdot\vec{n})dA = \int_0^\delta \rho v_x^2 dy\Big|_{x+\Delta x}$

$- \int_0^\delta \rho v_x^2 dy\Big|_x - \cancel{v_\infty}\, m_{OUTSIDE}$ $\quad \overset{\nearrow}{0}$

EQUATING: (LHS) = (RHS) & DIV. BY Δx:

$\beta g \rho \int_0^{\delta_t}(T - T_\infty)dy - \mu\frac{\partial v_x}{\partial y}(0)$

$= \frac{\int_0^\delta \rho v_x^2 dy\Big|_{x+\Delta x} - \int_0^\delta \rho v_x^2 dy\Big|_x}{\Delta x}$

IN LIMIT AS $\Delta x \to 0$ & ρ = CONSTANT

$\beta g \int_0^{\delta_t}(T_s - T_\infty)dy - \nu\frac{\partial v_x}{\partial y}(0) = \frac{d}{dx}\int_0^\delta v_x^2 dy$

ENERGY EQN: SAME FOR BOTH NATURAL & FORCED CONVECTION

$\alpha\frac{\partial T}{\partial y}(0) = \frac{d}{dx}\int_0^{\delta_t}(T_\infty - T)v_x dy$

19.16 $Re_L = \frac{(2\ ft)(10\ ft/s)}{\nu} = 20/\nu$

LAMINAR FLOW FOR ALL VALUES OF ν

$C_{f_L} = \frac{1.328}{Re_L^{1/2}}$ $\quad F_D = A C_f \rho V^2/2$

$F_D = \frac{4(1.328)(\rho)(100)}{Re^{1/2}} = 8.25\frac{\rho}{Re^{1/2}}$

$Nu_L = 0.664\, Re_L^{1/2} Pr^{1/3}$

$h = \frac{k}{L}(0.664) Re_L^{1/2} Pr^{1/3}$

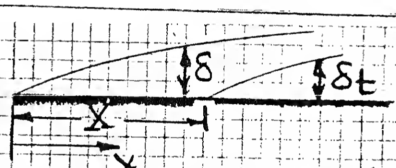
19.16 CONTINUED –

T, F	δ	ν	Re	f_D
30	79.5	6.61×10^{-2}	303	37.6 lb_f
50	79.0	1.52×10^{-2}	1320	17.9 "
80	78.2	0.13×10^{-2}	15400	5.2 "

T_L	T_R	T_f	ν	$Re^{1/2}$	$Pr^{1/3}$
30	80	55	0.0419	21.9	7.25
50	100	75	0.0101	44.6	4.64
80	130	105	0.0011	134.8	2.16

T	h_L		q/A	
30	8.74	$\frac{Btu}{hr\,ft^2\,F}$	437	$Btu/hr\,ft^2$
50	11.4	"	570	"
80	16.0	"	800	"

19.17



Assuming $T - T_s = \alpha + \beta y + \gamma y^2 + \delta y^3$

B.C. $T(0) = T_s$ $\quad \frac{\partial T}{\partial y}(\delta) = 0$

$T(\delta) = T_\infty$ $\quad \frac{\partial^2 T}{\partial y^2}(0) = 0$

Temp Profile Becomes

$$\frac{T - T_s}{T_0 - T_s} = \frac{3}{2}\frac{y}{\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3 \qquad (1)$$

Similarly for Velocity:

$$\frac{v}{v_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad (2)$$

Into Integral Expression:

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{d}{dx}\int_0^{\delta_t}(T_\infty - T)\, v\, dy$$

(LHS) ~ Left-Hand Side:

$$\alpha \frac{\partial T}{\partial y}(0) = \alpha (T_\infty - T_s)\left(3/2\delta_t\right)$$

19.17 CONTINUED

$$(RHS) \int_0^{\delta_t}(T_\infty - T_s)\, v\, dy = (T_\infty - T_s)\, v_\infty \int_0^{\delta_t} v\left(1 - \frac{T - T_s}{T_\infty - T_s}\right) dy$$

Subst Eqns (1) & (2):

$$= (T_\infty - T_s)\, v_\infty \left[\frac{3}{20}\frac{\delta_t^2}{\delta} - \cancel{\frac{3}{80}\frac{\delta_t^4}{\delta_t^3}}\right]$$

NEGLECT

Giving: $(RHS) = (T_\infty - T_s)\, v_\infty \frac{d}{dx}\left(\frac{3}{20}\frac{\delta_t^2}{\delta}\right)$

Equating: (LHS) = (RHS)

$$\delta_t \frac{d}{dx}\frac{\delta_t^2}{\delta} = 10\frac{\alpha}{v_\infty}$$

& Letting $\xi = \delta_t/\delta$:

$$\delta \xi\, d(\xi^2 \delta) = 10\frac{\alpha}{v}\, dx$$

$$\delta = \frac{4.64}{\sqrt{v_\infty/\nu}}\, x^{1/2}$$

Substitution & Some Algebra Give

$$4.31\, x\, \xi^2\, d\xi = \left(\frac{\alpha}{\nu} - 1.077\xi^3\right) dx$$

Separating Variables:

$$\frac{4.31\, \xi^2\, d\xi}{\frac{1}{Pr} - 1.077\xi^3} = \frac{dx}{x}$$

Solving

$$\xi^3 = \frac{1}{1.077}\frac{1}{Pr}\left[1 - \left(\frac{X}{x}\right)^{3/4}\right]$$

$$\xi = \frac{\delta_t}{\delta} \simeq \frac{1}{Pr^{1/3}}\left[1 - \left(\frac{X}{x}\right)^{3/4}\right]^{1/3}$$

19.17 CONTINUED -

NUSSELT No. "

$$\frac{q}{A} = -k\frac{dT}{dy}(0) = -\frac{3}{2}k\frac{T_\infty - T_s}{\delta_t}$$

$$= \frac{3}{2}k\frac{T_s - T_\infty}{\delta}Re_x^{1/2}\left[\frac{Pr}{1-(X/x)^{3/4}}\right]^{1/3}$$

$$= h(T_s - T_\infty)$$

$$\therefore\ Nu_x = \frac{hx}{k} = 0.323\,Re_x^{1/2}\left[\frac{Pr}{1-(X/x)^{3/4}}\right]^{1/3}$$

19.18 $Nu_x = \frac{h_x x}{k} = 0.508\,Pr^{1/2}(Pr+0.954)^{-1/4}Gr_x^{1/4}$

FOR AIR: $Pr = 0.72$ $k = 0.015$ $Btu/hr\,ft\,F$

$$h = k(0.38)\left(2\times10^6\frac{\Delta T}{x}\right)^{1/4} = 0.2445\left(\frac{\Delta T}{x}\right)^{1/4}$$

$$q = h_L A\Delta T = \int_0^L h_x \Delta T\,dx$$

$$h_L = \frac{1}{L}\int_0^L h_x\,dx = 0.286\left(\frac{\Delta T}{L}\right)^{1/4} = \alpha\left(\frac{\Delta T}{L}\right)^{\beta}$$

$$\Rightarrow\ \alpha = 0.286 \qquad \beta = 1/4$$

19.19 $\frac{v}{v_x} = \frac{y}{\delta}\left(1-\frac{y}{\delta}\right)^2 \qquad \frac{T-T_\infty}{T_s-T_\infty} = \left(1-\frac{y}{\delta}\right)^2$

INTO ENERGY EQN:

$$\alpha\frac{\partial T}{\partial y}(0) = \frac{d}{dx}\int_0^{\delta_t} v_x(T_\infty - T)\,dy$$

CONT. $LHS = -\frac{2\alpha(T_s-T_\infty)}{\delta}$

$$RHS = (T_\infty - T_s)\frac{d}{dx}\left(\frac{\delta v_x}{30}\right)$$

EQUATING: $\frac{2\alpha}{\delta} = \frac{d}{dx}\frac{\delta v_x}{30}$

19.19 CONTINUED -

INTO MOMENTUM EQN:

$$\beta g\int_0^{\delta_t}(T_s - T_\infty)\,dy - \nu\frac{\partial v_x}{\partial y}(0) = \frac{d}{dx}\int_0^{\delta} v_x^2\,dy$$

$$LHS = \frac{\beta g(T_s-T_\infty)\delta}{3} - \frac{\nu v_x}{\delta}$$

$$RHS = \frac{d}{dx}\left(\frac{\delta v_x^2}{105}\right)$$

EQUATING:

$$\frac{\beta g(T_s-T_\infty)\delta}{3} - \frac{\nu v_x}{\delta} = \frac{d}{dx}\int_0^{\delta} v_x^2\,dy$$

LETTING $\delta = Ax^a$ $\quad v_x = Bx^b$

PREVIOUS TWO EQNS BECOME:

$$\frac{2\alpha}{A}x^{-a} = \frac{AB(a+b)}{30}x^{a+b-1}$$

$$-\frac{\nu B}{A}x^{-a+b} + \beta g\Delta T\frac{A}{3}x^{a} = \frac{AB^2(a+2b)}{105}x^{a+2b-1}$$

EXPONENTS ON X MUST AGREE

$$\Rightarrow\quad -a = a+b-1$$

$$-a+b = a = a+2b-1$$

GIVING $\quad a = 1/4 \quad b = 1/2$

SO EQNS FOR A & B BECOME

$$\frac{2\alpha}{A} = \frac{AB}{30}\left(\frac{3}{4}\right)$$

$$\frac{\nu B}{A} + \frac{\beta g\Delta T A}{3} = \frac{AB^2}{105}\left(\frac{5}{4}\right)$$

SO WE HAVE

$$A = \left[240\left(\frac{\nu^2}{\beta g\Delta T}\right)\left(\frac{\alpha^2}{\nu^2}\right)\left(\frac{20}{21}+\frac{\nu}{\alpha}\right)\right]^{1/4}$$

$$B = \frac{80\alpha}{A^2}$$

19.19 CONTINUED -

& FINALLY - UPON SUBSTITUTION:

$$\delta = A x^{1/4} = 3.94\, Pr^{-1/2} (Pr + 0.954)^{1/4}\, Gr_x^{-1/4}\, x$$

$$\frac{q}{A} = -k\frac{\partial T}{\partial y}(0) = \frac{2k}{\delta}(T_s - T_\infty) = h(T_s - T_\infty)$$

$$\Rightarrow Nu_x = 0.508\, Pr^{1/2} (Pr + 0.954)^{-1/4}\, Gr_x^{1/4}$$

19.20

$$\frac{\delta_T}{\delta} = \frac{1}{Pr^{1/3}}\left[1 - \left(\frac{X}{x}\right)^{3/4}\right]^{1/3}$$

$$Nu_x = 0.33\left[\frac{Pr}{1 - (X/x)^{3/4}}\right]^{1/3} Re_x^{1/2}$$

$$Re_x = \frac{0.4(5)}{1.569\times10^{-5}} = 127{,}500$$

$$\delta = \frac{5x}{Re_x^{1/2}} = \frac{5(40)}{(1.275\times10^5)^{1/2}} = \underline{0.56\,\text{cm}}$$

$$\delta_t = \frac{0.56}{0.708^{1/3}}\left[1 - \left(\tfrac{1}{2}\right)^{3/4}\right]^{1/3} = \underline{0.465\,\text{cm}}$$

$$C_{f_x} = \frac{0.664}{Re_x^{1/2}} = \underline{1.86\times10^{-6}}$$

$$Nu_x = 0.33\left[\frac{0.708}{1 - (1/2)^{3/4}}\right]^{1/3} (1.275\times10^5)^{1/2}$$

$$= 143$$

$$h_x = k/x\,(143) = \underline{9.38\ \text{W/m}^2\cdot\text{K}}$$

19.21 $\quad v = a + by \quad$ BC, $v(0) = 0$, $v(\delta) = v_\infty$

$$\therefore \frac{v}{v_\infty} = \frac{y}{\delta}$$

$T - T_s = \alpha + \beta y \quad$ B.C. $(T - T_s)_0 = 0$, $(T - T_s)_{\delta_t} = T_\infty - T_s$

$$\therefore \frac{T - T_s}{T_\infty - T_s} = \frac{y}{\delta_t}$$

19.21 CONTINUED

INTO MOMENTUM EQN:

$$\nu \frac{\partial v}{\partial y}(0) = \frac{d}{dx}\int_0^{\delta} (v_\infty - v)\, v\, dy$$

$$LHS = \nu v_\infty / \delta$$

$$RHS = -v_\infty^2 \frac{d}{dx}\int_0^{\delta}\left(1 - \frac{v_x}{v_\infty}\right)\left(\frac{v_x}{v_\infty}\right) dy$$

EQUATING & SOLVING:

$$\delta^2 = \frac{12\nu x}{v_\infty} \qquad (1)$$

ENERGY EQN:

$$\alpha \frac{\partial T}{\partial y}(0) = \frac{d}{dx}\int_0^{\delta_t} (T_\infty - T)\, v\, dy$$

SUBSTITUTING & SOLVING:

$$\frac{6\alpha}{v_\infty \delta \xi} = \frac{d}{dx}(\delta \xi^2) \quad (2) \quad \left\{\xi = \frac{\delta_t}{\delta}\right\}$$

$\delta_t = 0$ FOR $x = X$

$$\xi^3 = \frac{\alpha}{\nu}\left[1 - \left(\frac{X}{x}\right)^{3/4}\right]$$

$$\frac{\delta_t}{\delta} = Pr^{-1/3}\left[1 - \left(\frac{X}{x}\right)^{3/4}\right]^{1/3}$$

$$\frac{q}{A} = -k\frac{dT}{dy}(0) = -\frac{k(T_s - T_\infty)}{\delta_t} = h(T_s - T_\infty)$$

GIVING:

$$Nu_x = \frac{hx}{k} = 0.288\left[\frac{Pr}{1 - (X/x)^{3/4}}\right]^{1/3} Re_x^{1/2}$$

IF $X = 0$

$$Nu_x = 0.288\, Pr^{1/3}\, Re_x^{1/2}$$

19.22 $v = a \sin by \quad T - T_s = \alpha \sin \beta y$

B.C. $v(0)=0 \qquad (T-T_s)|_0 = 0$

$v(\delta) = v_\infty \qquad (T-T_s)|_{\delta_t} = T_\infty - T_s$

$$\Rightarrow \frac{v}{v_\infty} = \sin\frac{\pi y}{2\delta} \qquad \frac{T-T_s}{T_\infty - T_s} = \sin\frac{\pi y}{2\delta_t}$$

INTO ENERGY EQN.

$$\delta_t \frac{d\delta_t}{dx} = \frac{\alpha\pi}{v_\infty}\left(\frac{\pi}{4-\pi}\right)$$

{PRESUMES $\delta = \delta_t$ FOR INTEGRATION}

$$\frac{q}{A} = -k\frac{dT}{dy}(0) = h(T_s - T_\infty)$$

$$\frac{k\pi}{2\delta_t} = h \quad \text{OR} \quad \frac{h}{k} = \frac{\pi}{2\delta_t}$$

$$\Rightarrow Nu_x = \frac{hx}{k} = 0.327\, Pr^{1/3} Re_x^{1/2}$$

19.23

$$\frac{v}{v_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \quad \frac{T-T_s}{T_\infty - T_s} = \left(\frac{y}{\delta_t}\right)^{1/7}$$

ENERGY EQN:

$$\alpha\frac{dT}{dy}(0) = v_\infty(T_\infty - T_s)\frac{d}{dx}\int_0^{\delta_t}\frac{v_x}{v_\infty}\left(1 - \frac{T-T_s}{T_\infty - T_s}\right)dy$$

$$LHS = \frac{0.0225(T_\infty - T_s)v_\infty}{\nu/\alpha}\left(\frac{\nu}{\delta v_\infty}\right)^{1/4}$$

$$RHS = v_\infty(T_\infty - T_s)\frac{7}{72}\frac{d\delta}{dx}$$

{ASSUMES $\delta = \delta_t$ FOR INTEGRATION}

EQUATING & SOME ALGEBRA:

$$\frac{\delta}{x} = 0.371\, Pr^{-4/5} Re_x^{-1/5}$$

19.23 CONTINUED -

$$\frac{q}{A} = -k\frac{dT}{dy}(0) = -\frac{k(0.0225)(\Delta T)v_\infty}{\nu}\left(\frac{\nu}{v_\infty\delta}\right)^{1/4}$$

$$= h\Delta T$$

$$Nu_x = \frac{hx}{k} = 0.0288\, Re_x^{19/20} Pr^{1/5}$$

19.24 $q = hA\Delta T$

$q/A = 284 - 95 = 189\ W/m^2$

$\Delta T = 8\,K \qquad A = (1)(18.3) = 18.3\ m^2$

$$h = \frac{189}{8} = 23.63\ W/m^2\cdot K$$

FOR CONDITIONS SPECIFIED:

$$Re_L = \frac{(18.3\,m)\,v}{1.5689\times10^{-5}\ m^2/s} = 1.166\times10^6\, v$$

PROBABLY TURBULENT B.L.

USE COLBURN ANALOGY: $St = \frac{C_f}{2} Pr^{-2/3}$

FROM CH 13 - FOR TURB. B.L.

$$C_{fx} = 0.0576\, Re_x^{-1/5}$$

$$C_{fL} = \frac{1}{L}\int_0^L C_{fx}\,dx$$

$$= 0.072\, Re_L^{-1/5}$$ (ASSUMING ALL SURFACE EXPOSED TO TURB. B.L.)

$$St = \frac{h}{\rho c_p v_\infty} = \frac{0.072}{2} Re_L^{-1/5} Pr^{-2/3}$$

$$= 0.036\left[1.166\times10^6\, v\right]^{-1/5}(0.708)^{-2/3}$$

$$= 0.00277\, v^{-1/5}$$

$$h = \rho c_p v\,(0.00277\, v^{-1/5})$$

$$= (1.177)(1006)(0.00277)\, v^{4/5}\ W/m^2\cdot K$$

$$= 3.280\, v^{4/5} = 23.63$$

$$v = 11.8\ m/s$$

19.25

$$\frac{T-T_s}{T_0-T_s} = \exp\left(-St\,\frac{4x}{D}\right)$$

$$\frac{T-300}{60-300} = \exp\left[-St\,\frac{4(15)}{\frac{1}{12}}\right] = \exp(-720\,St)$$

$V = 12.25$ FT/S

REYNOLDS ANALOGY: ASSUME $T_L = 240$ F

$T_{AVG} = 150$ F $T_f = 225$ F

$$Re = \frac{\frac{1}{12}(12.25)}{3.07\times10^{-5}} = 3.32\times10^5$$

$f = 0.0036$ $St = 0.0018$

$$T = 300 - (240)e^{-(0.0018)(720)}$$

$= 234.5$ F

CLOSE ENOUGH — DOING OVER WITH $T_L = 234.5$ WILL YIELD $T_L \cong 234.5$ AS A RESULT.

COLBURN ANALOGY: ASSUME $T_L = 200$ F

$T_{AVG} = 130$ $T_f = 215$

$$Re = \frac{\frac{1}{12}(12.25)}{0.321\times10^{-5}} = 3.18\times10^5$$

$f = 0.0036$ $St = 0.0018\,(1.79)^{-2/3}$

$= 0.00122$

$$T = 300 - (240)e^{-(0.00122)(720)}$$

$= 201$ F

$$q = \dot m c_p \Delta T = \frac{30}{7.48}(62.3)(0.999)\Delta T$$

$= 250\,\Delta T$ BTU/MIN

SUMMARY	ΔT, F	q, BTU/MIN.
REYNOLDS	174	43,500
COLBURN	141	35,300

19.26

$$q = 500\,\frac{\text{BTU}}{\text{HR FT}^2}(\pi)\left(\frac{15}{12}\right)\text{FT}^2$$

$= 1960$ BTU/HR

$$q = \dot m c_p \Delta T = 1960$$

$$\Delta T = \frac{1960}{(30/7.48)(62.3)(60)(0.999)}$$

$= 0.131$ F

$T_{w\,EXIT} = 60.13$ F

FROM COLBURN ANALOGY: $T \cong 60$ F

$$Re = \frac{\left(\frac{1}{12}\right)\left(\frac{30}{7.48}\right)\left(\frac{144\times4}{\pi}\right)\left(\frac{1}{60}\right)}{0.76\times10^{-3}}$$

$= 1344$ {LAMINAR}

$$f = \frac{16}{1344} = 0.0119$$

$$St = \frac{0.0119}{2}(8.07)^{-2/3} = 0.00148$$

$$h = 62.3\left(\frac{30\times144\times4}{7.48\times\pi\times60}\right)(0.24)(3600)(St)$$

$= 976$ BTU/HR FT^{2}F

$$T_{WALL} = 60.13 + \frac{500}{976} = 60.6\text{ F}$$

19.27

q = SAME AS IN PROB 19.26

$= 1960$ BTU/HR

$$T = T_0 + \frac{q}{\rho A v c_p}$$

$$= 60 + \frac{1960}{(0.0764)\left(\frac{\pi}{4}\times\frac{1}{144}\right)(15\times3600)(0.24)}$$

$= 423$ F

19.28

$$T_L = T_0 - \Delta T\, e^{-St\, 4L/D}$$
$$= 300 - 100\, e^{-St\, 4L/D}$$

$$v = \frac{30(144)}{7.48(60)(\pi/4)} = 12.25 \text{ ft/s}$$

$$Re = \frac{Dv}{\nu} = \frac{(1/12)(12.25)}{7.43\times10^{-6}} = 137{,}100$$

$Pr = 0.0118$ $\quad f_F = 0.0044$

REYNOLDS ANALOGY: $\quad St = 0.0022$

$$T_L = 300 - 100\, e^{-1.585} = \underline{279.5 \text{ F}}$$

$$q = \rho A v c_p \Delta T = \underline{\underline{100.5 \text{ Btu/s}}}$$

COLBURN ANALOGY: $\quad St = 0.0022(0.0118)^{-2/3}$

$$= 0.0424$$

$$T_L = 300 - 100\, e^{-30.1} \hat{=} 300 \text{ F}$$

$$\underline{\underline{q = 126.4 \text{ Btu/s}}}$$

19.29

FOR CONSTANT q/A

$$T = T_0 + \frac{q/A}{\rho v c_p}$$

$$= 200 + \frac{500}{(58.1)(12.25\times3600)(0.332)}$$

$$\cong \underline{\underline{200 \text{ F}}}$$

CHAPTER 20

20.1 $\frac{q}{A} = \frac{750(3413)}{\pi(3/48)(1/2)} = 26100 \frac{Btu}{HR\,FT^2}$

ENDS ARE NEGLECTED

$$h = \frac{k}{L} Nu_L = \frac{k}{L}\left[0.825 + \frac{0.387\,Ra^{1/6}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

FOR VERTICAL ORIENTATION

BY TRIAL & ERROR: $\Delta T \cong 103$ F

HTR SURFACE TEMP = 198 F

HORIZONTAL ORIENTATION:

$$h = \frac{k}{D}\left[0.60 + \frac{0.387\,Ra_D^{1/6}}{\left\{1+\left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

TRIAL & ERROR: $\Delta T = 99$ F

HTR SURFACE TEMP = 194 F

20.2 BISMUTH $T_\infty = 700$ F

AS IN PROBLEM 20.1 $\frac{q}{A} = 26100 \frac{Btu}{HR\,FT^2}$

VERTICAL - SAME FORMULA AS ABOVE

TRIAL & ERROR: $\Delta T \cong 57$ F

$T_{SURF} \cong$ 757 F

HORIZONTAL - SAME FORMULA AS ABOVE

TRIAL & ERROR: $\Delta T \cong 44$F

$T_{SURF} \cong$ 744 F

20.2 CONT. HYDRAULIC FLUID

SAME FORMULAS & PROCEDURES

VERTICAL: $\Delta T \cong 630$ $T_{SURF} \cong$ 630 F

HORIZONTAL $\Delta T \cong 580$ $T_{SURF} \cong$ 580 F

{ PROPERTIES USED AT 200 F — HIGHEST TEMP IN TABLES }

20.3 $\frac{q}{A} = \frac{3413}{0.344} = 9900$ Btu/HR FT^2

16 cm (0.525 FT) - VERTICAL HT.

$T_\infty = 71$ F

$$h = \frac{k}{L}\left[0.825 + \frac{0.387\,Ra^{1/6}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

TRIAL & ERROR: $\Delta T = 62$ F $T_S = 133$F

FOR 10 cm (0.328 FT) - HEIGHT

TRIAL & ERROR: $\Delta T = 60$ F $T_S = 131$F

ENGLISH UNITS USED - TABLES EASIER TO USE

20.4 FOR $T_f = 100$ F

$Gr_L = (107 \times 10^6)(1/2)^3(100) = 1.337 \times 10^9$

$Pr = 4.51$ $Ra = 6.03 \times 10^9$

$$h_{SIDES} = \frac{k}{L}\left[1 + \frac{0.387\,Ra^{1/6}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

$= 190$ Btu/HR FT^2 F

$Bi = \frac{hV/A}{k} = \frac{190}{220}(0.0357) = 0.0308$

USE LUMPED PARAMETER

20.4 CONT. FOR $T_{S\,AVG} = 150\,F$

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{50}{150} = \frac{1}{3} = e^{-Bi\,Fo}$$

$$Fo = \frac{\alpha t}{(V/A)^2} = \frac{3.98\,t}{(0.0357)^2} = 3120\,t$$

t IN HOURS

$$-Bi\,Fo = \ln 1/3$$

$$t = 0.0114\ \text{HR} = 0.686\ \text{MIN} = 41.1\ \text{S}$$

SINCE LUMPED PARAMETER SOLN IS VALID – ANSWERS TO PARTS (a) & (b) ARE THE SAME

WHEN $T_C = 100\,F$ $\underline{T_{SURF} \cong 100\,F}$

20.5 $T_S = 140\,C$ $T_\infty = 25\,C$ $T_f = 82.5\,C$

AIR @ 355 K: $\frac{\beta g}{\nu^2} = 0.625\times10^8\ (m^3\cdot K)^{-1}$

$$Ra = (0.625\times10^8)(0.035)^3(115) = 3.08\times10^5$$

HORIZ. CYLINDER;

$$Nu = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

– FOR $Pr = 0.696$ $Nu_D = 10.47$

$$q = hA\Delta T = \frac{k}{\cancel{D}}(\pi \cancel{D} L)(\Delta T)$$

$$= (0.0304)(\pi)(0.8)(115)(10.47)$$

$$= \underline{92.0\ W}$$

REMAINDER OF 100 W INPUT GOES TO ELECTRICAL & CONDUCTION LOSSES & TO ILLUMINATION

20.6 FOR A HORIZ. CYLINDER

$$Nu = \left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$q = 27\ W/m = hA\Delta T = \frac{k}{D} Nu\,A\,\Delta T$$

$$= \frac{k}{\cancel{D}} Nu (\pi \cancel{D}) \Delta T$$

$$27 = \pi k\,Nu\,\Delta T$$

TRIAL & ERROR ~ $\Delta T \cong 9.8\,K$

$$\underline{T_{SURF} \cong 39.8\ C}$$

20.7 FOR Cu CYLINDER WITH HT = 20.3 cm, DIAM = 2.54 cm

$$\frac{V}{A} = \frac{(\cancel{\pi} D^2/4)L}{\cancel{\pi} D L + \cancel{\pi} D^2/2} = \frac{DL/4}{L+D/2} = 0.598\ cm$$

FOR $Bi = \frac{h\,V/A}{k} = 0.1$ $T_f \cong 16\,C$

$$h = \frac{0.1(379)}{0.00598} = 6340\ W/m^2\cdot K$$

FOR AN h VALUE < 6340

Bi < 0.1 ∴ LUMPED PARAM.

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-Bi\,Fo}$$

$$Fo = \frac{\alpha t}{(V/A)^2} = \frac{(10.27\times10^{-5})(180)}{(0.00598)^2}$$

$$= 516.9$$

$$\frac{4.8-(-1)}{32.5-(-1)} = 0.173 = e^{-Bi\,Fo}$$

$Bi\,Fo = 1.754$ $Bi = 3.393\times10^{-3}$

$$h = \frac{Bi(379)}{0.598} = \underline{2.15\ W/m^2\cdot K}$$

20.8 FOR A SPHERE: $Nu_D = 2 + 0.43 Ra_D^{1/4}$

$T_{S\,AVG} = 340\,K$ $\quad T_f \cong 320\,K$ $\quad k = 2.78 \times 10^{-2}$

$T_\infty = 295\,K$ $\quad Pr = 0.703$

$$Ra_D = (0.994 \times 10^8) D^3 (45)(0.703)$$
$$= 3.144 \times 10^9 \; D^3$$

D, cm	h	k/hx₁
7.5	615	0.0104
5	710	0.0135
1.5	1180	0.0271

I.E., SURFACE RESISTANCE IS VERY SMALL ∴ $T_S = T_\infty$ & FALLS TO THIS VALUE ALMOST INSTANTANEOUSLY

$\sim$ TIME $\cong 0$

20.9 FOR T_c TO REACH 320 K — USE VALUES CALCULATED IN PROB 20.8

$$\alpha = k/\rho c_p = 2.1 \times 10^{-7}\ m^2/s$$

D, cm	h	$\alpha t/x_1^2$	t
7.5	615	0.16	1.19 HR
5	710	0.16	31.7 MIN
1.5	1180	0.16	2.86 "

$T_{SURF} \cong 295\,K$ AT ALL TIMES

20.10 $T_S = 240\,F$ $\quad T_\infty = 60\,F$ $\quad T_f = 150\,F$

$\Delta T = 180\,F$ $\quad \frac{\beta g}{\nu^2} = 1.22 \times 10^6$ $\quad Pr = 0.698$

$k = 0.0167$

a) HORIZONTAL:

$$h = \frac{k}{D}\left[0.6 + \frac{0.387\, Ra_D^{1/6}}{\left\{1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

20.10 CONT.

$$h = 1.51\ \frac{Btu}{HR\,FT^2\,F}$$

$$\frac{q}{A} = h\Delta T = \underline{\underline{271\ Btu/HR\,FT^2}}$$

VERTICAL:

$$h = \frac{k}{L}\left[0.825 + \frac{0.387\, Ra_L^{1/6}}{\left\{1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

$h = 1.0$ $\quad \underline{\underline{\frac{q}{A} = 180\ Btu/HR\,FT^2}}$

20.11 SAME CONDITIONS AS PROB 20.10 EXCEPT FLUID IS H_2O @ 60 F

$\frac{\beta g}{\nu^2} = 403 \times 10^6$ $\quad Pr = 2.72$ $\quad k = 0.383$

HORIZ: $h = 316\ \frac{Btu}{HR\,FT^2\,F}$ $\quad \underline{\underline{\frac{q}{A} = 57000\ \frac{Btu}{HR\,FT^2}}}$

VERT: $h = 281$ " $\quad \underline{\underline{\frac{q}{A} = 50{,}600\ \frac{Btu}{HR\,FT^2}}}$

20.12 SPHERICAL TANK $D = 0.6\,m$

$T_i = 78\,K$ $\quad T_\infty = 278\,K$ $\quad q = \Delta T / \Sigma R$

FOR SPHERE:

$$h = \frac{k}{D} Nu = \frac{k}{D}\left[2 + 0.43\, Ra^{1/4}\right]$$

$$h = \frac{0.0247}{.70}\left[2 + 0.43\left\{(2.57 \times 10^8)(0.7)^3(18)(0.72)\right\}^{1/4}\right]$$

$= 2.86\ W/m^2 \cdot K$ — PROPERTIES @ 260 K

$$R_{CONV} = \frac{1}{2.86(4\pi)(0.35)^2} = 0.227$$

20.12 CONTINUED

$$R_{COND} = \frac{r_o - r_i}{4\pi k r_o r_i} = \frac{0.05}{4\pi(0.04)(0.3)(0.35)}$$

$$= 0.947$$

$$\Sigma R = 1.174 \quad q = \frac{200}{1.174} = \underline{\underline{170\ W}}$$

$\Delta T_{CONV} \cong 39\ K$ $\quad T_{SURF} \cong 239\ K$, $\quad T_f \cong 259 K$ $\quad$ O.K.

20.13

ASSUMING EACH PLATE IS INDEPENDENT

$$h = \frac{k}{L} Nu = \frac{k}{L}\left[0.825 + \frac{0.387\, Ra^{1/6}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

$T_{PLATE} = 200 F \quad T_\infty = 80 F \quad T_f = 140 F$

$Pr = 3.08 \quad Ra = (540\times10^6)(3)^3(120)(3.08)$

$$= 5.39\times10^{12}$$

$$h = 287\ Btu/HR\,FT^2 F$$

$$q = hA\Delta T = 287(30\times1\times3\times2)(120)$$

$$= \underline{\underline{6.19\times10^6\ Btu/HR}}$$

$$= \underline{1.81\ kW}$$

20.14 $\quad q/A = h\Delta T$

$$h = \frac{k}{L}\left[0.14\, Ra_L^{1/3}\right] \text{ IF } Ra > 2\times10^7$$

$T_{SURF} = 150 F \quad T_\infty = 50 F \quad T_f = 100 F$

$Pr = 0.703 \quad Ra = (1.76\times10^6)(20)^3(100)(0.703)$

$k = 0.0156 \quad = 9.90\times10^{11}$

$$h = \frac{0.0156}{20}(0.14)(9.90\times10^{11})^{1/3} = 1.09\ \frac{Btu}{HR\,FT^2 F}$$

$$\frac{q}{A} = 1.09(100) = 109\ Btu/HR\,FT^2$$

20.14 CONTINUED -

$$\text{FRACTION OF TOTAL} = \frac{109}{200} = \underline{\underline{0.54}}$$

WITH 1 FT x 1 FT RIDGES

$$Ra = (1.76\times10^6)(1)^3(100)(0.703) = 1.237\times10^9$$

$$h = \frac{0.0156}{1}(0.14)(1.237\times10^9)^{1/3} = 1.09\ \frac{Btu}{HR\,FT^2 F}$$

SAME AS IN PART (a) FRACT. = $\underline{\underline{0.54}}$

20.15 FOR FORCED CONVECTION

$$Re = \frac{(20\ FT)(6.1\times3.281\ FT/S)}{0.181\times10^{-3}\ FT^2/S} = 2.21\times10^6$$

FLOW IS LAMINAR UP TO $Re = 2\times10^5$, TURBULENT PAST 3×10^6

PLATE IS MOSTLY IN TRANSITION REGIME

ASSUME LAMINAR B.L.

$$h = \frac{k}{L}Nu = \frac{0.0156}{20}\left(0.664\, Re_L^{1/2}\, Pr^{1/3}\right)$$

$$= 0.685\ Btu/HR\,FT^2 F$$

$$\frac{q}{A} = h\Delta T = 68.5\ Btu/HR\text{-}FT^2$$

FRACTION DUE TO F.C. = $\underline{0.34}$

IF B.L. IS TURBULENT

$$h = \frac{0.0156}{20}\left[0.036\, Re^{0.8}\, Pr^{1/3}\right]$$

$$= 2.97\ Btu/HR\,FT^2 F$$

$\frac{q}{A} = 297\ Btu/HR\text{-}FT^2 \quad$ FRAC = $\underline{1.49}$

FOR THIS CASE THERE IS MORE CAPACITY TO TRANSFER HEAT THAN THERE IS SOLAR ENERGY SUPPLIED. SURFACE TEMP WILL BE <150 F

20.16

$$T_{surf} = 1300\text{ K}, \quad T_\infty = 270\text{ K} \quad \Big\} \; T_f = 785\text{ K}$$

$$\frac{q}{A} = h\Delta T \qquad h = \frac{k}{D}\left(2 + 0.43\, Ra^{1/4}\right)$$

$$k = 5.69\times10^{-2}\ \text{W/m·K} \qquad Pr = 0.688$$

$$Ra = (2.015\times10^{6})(0.15)^3(1030)(0.688)$$

$$= 4.82\times10^{6}$$

$$h = 8.4\ \text{W/m}^2\text{·K}$$

$$\frac{q}{A} = 8.4(1030) = \underline{\underline{8650\ \text{W/m}^2}}$$

20.17 From Prob 20.16

$$h = \frac{k}{D}Nu = \frac{k}{D}\left[2 + 0.43\, Ra^{1/4}\right]$$

T_{surf}	$T-T_\infty$	T_f	$Ra^{1/4}$	h
1300	1030	785	46.8	8.40
1000	730	685	50.3	7.65
700	430	485	63.1	8.10
420	150	345	57.2	6.69

$$h_{AVE} \cong 7.71\ \text{W/m}^2\text{·K}$$

$$Bi = \frac{hV/A}{k} = \frac{7.71(0.15/6)}{39.8} = 0.00484$$

{Lumped parameter is ok}

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{600-270}{1300-270} = 0.32 = e^{-Bi\,Fo}$$

$$Bi\,Fo = 1.1394 \qquad Fo = 235.4 = \frac{\alpha t}{(V/A)^2}$$

$$t = \frac{(0.15/6)^2(235.4)}{0.001125} = \underline{\underline{130.8\ \text{s}}}$$

20.18

$$q = I^2R = (400)^2\, 8L/A$$

$$R = \frac{(1.72\times10^{-6})(100)}{\pi/4\,(0.5)^2} = 8.76\times10^{-4}\ \Omega/\text{m}$$

$$\frac{q}{A} = \frac{400^2(8.76\times10^{-4})}{\pi(0.018)(1)} = 2480\ \text{W/m}$$

$$= h\Delta T \qquad q = 140\ \text{W/m}$$

$$h = \frac{k}{D}C\,Ra^n$$

$$q = \frac{k}{D}C\left[\left(\frac{\beta g}{\nu^2}\right)D^3 Pr\right]^n \Delta T^{1+n}$$

By trial & error: $\Delta T \cong 220\text{ K}$

$$h = \underline{\underline{11.0\ \text{W/m}^2\text{·K}}}$$

$$T_{surf} = 290 + 220 = \underline{\underline{510\ \text{K}}}$$

Resistance of insulation

$$= \frac{\ln D_o/D_i}{2\pi k} = \frac{\ln\frac{0.018}{0.005}}{2\pi(0.242)} = 0.842$$

$$q = \frac{\Delta T}{R} \qquad \Delta T = 140/0.842 = 166\text{ K}$$

$$T_{interface} = 510 + 166 = \underline{\underline{676\ \text{K}}}$$

20.19

$$R_{Al} = \frac{2.83\times10^{6}}{1.72\times10^{6}}(8.76\times10^{-4})$$

$$= 1.44\times10^{-3}\ \Omega/\text{m}$$

$$q = (400)^2(1.44\times10^{-3}) = 231\ \text{W/m}$$

$$\frac{q}{A} = \frac{231}{\pi(0.018)} = 4080\ \text{W/m}^2$$

$$= \frac{k}{D}C\left[\frac{\beta g}{\nu^2}D^3 Pr\right]^n \Delta T^{1+n}$$

20.19 CONTINUED -

TRIAL & ERROR: $\Delta T \approx 336\,K$

$h = \underline{12.1\ W/m^2 \cdot K}$

$T_{SURF} = 290 + 336 = \underline{626\,K}$

$R_{INSUL} = 0.842$ {PROB 20.43}

$\Delta T = q/R = \dfrac{231}{0.842} = 274\,K$

$T_{INTERFACE} = 626 + 274 = \underline{900\,K}$

20.20 $q_{TOTAL} = q_{CONV} + q_{RAD}$

ASSUME $T_{INSIDE} = T_{SURFACE}$

$$q = h_i A_i (T_{STM} - T) = h_o A_o (T - T_\infty) + \sigma A_o \left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_\infty}{100}\right)^4\right]$$

T IN R

TRIAL & ERROR: $T = 1147\,R$
$= 687\,F$

$$h_o = \frac{k}{D}\left[0.6 + \frac{0.387\, Ra_D^{1/6}}{\left\{1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$$

$\frac{q}{A} = 33(1260 - 1147) = 3730\ Btu/hr\,ft^2$

$q = 3730(\pi)\left(\frac{8.625}{12}\right)(20)$
$= \underline{168{,}000\ Btu/hr}$

20.21 FORCED CONVECTION OUTSIDE

$$\frac{q}{A} = 33(1260 - T) = h_o(T - 530) + \sigma\left[\left(\frac{T}{100}\right)^4 - 5.3^4\right]$$

$h_o = \frac{k}{D} B\, Re^n\, Pr^{1/3}$ {B, n - FUNCTIONS OF Re}

ASSUME $T \approx 650\,F = 1110\,R$ $T_f = 360\,F$

$$Re = \frac{\left(\frac{8.625}{12}\right)(6.5)(3.281)}{0.348 \times 10^{-3}} = 4.40 \times 10^4$$

TABLE 20.3 $B = 0.027$ $n = 0.805$

@ THIS TEMP $h_o = 4.31$

LHS = 4950 RHS = 4923 {PRETTY GOOD}

$q \simeq 4950\ Btu/hr\,ft^2 \left(\frac{8.625}{12}\right)(\pi)(20)$
$= \underline{224000\ Btu/hr}$
$= \underline{65.5\ kW}$

20.22 INSULATION ON OUTSIDE W/ NATURAL CONV. ON SURFACE

$R_{INSULATION} = \dfrac{\ln r_o/r_i}{2\pi k} = 1.401$ ~ PER FT

$$\Sigma R = \frac{1}{A_i h_i} + 1.401 + \frac{1}{A_o h_o}$$

$$= \frac{1}{33\pi\left(\frac{8.625}{12}\right)} + 1.401 + \frac{1}{h_o \pi \frac{14.625}{12}}$$

$= 1.414 + 0.261/h_o$

$$\frac{q}{L} = \frac{\Delta T}{\Sigma R} = \frac{730}{1.414 + \frac{0.261}{h_o}} = \frac{800 - T}{1.414}$$

WITH $h_o = \dfrac{k}{D}\left[0.6 + \dfrac{0.387\, Ra^{1/6}}{\left\{1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2$

20.22 CONTINUED

TRIAL & ERROR: $T \cong 190 F$

$$q = \frac{800-190}{1.414} \frac{Btu}{HR\,FT} (20\ FT)$$

$$= \underline{\underline{8630\ Btu/HR}} = \underline{\underline{253\ kW}}$$

20.23 $\quad q = \Delta T / R$

$$= \frac{800 - T_i}{1/\pi D_i(33)} = \frac{2\pi k}{\ln r_o/r_i}(T_i - 250) = \frac{800-250}{\Sigma R}$$

$$R_{STM} = \frac{1}{33(\pi)\left(\frac{8.625}{12}\right)} = 0.0134$$

$$R_{INSUL} = \frac{\ln D_o/8.625}{2\pi(0.06)} = 2.65 \ln D_o/8.625$$

EQUATIONS TO BE SOLVED ARE:

$$q = \frac{800-T}{0.0134} = \frac{T-250}{2.65 \ln D_o/8.625} = \frac{550}{\Sigma R}$$

TRIAL & ERROR; $T \cong 750 F$

$D_o \cong 9.08$ IN.

INSULATION THICKNESS

$$= \frac{9.08-8.625}{2} = \underline{\underline{0.228\ IN.}}$$

20.24

$$q = \frac{800-T_1}{R_{INS}} = \frac{T_1 - T_2}{\frac{\ln D_o/D_i}{2\pi k}} = \pi D_o h_o (T_2 - 70)(1/0.85)$$

$$\frac{800-T_1}{0.0134} = (T_1 - T_2)\frac{0.377}{\ln D_o/D_i} = 3.70 D_o h_o (T_2 - 70)$$

$$= \frac{730}{0.0134 + 2.65 \ln D_o/8.625 + \frac{0.27}{D_o h_o}}$$

TRIAL & ERROR PROBLEM (LENGTHY)

20.24 CONTINUED -

ANSWER - APPROXIMATELY

$T_1 = 793.8 F \quad \underline{\underline{T_2 = 178.4 F}}$

$$q = 465 \frac{Btu}{HR\,FT}(20\ FT) = 9290\ Btu/HR$$

$$= 2.72\ kW$$

$$\frac{800-793.8}{0.0134} = \frac{(793.8-178.4)\ 0.377}{\ln D_o/D_i}$$

$$\ln D_o/D_i = 0.50 \qquad \frac{D_o}{D_i} = 1.651$$

$$D_o = 1.651(8.625) = 14.24\ IN$$

$$\text{THICKNESS} = \frac{14.24 - 8.625}{2}$$

$$= \underline{\underline{2.81\ \text{INCHES}}}$$

20.25 FOR NATURAL CONVECTION CASE PLANE UPWARD-FACING HOT SURFACE

$Nu_L = 0.14\ Ra_L^{1/3}$ IF $2\times10^7 < Ra < 10^{10}$

ASSUME TOP SURFACE IS SQUARE

$\sim A = L^2$

$$q = hA\Delta T = hL^2\Delta T$$

$$= \frac{k}{L} Nu_L L^2 \Delta T$$

$$= k\left[0.14\ Ra_L^{1/3}\right] L \Delta T$$

$$Ra_L = \frac{\beta g}{\nu^2} L^3 \Delta T\ Pr$$

$$q = k\left[0.14\left(\frac{\beta g}{\nu^2}\Delta T\right)^{1/3} Pr^{1/3} L^2 \Delta T\right]$$

@ $T_s = 45 C \quad T_\infty = 20 C \quad T_f = 32.5 C$

$k = 0.02663\ \frac{W}{m\,K} \qquad \frac{\beta g}{\nu^2} = 1.244\times10^{8}\ (m^3 K)^{-1}$

20.25 CONTINUED

$Ra = (1.244 \times 10^{8})(25)(0.707) L^3$

$= 2.199 \times 10^9 L^3$

$40\,W = 0.02463 \left[0.14 (1.244 \times 10^8)^{1/3} (0.707)^{1/3} L^2 (25)\right]$

$L^2 = 0.965 \quad L \cong 0.982\,m$

$Ra = 2.08 \times 10^9$ ~ OK ~

Now — For Same Ht Loss & $L = 0.982\,m$

$q = hA\Delta T = \frac{k}{L} Nu_L A \Delta T$ {FORCED CONV.}

Assume $T_{SURF} \cong 20\,C$

$k = 2.569 \times 10^{-2}$

$\nu = 1.506 \times 10^{-5}$

$Re = \frac{LV}{\nu} = \frac{0.965(20)}{1.506 \times 10^{-5}} = 1.28 \times 10^6$

Transition Regime —

Assume Laminar B.L. —

$Nu = 0.664\, Re_L^{1/2} Pr^{1/3}$

$40\,W = \frac{0.02569}{0.965} (0.965)^2 (0.664)(1.28 \times 10^6)^{1/2} \times (0.71)^{1/3} \Delta T$

$\Delta T = 2.4\,F$ $\quad \underline{T_{SURF} \cong 22.4\,C}$

20.26 $q = \frac{\Delta T}{\Sigma R} = \frac{319 - 301\ K}{\Sigma R}$

$R_o = \frac{1}{h_o \pi D_o L} = \frac{1}{(6800)\pi(0.019)} = 2.464 \times 10^{-3}$

$R_i = \frac{1}{h_i \pi D_i L} = \frac{1}{(5200)\pi(0.0165)} = 3.71 \times 10^{-3}$

$R_{cu} = \frac{\ln D_o/D_i}{2\pi k L} = \frac{\ln 1.9/1.65}{2\pi(385)} = 5.83 \times 10^{-5}$

20.26 CONTINUED —

$\Sigma R = 6.232 \times 10^{-3}$

$\dot{m} = \frac{18}{(\Sigma R)(h_{fg})} = \frac{18}{6.232 \times 10^{-3}(2390)}$

$= \underline{1.21\ kg/s}$

20.27 $\dot{m}$ per tube $= 0.49\ kg/s$

$Re = \frac{0.49(4)}{\pi(0.0209)(79 \times 10^{-3})} = 3780$

Use Analogy or Assume Turbulent

$\ln \frac{T_L - T_S}{T_0 - T_S} = -4 \frac{L}{D} St \qquad St = \frac{f}{2} Pr^{-2/3}$

Assume $T_{EXIT} = 314\,K \quad T_{bAVG} = 307\,K$

$Pr \cong 121$

$f \cong 0.01$ $\qquad St = \frac{0.01}{2}(121)^{-2/3} = 2.04 \times 10^{-4}$

$T_L = 372 - 72\, e^{-2.04 \times 10^{-4}(4)(5)/0.0209}$

$= 313\,K$ ~ CHECK

$q = 1.47(2000)(13) = \underline{38.2\ kW}$

20.28 Assume $T_L = 235\,F \quad T_{AVG} = 148\,F$

$Re = \frac{DV}{\nu} = \frac{(0.87/12)(40)}{0.209 \times 10^{-3}} = 1.39 \times 10^4$

{TURBULENT}

$St = 0.023\, Re_D^{-0.8} Pr^{-0.2} = 4.33 \times 10^{-3}$

$\frac{T_L - T_S}{T_0 - T_S} = e^{-4 \frac{L}{D} St}$

$T_L = 240 - 180(0.0083)$

$= \underline{239\ F}$ {CLOSE ENOUGH}

20.29 $\frac{q}{A} = h\Delta T = 180\,h$

a) FLOW PARALLEL TO TUBE

$$Re = \frac{Lv}{\nu} = \frac{L(40)}{0.201\times10^{-3}} = 1.91\times10^{5}\,L$$

IF $X \leq 1.5$ B.L. IS LAMINAR

IF $X = 10$ B.L. IS IN TRANSITION

IF LAMINAR OVER TOTAL LENGTH:

$$h = \frac{k}{L}(0.664)\,Re_L^{1/2}\,Pr^{1/3}$$

$$= \frac{0.167}{10}(0.664)(1.91\times10^{5})^{1/2}(0.72)^{1/3}$$

$$= 13.74 \text{ BTU/HR FT}^2\text{ F}$$

$$\frac{q}{A} = 13.74(180) = \underline{\underline{2470 \text{ BTU/HR FT}^2}}$$

b) CROSSFLOW CASE

$$Re = \frac{Dv}{\nu} = 1.59\times10^{4}$$

$$h = \frac{k}{D}\left[0.193\,(1.59\times10^{4})^{0.618}(0.72)^{1/3}\right]$$

$$= 137 \text{ BTU/HR FT}^2\text{ F}$$

$$\frac{q}{A} = 137(180) = \underline{\underline{24{,}600 \text{ BTU/HR FT}^2}}$$

20.30 WATER:

a) PARALLEL TO TUBE $T_f = 150\text{F}$

$$Re = \frac{10(40)}{0.40\times10^{-5}} = 8.44\times10^{7}$$

{TURBULENT}

$$h = \frac{k}{L}(0.036)\,Re_L^{4/5}\,Pr^{1/3}$$

$$= \frac{0.383}{10}(0.036)(8.44\times10^{7})^{4/5}(2.72)^{1/3}$$

$$= 4220 \text{ BTU/HR FT}^2\text{ F}$$

$$\frac{q}{A} = 4220(180) = \underline{\underline{7.6\times10^{5} \text{ BTU/HR FT}^2}}$$

20.30 CONTINUED -

b) CROSSFLOW

$$Re = 7.03\times10^{5}$$

$$h = \frac{k}{D}(0.027)(7.03\times10^{5})^{0.805}(2.72)^{1/3}$$

{TABLE 20.3 VALUES @ HIGHEST Re}

$$h = 8820 \text{ BTU/HR FT}^2\text{ F}$$

$$\frac{q}{A} = 8820(180) = 1.59\times10^{6} \text{ BTU/HR FT}^2$$

20.31 a) PARALLEL

$$Re = \frac{10(40)}{5.45\times10^{-5}} = 7.34\times10^{6}$$

{TURB}

$$h = \frac{k}{L}(0.036)(7.34\times10^{6})^{4/5}(80.5)^{1/3}$$

$$= 312 \text{ BTU/HR FT}^2\text{ F}$$

$$\frac{q}{A} = 312(180) = \underline{\underline{56{,}100 \text{ BTU/HR FT}^2}}$$

b) CROSSFLOW $Re = 61{,}000$

$$h = \frac{k}{D}(0.027)(61{,}000)^{0.805}(80.5)^{1/3}$$

$$= 642 \text{ BTU/HR FT}^2\text{ F}$$

$$\frac{q}{A} = 642(180) = \underline{\underline{115{,}600 \text{ BTU/HR FT}^2}}$$

20.32 $Re = \frac{GD}{\mu}$ $T_f = 186\text{F}$

$$Re = \frac{(0.385/12)(20)}{0.379\times10^{-5}} = 169{,}000$$

FROM FIGURE 20.13 $j \cong 10^{-3}$

{EXTRAPOLATION}

20.32 CONTINUED

$$\frac{h}{3c_p V} = 10^{-3}\,Pr^{-2/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$= 10^{-3}(2.14)^{-2/3}\left(\frac{0.273}{0.185}\right)^{0.14} = 0.631\times10^{-3}$$

$$h = (0.631\times10^{-3})(1.01)(60.4)(3600)$$

$$= \underline{135\ \text{Btu/hr ft}^2\,\text{F}}$$

20.33 $q = hA\Delta T$

$$= 135(48)(\pi)\left(\frac{0.387}{12}\right)(5)$$

$$= \underline{3280\ \text{Btu/hr}}$$

20.34 $T_{surf} = 380\text{K}$ $T_f = 337.5\text{K}$
$T_\infty = 295\text{K}$

$\rho = 980.6\ \text{kg/m}^3$ $\nu = 0.453\times10^{-6}\ \text{m}^2/\text{s}$

$k = 0.661\ \text{W/m}\cdot\text{K}$ $Pr = 2.90$

a) HORIZONTAL NATURAL CONV.

$$h = \frac{k}{D}\left\{0.60 + \frac{0.387\,Ra_D^{1/6}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$Ra_D = (27.54\times10^{9})(0.0126)^3(85)(2.9)$$

$$= 1.358\times10^{7}$$

$$h = 1898\ \text{W/m}^2\cdot\text{K}$$

$$q = hA\Delta T = (1898)\pi(0.0126)(0.075)(85)$$

$$= \underline{479\ \text{W}}$$

20.34 CONTINUED -

b) VERTICAL NATURAL CONV.

$$h = \frac{k}{L}\left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$Ra_L = (27.54\times10^{9})(0.075)^3(85)(2.9)$$

$$= 2.864\times10^{9}$$

$$h = 1351\ \text{W/m}^2\cdot\text{K}$$

$$q = (1351)\pi(0.0126)(0.075)(85)$$

$$= \underline{341\ \text{W}}$$

c) CROSSFLOW

$$Re = \frac{DV}{\nu} = \frac{(0.0126)(1.5)}{0.453\times10^{-6}}$$

$$= 41700$$

$$h = \frac{k}{D}\left\{0.3 + \frac{0.62\,Re^{1/2}\,Pr^{1/3}}{\left[1+\left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

$$= 11030\ \text{W/m}^2\cdot\text{K}$$

$$q = (11030)\pi(0.0126)(0.075)(85) = \underline{2.78\ \text{kW}}$$

20.35 $$Re = \frac{0.15(150)}{7.98\times10^{-5}} = 282{,}000$$

$$h = \frac{k}{D}Nu = \frac{0.0566}{0.15}(400)$$

FROM FIG 20.11

$$= 151\ \text{W/m}^2\cdot\text{K}$$

$$\frac{q}{A} = 151(1030) = \underline{155\ \text{kW/m}^2}$$

20.36 $q = 140$ W/m
$q/A = 2480$ W/m^2 {FROM PROB 20.18}

$$Re = \frac{0.018(9)}{1.569\times10^{-5}} = 10300 \quad \{T_f \cong 300K\}$$

$Pr = 0.708 \quad k = 0.0262$

$$h = \frac{k}{D}\left[0.193\,(10300)^{0.618}(0.708)^{1/3}\right]$$

$= 75.6$ W/m$^2\cdot$K

$\Delta T = 2480/75.6 = 32.8$

{$T_s = 323$, $T_f = 306$, CLOSE ENOUGH}

$\underline{h = 75.6 \text{ W/m}^2\cdot\text{K}}$

$\Delta T = 32.8 \quad \underline{T_{SURF} = 323\text{ K}}$

INSUL. RESISTANCE $= 0.842$ {PROB 20.43}

$\Delta T = 140/0.842 = 166$ K

$T_{INTERF.} = 323 + 166 = \underline{489\text{ K}}$

20.37 SPHERE: $D = 0.075$ m

$T_\infty = 25C \quad T_s = 145C$

$\nu = 1.551\times10^{-5} \quad \mu_\infty = 1.837\times10^{-5}$

$k = 0.0261 \quad \mu_s = 2.429\times10^{-5}$

$Pr = 0.708$

$$q = hA\Delta T = h(\pi)(0.075)^2(120)$$

$$Re = \frac{Dv}{\nu} = \frac{(0.075)(0.5)}{1.551\times10^{-5}} = 2418$$

$$h = \frac{k}{D}\left[2 + (0.4Re^{1/2} + 0.06Re^{2/3})Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}\right]$$

$$= \frac{0.0261}{0.075}\left\{2 + \left[0.4(2418)^{1/2} + 0.06(2418)^{2/3}\right] \times (0.708)^{0.4}\left(\frac{1.837}{2.429}\right)^{1/4}\right\}$$

20.37 CONTINUED -

$= 8.99$ W/m$^2\cdot$K

$q = 8.99(\pi)(0.075)^2(120)$

$\underline{= 19.07\text{ W}}$

20.38 $G = \rho v = 3.64$ Lbm/s-FT^2

$$Re = \frac{GD}{\mu} = \frac{3.64(0.622/12)}{0.29\times10^{-3}} = 650$$

(LAMINAR)

USE SIEDER-TATE EQN. ASSUME $T_{b,AVG} = 150F$

$$Nu = 1.86\left(Re\,Pr\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$h = \frac{k}{D}Nu = \frac{0.383(1.86)}{0.622/12}\left[(650)(2.72)\frac{0.622}{12(5)}\right]^{1/3} \times \left(\frac{0.29}{0.578}\right)^{0.14}$$

$= 32.9$ BTU/HR FT^{2}F

$St = Nu/Re\,Pr = 0.00252$

$$\frac{T_L - T_s}{T_0 - T_s} = e^{-4\frac{L}{D}St} = e^{-0.972} = 0.378$$

$T = 80 + 0.378(100) = 117.8$ F

$$T_{b,AVG} = \frac{117.8 + 180}{2} = 149\text{ F} - \text{OK}$$

$q = \dot{m}c_p\Delta T = 3.64(0.0021)(149.7 - 85.7)$

FROM STEAM TABLES

$\underline{= 1710\text{ BTU/HR}}$

20.39 $G = 60.6(35) = 2120$ Lbm/s-FT^2

$Re = 307{,}000$ {TURBULENT}

$T_f \cong 130F$ - USE COLBURN EQ:

$St = 0.023(307000)^{-0.2}(3.44)^{-2/3}$

$= 0.000807$

20.39 CONTINUED

$$T = 80 + 100\, e^{-St(4)(5)/\frac{0.622}{12}}$$

$= 153.3$ F — FIRST GUESS

$$T_f = \left[\frac{80+154}{2} + 180\right]/2 \cong 149 \text{ F}$$

AT THIS TEMP: $Re = 443{,}000$ $Pr = 4.51$

$St = 0.000625$ $\underline{\underline{T \cong 155 \text{ F}}}$

20.40 GA = 10,000 Lbm/HR

$$G = \frac{10{,}000}{0.276} = 37400 \text{ Lbm/HR FT}^2$$

$$Re = \frac{37400\left(\frac{7.001}{12}\right)}{1.63\times10^{-5}(3600)} = 3.72\times10^{5}$$

{TURBULENT}

USE DITTUS-BOELTER EQN:

$$h = \frac{k}{D}(0.023)\,Re^{0.8}\,Pr^{0.3}$$

$$= \frac{0.0321}{(7.001/12)}(0.023)(3.72\times10^{5})^{0.8}(0.912)^{0.3}$$

$$= \underline{\underline{35.2 \text{ BTU/HR FT}^2 \text{ F}}}$$

20.41 $\dot{m}_{TOTAL} = 1.47 \frac{kg}{s}$ ~ 0.245 kg/s PER TUBE

$$Re = \frac{DV\rho}{\mu} = \frac{\dot{m}4}{\pi D\mu} = \frac{0.245(4)}{\pi(0.0209)(7.9\times10^{-3})}$$

$= 1890$ {LAMINAR}

$$\ln\frac{T_L - T_S}{T_0 - T_S} = -\frac{4L}{D}St$$

$$St = 1.86\left(\frac{D}{L}\right)^{1/3}(Re\,Pr)^{-2/3}\left(\mu_b/\mu_w\right)^{0.14}$$

20.41 CONTINUED -

ASSUME $T_{EXIT} = 305$ K $T_{AVG} \cong 302$ K

$$St = 1.86\left(\frac{0.0209}{2.5}\right)^{1/3}(1890\times121)^{-2/3}\left(\frac{0.0414}{3.72\times10^{-3}}\right)^{0.14}$$

$= 1.414\times10^{-3}$

$$T_L = 372 - 72\, e^{-(1.414\times10^{-3})(4)(2.5)/0.0209}$$

$= 304.7$ ~ GOOD AGREEMENT

$$q = \dot{m}c_p\Delta T = 1.47(1.84\times10^{3})(4.7)$$

$= 12740$ J/s $= \underline{\underline{12.74 \text{ kW}}}$

20.42 $$\frac{T - T_S}{T_0 - T_S} = e^{-4\frac{L}{D}St}$$

$T_0 = 160$ C ASSUME $T \cong 140$ C

$T_S = 100$ C $T_{bAVG} = 150$ C

$$Re = \frac{4\dot{m}}{\pi D\mu} = \frac{4(136)}{(3600)(0.075)\pi\mu}$$

@ 423 K $\mu = (0.0068\times10^{-3})(812)$

$Re = 116$ ~ LAMINAR

$$St = \frac{Nu}{Re\,Pr} = \frac{1.86}{(Re\,Pr)^{2/3}}\left(\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$= \frac{1.86}{[(116)(100)]^{2/3}}\left(\frac{0.075}{15}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$= 0.00053$

$e^{-440\,St} = 0.654$

$T = 100 + 0.654(60) = 139.3$ C

GOOD!

$\underline{\underline{T_{EXIT} \cong 139 \text{ C}}}$

20.43 FOR THIS CASE

$$\frac{T-T_\infty}{T_0-T_\infty} = e^{-4\frac{L}{D}\frac{U}{\rho V c_p}}$$

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{30}{40} = 0.75$$

$$0.2877 = 4\frac{L}{D}\frac{U}{\rho V c_p}$$

$$U = \frac{1}{\frac{1}{h_iA_i}+\frac{1}{h_oA_o}} = \frac{\pi D L}{\frac{1}{h_i}+\frac{1}{h_o}}$$

$h_o = 500\ W/m^2\cdot K$ $\quad h_i = \frac{k}{D}Nu_i$

$$Re_D = \frac{4\dot{V}}{\pi D \nu} = \frac{4(0.006)}{\pi(0.0025)(7\times10^{-7})(3600)}$$

$$= 1213 \quad \{\text{LAMINAR}\}$$

$$Pr = \frac{\mu c_p}{k} = \frac{(1000)(7\times10^{-7})(4000)}{0.5} = 5.6$$

$$h_i = \frac{k}{D}(1.86)\left(Re\,Pr\,\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$= \frac{0.5}{0.0025}(1.86)\left[(1213)(5.6)\left(\frac{0.0025}{L}\right)\right]^{1/3}$$

$$= 956\,L^{-1/3}$$

$$U = \frac{\pi D L}{1/500 + L^{1/3}/956} = \frac{L}{0.255+0.133\,L^{1/3}}$$

PUTTING EVERYTHING TOGETHER:

$$0.2877 = \frac{0.001178\,L^2}{0.255+0.133\,L^{1/3}}$$

BY TRIAL & ERROR:

$$\underline{\underline{L \hat{=} 11.7\ m}}$$

20.44 $T_0 = 320\ K$

$$\frac{q}{L} = \frac{\Delta T}{\Sigma R} = \frac{\Delta T}{\frac{1}{h_iA_i}+\frac{\ln r_o/r_i}{2\pi k}}$$

{ASSUME T_0 IS OUTSIDE TUBE TEMP}

$$\frac{1}{h_iA_i} = \frac{1}{(1700)(\pi)(0.01656)} = 1.13\times10^{-2}$$

$$\frac{\ln r_o/r_i}{2\pi k} = \frac{\ln 1.905/1.656}{2\pi(110)} = 2.03\times10^{-4}$$

$$\Sigma R = 1.15\times10^{-2}$$

$$\frac{q}{L} = \frac{320-290}{1.15\times10^{-2}} = 2.606\ kW/m$$

$$\dot{m}_{COND} = \frac{2.606\ kW/m}{2393\ kJ/kg} = 1.09\ g/s$$

$$= \underline{\underline{3920\ g/HR}} = \underline{\underline{3.92\ kg/HR}}$$

20.45 OUTSIDE OF TUBE INSULATED

∴ ALL HEAT GENERATED GOES INTO H_2O

$$q = \dot{m}c_p\Delta T = (0.12\ kg/s)(4177\ J/kg\cdot K)(70-25)K$$

$$= 22.56\ kW$$

$$\left\{ q = \dot{q}V = 1.5\times10^{6}\ W/m^3\left[\frac{\pi}{4}(0.045^2-0.025^2)\right]L\right.$$

$$= 1.649\ L\ kW$$

$$L = \underline{13.68\ m}$$

$$\text{CONSTANT HT FLUX} = \frac{22560\ W}{\pi(0.025)(13.68)\ m^2}$$

$$= 21000\ W/m^3$$

$$h = \frac{21000}{110-70} = \underline{\underline{525\ W/m^2\cdot K}}$$

20.46 $\dfrac{T-T_s}{T_0-T_s} = e^{-\frac{4L}{D}St}$

$St = \dfrac{D}{4L} \ln \dfrac{60-120}{100-120} = \dfrac{0.0717}{L}$

a) $v = 25$ ft/s

$Re = \dfrac{(0.75/12)(25)}{0.181 \times 10^{-3}} = 8640$

Use Colburn Analogy: $St = \dfrac{C_f}{2} Pr^{-2/3}$

$St = \dfrac{0.0078}{2}(1.257) = 0.0049$

$L = \dfrac{0.0717}{0.0049} = \underline{3.5 \text{ ft}}$

b) $v = 15$ ft/s $\quad Re = 5190$

$St = 0.00566 \quad \underline{L = 3.03 \text{ ft}}$

20.47 $D_{equiv} = 4\dfrac{(2)(4)}{(2)(6)} = \dfrac{16}{6}$ ft

$Re = \dfrac{(16/6)(6)}{1.28 \times 10^{-5}} = 1.25 \times 10^6$

$St = 0.023\, Re^{-0.2} Pr^{-2/3}$

$= 0.023\,(1.25 \times 10^6)^{-0.2}(0.703)^{-2/3}$

$= 0.00176$

For the short distance involved:

$h = St(\rho v c_p) = (0.00176)(6)(0.24)$

$= 2.53 \times 10^{-3} \dfrac{\text{Btu}}{\text{s ft}^2 \text{ F}}$

$q = hA\Delta T = (2.53 \times 10^{-3})(12)(40)$

$= 1.215 \dfrac{\text{Btu}}{\text{s}}$ per ft $= \underline{4370 \dfrac{\text{Btu}}{\text{hr}} \text{ per ft}}$

$\Delta T = q/\dot{m}c_p = \dfrac{1.215}{(0.24)(6)(8)} = \underline{0.105 \text{ F per ft.}}$

20.48

$T_{IN} = 290$ K $\quad$ Assume $T_{OUT} = 350$

$T_{SURF} = 370$ K $\quad T_{bAVG} = 320$ K

$\nu = 0.596 \times 10^{-6}$ m²/s $\quad Pr = 3.87$

$\dfrac{T-T_s}{T_0-T_s} = e^{-4\frac{L}{D}St}$

$Re = \dfrac{(0.0254)(1.5)}{0.596 \times 10^{-6}} = 63900$

Use Dittus-Boelter Eqn:

$St = 0.023\, Re^{-0.2} Pr^{-0.6}$

$= 0.023(63900)^{-0.2}(3.87)^{-0.6}$

$= 0.00112$

$e^{-4L/D(St)} = 0.414$

$T_{OUT} = 370 - (0.414)(80)$

$\cong 337$

Second try - $T_{OUT} = 337$

$T_{bAVE} = 313.5$

$\nu = 0.663 \times 10^{-6} \quad Pr = 4.33$

$Re = 57460 \quad St = 0.00107$

$e^{-4L/D(St)} = 0.432$

$T_{OUT} = 370 - (0.432)(80) = \underline{335 \text{ K}}$

$q = \dot{m} c_p \Delta T$

$= (992)\dfrac{\pi}{4}(0.0254)^2(1.5)(4175)(45)$

$= \underline{141.6 \text{ kW}}$

20.49 RECT. DUCT 0.61 m × 1.22 m

$$D_{EQUIV} = \frac{4(0.61)(1.22)}{2(0.61+1.22)} = 0.813\ m$$

$q = hA\Delta T$

USE DITTUS BOELTER EQN.

$$Re = \frac{DG}{\mu} = \frac{(0.813)(29.4)}{1.948\times10^{-5}} = 1.227\times10^{6}$$

$Pr = 0.703$

$$h = \frac{k}{D}(0.023)Re^{0.8}Pr^{0.3}$$

$$= \frac{0.0279}{0.813}(0.023)(1.227\times10^{6})^{0.8}(0.703)^{0.3}$$

$$= 52.8\ W/m^2\cdot K$$

$$q = hA\Delta T = 52.8(2)(0.61+1.22)(22)$$

$$= \underline{\underline{4250\ W/m}}$$

$$q = \dot{m}c_p\Delta T = GAc_p\Delta T$$

$$\Delta T = \frac{4250}{29.4(0.61)(1.22)(1007)}$$

$$= \underline{\underline{0.193\ K}}\ \text{PER M}$$

20.50 DUCT: 7.5 cm × 15 cm

L = 6 m

$$\frac{T-T_s}{T_0-T_s} = e^{-4\frac{L}{D}St}$$

$D_{EQUIV} = 10\ cm$

$$\frac{T-T_s}{T_0-T_s} = \frac{30-70}{10-70} = 0.667$$

$$4\frac{L}{D}St = 0.405$$

$$St = \frac{0.405(0.10)}{4(6)} = 0.00169$$

20.50 CONTINUED -

USE COLBURN EQ.

$$St = 0.023\,Re^{-0.2}Pr^{-2/3}$$

$$Re = \frac{DV}{\nu} = \frac{(0.10)(V)}{1.739\times10^{-5}} = 5750\,V$$

$Pr = 0.704$

$$0.00169 = 0.023(5750V)^{-0.2}(0.704)^{-2/3}$$

$$\underline{\underline{V = 261\ m/s}}$$

UNREALISTIC BUT MATHEMATICALLY CORRECT

20.51 FIGS 20.12 & 20.13 APPLY STRICTLY FOR LIQUIDS FLOWING THROUGH TUBE BANKS BUT THEY WILL BE USED FOR LACK OF OTHER RESOURCES. —

USING FIG 20.13 ~

$$Re = \frac{D_t V}{\nu} = \frac{(0.018)(60)}{1.505\times10^{-5}}$$

$$= 7.17\times10^{3}$$

AT THIS Re: $j \doteq 0.01$

$$h = 0.01\,c_pG\,Pr^{-2/3}\left(\frac{\mu_w}{\mu_b}\right)^{-0.14}$$

$$= 0.01(1005.5)(1.205)(6)(0.707)^{-2/3}\left(\frac{2.117}{1.813}\right)^{-0.14}$$

$$= 89.6\ W/m^2\cdot K$$

FOR A BANK OF 10 TUBES, 10 ROWS DEEP

$$A = 100(\pi)(0.018)(1.8) = 10.18\ m^2$$

$$= (89.6)(10.18)(65) = \underline{\underline{59.3\ kW}}$$

20.52 For Same Conditions As Prob. 20.51 Except for Staggered Tube Arrangement —

$Re = 7.17 \times 10^3$

& Both Arrangements Give Same Value for j

$\therefore \quad \underline{q = 59.3 \text{ kW}}$

20.53 Using Fig 20.12

Same Caveats As for Prob 20.51 —

$$D_{equiv} = \frac{4}{\pi(0.013)}\left[(0.032)(0.032) - \frac{\pi}{4}(0.013)^2\right]$$

$$= 0.0873 \text{ m}$$

$$Re = \frac{(0.0873)(1.25)}{1.569 \times 10^{-5}} = 6.95 \times 10^3$$

— Out of Laminar Range — Must Use Fig 20.13

$$Re = \frac{0.013(1.25)}{1.569 \times 10^{-5}} = 1.04 \times 10^3$$

For In-Line Configuration —

$$j \cong 0.017$$

$$h = 0.017(1006.3)(1.177)(1.25) \times (0.708)^{-2/3}\left(\frac{2.143}{1.813}\right)^{-0.14}$$

$$= 30.95 \text{ W/m}^2 \cdot \text{K}$$

20.53 Continued —

$$A = 64(\pi)(0.013)(18) = 4.70 \text{ m}^2$$

$$q = hA\Delta T$$

$$= 30.95(4.70)(63)$$

$$= \underline{9.164 \text{ kW}}$$

20.54 Same Conditions As Prob 20.53 Except Tubes Are in Staggered Configuration.

All Calculations The Same As in Prob 20.54 Except $j = 0.1035$

Giving $h = 63.7 \text{ W/m}^2 \cdot \text{K}$

& $q = 63.7(4.70)(63)$

$= \underline{18.87 \text{ kW}}$

CHAPTER 21

21.1 PLATE IS ASSUMED TO BE COPPER

FOR H_2O @ 323 K L = 0.565 FT

$\frac{\beta g}{\nu^2} = 1.26 \times 10^8\,(F\cdot FT)^{-1}$ Pr = 1.81

$\mu_L = 0.702\ LB_m/HR\,FT$ $C_L = 1.01\ BTU/LB_m F$

$k = 0.393\ BTU/HR\,FT^2 F$ $h_{fg} = 970\ BTU/LB_m$

$\rho_L - \rho_V \cong \rho_L = 60\ LB_m/FT^3$

NATURAL CONVECTION: $\frac{q}{A} = h\Delta T$

$$\frac{q}{A} = \frac{k}{L}\left[0.68 + \frac{0.67\,Ra^{1/4}}{\left\{1+\left(\frac{0.492}{Pr}\right)^{9/16}\right\}^{4/9}}\right]\Delta T$$

$$= 60\left[0.68 + 89.6\,\Delta T^{1/4}\right]\Delta T \qquad (1)$$

NUCLEATE BOILING:

$$\frac{C_L\Delta T}{h_{fg}\,Pr^{1.7}} = C_{sf}\left[\frac{q/A}{\mu_L h_{fg}}\left(\frac{g_c\,\sigma}{g[\rho_L-\rho_V]}\right)^{1/2}\right]^{1/3}$$

$\sigma = 3.79 \times 10^{-3}\ LB_f/FT$ $C_{sf} = 0.013$

LHS: $\frac{C_L\Delta T}{h_{fg}\,Pr^{1.7}} = 3.80 \times 10^{-4}\,\Delta T$

RHS: $C_{sf}[\;]^{1/3} = 0.013\left[\frac{q/A}{0.702(970)}\sqrt{\frac{3.79\times10^{-3}}{60}}\right]^{1/3}$

$= 2.94 \times 10^{-4}\,(q/A)^{1/3}$

$$\frac{q}{A} = 2.14\,\Delta T^3 \qquad (2)$$

EQUATING: (1) = (2)

$2.14\,\Delta T^3 = 0.6\left[0.68 + 89.6\,\Delta T^{1/4}\right]$

$\Delta T \cong 6.3\ F$

PART (b): PLOT q/A FROM (1), q/A FROM (2), & THEIR SUM

21.2 $$\frac{C_L(\Delta T)}{h_{fg}\,Pr^{1.7}} = C_{sf}\left[\frac{q/A}{\mu_L h_{fg}}\left(\frac{\sigma}{g\Delta\rho}\right)^{1/2}\right]^{1/3}$$

IN ENGLISH UNITS:

$C_L = 1.03$ $Pr^{1.7} = 2.74$

$h_{fg} = 970$ $\mu_L = 0.195 \times 10^{-3}$

$T_{SAT} = 212$ $\Delta\rho = 60.2$

$A = \frac{C_L\Delta T}{h_{fg}}$ $B = \frac{1}{\mu_L h_{fg}}\sqrt{\frac{\sigma}{g\Delta\rho}}$

$$\frac{q}{A} = \left[\frac{A}{C_{sf}\,B\,(2.74)}\right]^3$$

FOR Ni & BRASS $C_{sf} = 0.006$

" Cu & Pt $C_{sf} = 0.013$

T_S(K)	ΔT	ΔT(F)	A	$\sigma \times 10^3$	B
390	17	31	0.033	5.04	0.364
420	47	85	0.090	4.67	0.360
450	77	139	0.148	3.79	0.355

T_S	q/A	h W/m²·K	q/A	h
390	168	2×10^5	165	0.533×10^5
420	3516	8.5 "	346	4.07 "
450	16340	24 "	1610	1610 "
	Ni, BRASS		Cu, Pt	

21.3 $\frac{C_L\Delta T}{h_{fg}} = 0.0709$

$\frac{q}{A} = \left(\frac{0.0709}{0.01235}\right)^3 = 190\ BTU/s\cdot ft^2$

$= 680{,}000\ BTU/HR\,FT^2$

$q = 680{,}000(\pi)(1/24)^2 = \underline{178{,}000\ BTU/HR}$

$h = \frac{680{,}000}{68} = \underline{10{,}000\ BTU/HR\,FT^2 F}$

21.4 BOILING H_2O @ 1 ATM; BURNOUT POINT IS $\Delta T \doteq 100F$, $T_S = 312F$
AS IT COOLS THE CYLINDER IS IN

FILM BOILING $500 < T_S < 312$
NUCLEATE " $312 < T_S < 240$

FILM BOILING PART:

$$h = 0.62\left[\frac{k_v^3 \rho_v(\Delta\rho)g(h_{fg} + 0.4c_{pv}\Delta T_S)}{D\mu_v \Delta T_S}\right]^{1/4}$$

$k_v = 0.0145$ Btu/HR FT F
$\rho_v = 0.0372$ LBm/FT³ $\quad h_{fg} = 970$ Btu/LBm
$\rho_L = 60.0$ " $\quad c_{pv} = 0.451$ Btu/LBm F
$\mu_v = 3.12 \times 10^{-3}$ LBm/HR FT

SUBSTITUTING INTO FORMULA:

$h = 35.9$ Btu/HR FT² F {AVG $\Delta T \doteq 194F$

$$\frac{q}{A} = h(194) = 35.9(194) = 6960 \ \frac{Btu}{HR\,FT^2}$$

NUCLEATE BOILING PART:

$$\frac{c_L \Delta T}{h_{fg}} = C_{sf}\left[\frac{q/A}{\mu_L h_{fg}}\left(\frac{\sigma}{g\Delta\rho}\right)^{1/2}\right]^{1/3} Pr^{1.7}$$

$$\Rightarrow \frac{q}{A} = \left(\frac{0.0168\,\Delta T}{C_{sf}}\right)^3 = 4.8\times10^{-6}\left(\frac{\Delta T}{C_{sf}}\right)^3$$

WITH $\bar{\Delta T} = 64F$ $\quad C_{sf} = 0.013$ ~ Cu
$\qquad = 0.006$ Br, Ni

$\frac{q}{A} = 5.72\times10^5$ ~ Cu
$\quad = 5.82\times10^6$ ~ BRASS, Ni

$$\frac{q}{A} = \frac{\rho V c_p}{A}\frac{dT}{dt}$$

$$t = \rho\left(\frac{V}{A}\right)c_p\left[\int_{312}^{500}\frac{dT}{q/A|_f} + \int_{240}^{312}\frac{dT}{q/A|_{nuc}}\right]$$

21.4 CONT.

$$t = \rho\frac{D}{4}c_p\left(\frac{188}{q/A|_f} + \frac{72}{q/A|_{nuc}}\right)$$

$\rho\frac{D}{4}c_p = 555(1/96)(0.092) = 0.532$ } Cu
$\quad = 532(1/96)(0.091) = 0.503$ } Br
$\quad = 556(1/96)(0.111) = 0.643$ Ni

COPPER: $t = 0.532\left[\frac{188}{6960} + \frac{72}{5.72\times10^5}\right] = \underline{\underline{52\,s}}$

Br: $t = 0.503\left[\frac{188}{6960} + \frac{72}{5.8\times10^5}\right] = \underline{\underline{48.9\,s}}$

Ni: $t = 0.643\left[\frac{188}{6960} + \frac{72}{5.8\times10^5}\right] = \underline{\underline{62.6\,s}}$

21.5 USING ENGLISH UNITS:

$$A = \pi DL + 2\frac{\pi D^2}{4} = \pi(0.02)(0.15) + \frac{\pi}{4}(2)(0.02)^2$$
$$= 0.1082\,FT^2$$

$$\frac{q}{A} = \frac{500(3.413)}{0.1082} = 15800\ \frac{Btu}{HR\,FT^2}$$

ASSUME NUCLEATE BOILING:

$$\frac{1\ \Delta T}{(1.8)^{1.7}(970)} = 0.006\left[\frac{15800}{(0.195\times10^{-3})(970)(3600)}\times\left(\frac{3.79\times10^{-3}}{60}\right)^{1/2}\right]^{1/3}$$

$\Delta T = 9.0\ F$

SURFACE TEMP. = $\underline{\underline{221\ F}}$

$$h = \frac{15800}{9} = \underline{\underline{1760\ Btu/HR\,FT^2\,F}}$$

21.6 $h = 0.62\left[\dfrac{k_v^3 \rho_v \Delta\rho g (h_{fg} + 0.4 c_{p_v} \Delta T)}{D \mu_v (T_s - T_{SAT})}\right]^{1/4}$

$= 0.62\left[\dfrac{(0.0153)^3(0.0341 \times 40/14.7)(584)}{1/12(0.914\times10^{-5})(933)} \times 32.2(3600)(934 + 0.4\times0.481\times933)\right]^{1/4}$

$= 26.9 \ BTU/HR\,FT^2 F$

$\dfrac{q}{A} = h\Delta T = 26.9(1200-267) = \underline{\underline{25{,}000 \ BTU/HR\,FT^2}}$

21.7 $\Delta T = 2200 - 240 = 1960 F$ {FILM BOILING}

$h = 0.62\left[\dfrac{(0.0155)^3(0.035)(58.9)(32.2)(3600)}{(0.02/12)(1.53\times10^{-5})(1960)} \times (952 + 0.4\times0.483\times1960)\right]^{1/4}$

$= 43.3 \ BTU/HR\,FT^2 F$

$q = hA\Delta T = 43.3(\pi)\left(\frac{0.2}{12}\right)(1)(1960)$

$= \underline{\underline{444 \ \frac{BTU}{HR} \ PER \ FT}}$

21.8 $2000 W = 6826 \ BTU/HR$

PER PLATE: $A = 2(0.05)(0.1) = 0.01 M^2$

$= 0.1076 \ FT^2$

$\dfrac{q}{A} = 20{,}000 \ W/m^2 = 63400 \ BTU/HR\,FT^2$

IT APPEARS THAT NUCLEATE BOILING ON ONE PLATE CAN ACHIEVE THIS

$T_{SAT} = 242 F$

$\dfrac{c_L \Delta T}{h_{fg} Pr^{1.7}} = C_{sf}\left[\dfrac{q/A}{\mu_L h_{fg}}\left(\dfrac{\sigma}{g\Delta\rho}\right)^{1/2}\right]^{1/3}$

21.8 CONT.

$\dfrac{\Delta T}{945(2.12)} = 0.013\left[\dfrac{63400}{0.167\times10^{-3}(970)(3600)} \times \left(\dfrac{3.79\times10^{-3}}{59.1}\right)^{1/2}\right]^{1/3}$

$\underline{\underline{\Delta T = 24.9 F}}$ - {OK 1 PLATE WILL DO IT}

21.9 PLOTS REQUIRED

PROCEDURE:

T_{SURF} (K)	ΔT (K)	
600	327	FILM BOILING *
500	127	" "
400	27	NUCLEATE **

* USE EQN (21-7)
** USE EQN (21-5)

$q = h\Delta T$

$h_{FILM\,B.}$ VARIES AS $\Delta T^{-1/4}$

$h_{NUC.B.}$ " " ΔT^2

$\dfrac{q}{A}\Big|_{FILM}$ VARIES AS $\Delta T^{3/4}$

$\dfrac{q}{A}\Big|_{NUC}$ " " ΔT^3

PLATE IS LUMPED

$\dfrac{q}{A} = \rho c_p \dfrac{V}{A}\dfrac{dT}{dt} \qquad \Delta T = \dfrac{q/A}{\rho c_p \frac{V}{A}}\Delta t$

21.10 Idea here is to retard boiling such that internal pressure will not be too large

$$q = \frac{\left(3\times10^6 \frac{Btu}{HR\,FT^2}\right)\pi\left(\frac{3}{48}\right)(4)}{3600} = 655\ Btu/s$$

For $P = 1$ ATM $h_{fg} = 970\ Btu/LBm$

$$\dot{m}_{evaporation} = \frac{655}{970} = 0.676\ LBm/s$$

$$= \frac{0.676}{0.0372} = 18.2\ FT^3/s$$

Volume of pipe {also volume of fluid}

$$= \frac{\pi}{4}\left(\frac{3}{48}\right)^2(4) = 0.0123\ FT^3$$

$$\text{Flow Area} = \frac{\pi}{4}\left(\frac{3}{48}\right)^2 = 0.0031\ FT^2$$

For $\dot{V}_w \{FT^3/s\}$ of fluid entering

$\dot{V}_w + 18.2$ total fluid exits

$$\dot{V}_w = 18.2\left(\frac{0.0372}{59.5}\right) \cong 0.0144\ FT^3/s$$

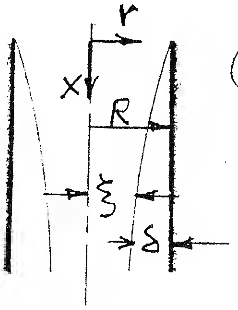
$$\dot{V}_w + 18.2 \cong 18.2\ FT^3/s \text{ out}$$

$$V_{exit} = \frac{18.2}{0.0031} = 5870\ FT/s$$

Supersonic!

~ Increase pipe diameter, or add additional pipes in parallel, or decrease q ~

21.11



{Nusselt film model for flow inside a cylinder}

21.11 cont.

At equilibrium $\sum F_x = 0$

Upward force (viscous) $= \tau \Delta x\, 2\pi r = -\mu \frac{dv}{dr} 2\pi r \Delta x$

Downward force $(\xi < r < R)$ $= \rho g \pi (r^2 - \xi^2)\Delta x$

$$\Rightarrow \rho g (r^2 - \xi^2) = -\mu \frac{dv}{dr} 2r$$

Separating variables: $dv = -\frac{\rho g}{2\mu}\left(r - \frac{\xi^2}{r}\right)dr$

$$v = -\frac{\rho g}{2\mu}\left(\frac{r^2}{2} - \xi^2 \ln r\right) + C$$

B.C. $v(R) = 0 \Rightarrow C = \frac{\rho g}{\mu}\left(\frac{R^2}{2} - \xi^2 \ln R\right)$

$$v = \frac{\rho g}{2\mu}\left[\frac{R^2 - r^2}{2} - \xi^2 \ln \frac{R}{r}\right]$$

At any x, mass flow rate is

$$\Gamma = \int_\xi^R \rho v (2\pi r)\, dr$$

$$= \frac{\pi \rho^2 g}{\mu}\int_\xi^R \left[\frac{R^2}{2}r - \frac{r^3}{2} - \xi^2 r \ln R + \xi^2 r \ln r\right] dr$$

$$= \frac{\pi \rho^2 g}{\mu}\left[\frac{R^4}{8} - \frac{\xi^2 R^2}{2} + \frac{3\xi^4}{8} + \frac{\xi^2}{2}\ln R - \frac{\xi^4}{2}\ln \xi\right]$$

Rate of heat flow to wall — through condensate —

$$= \frac{2\pi k}{\ln r/\xi}\, dx\, (T_v - T_w)$$

Amount of condensate in distance dx

$$= \frac{\partial}{\partial x}\Gamma\, dx = \frac{\partial}{\partial \xi}\Gamma \frac{d\xi}{dx} dx = \frac{d\Gamma}{\partial \xi}\, d\xi$$

21.11 CONT

$$\frac{d\Gamma}{dx} = \frac{2\pi\rho^2 g}{\mu} h_{fg}\left[-\xi R^2 + \xi^3 + 2\xi^3 \ln R - 2\xi^2 \ln \xi\right]$$

∴ RATE OF HEAT FLOW TO COOL WALL

$$= \frac{2\pi\rho^2 g}{\mu} h_{fg}\left[-\xi R^2 + \xi^3 + 2\xi^2 \ln R - 2\xi^2 \ln \xi\right] d\xi$$

EQUATING HEAT FLOW RATES -

$$\frac{k\, dx}{\ln r/\xi}(T_v - T_w) = \frac{\rho^2 g h_{fg}}{\mu}\left[-\xi R^2 + \xi^3 + 2\xi^2 \ln R - 2\xi^2 \ln \xi\right] d\xi$$

SEPARATING VARIABLES & INTEGRATING

$$A\int_0^x dx = \int_R^{\xi} [\ ]\, d\xi$$

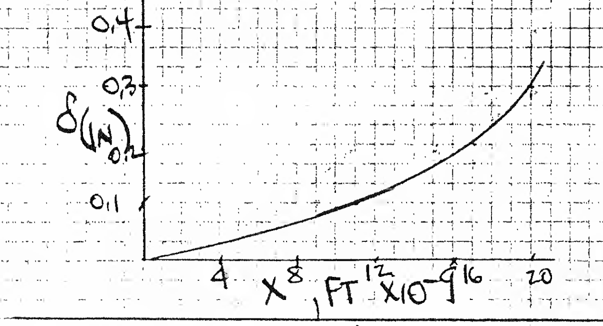
$$Ax = f(\xi/R) = f(\eta)$$

SOLVING {MESSY} WE GET, FOR $X(\eta)$

$$X(\text{FT}) = (2.42\times10^9)\left[(1.23 - 3.14\ln\eta)\eta^4 + (1.84 - 0.5\ln\eta)\eta^2 - 9.07\right]$$

η	$\ln\eta$	η^2	η^4	$X\times10^{-9}$ (FT)	$\delta_{(IN)}$
0.9	−0.1	0.81	0.654	6.3	0.05
0.8	−0.22	0.64	0.409	11.2	0.10
0.6	−0.508	0.36	0.1294	17.3	0.20
0.4	−0.913	0.16	0.0256	20.2	0.30
0.2	−1.607	0.04	0.0016	21.6	0.40
0				22.0	0.50

21.11 CONT.



21.12 a) VERTICAL TUBE

$$h = 0.943\left\{\frac{\rho_L g k^3 \Delta\rho\left[h_{fg} + \frac{3}{8}c_{pL}(\Delta T)\right]}{L\mu\,\Delta T}\right\}^{1/4}$$

$$= 0.943\left\{\frac{961.2(9.81)(0.679)^3(0.961)}{1(297\times10^{-6})(9)}\times\left[2.25\times10^6 + \frac{3}{8}(4206)(9)\right]\right\}^{1/4}$$

$$= 6600\ \text{W/m}^2\cdot\text{K}$$

$$\dot{m}_{\text{COND.}} = \frac{6600(9)(\pi)(15)(1)}{225\times10^6} = \underline{\underline{0.0125\ \text{kg/s}}}$$

b) HORIZONTAL

$$h = 0.725\left\{\frac{\rho_L g \Delta\rho k^3\left[h_{fg} + \frac{3}{8}c_{pL}\Delta T\right]}{\mu D\,\Delta T}\right\}^{1/4}$$

$$= 7100\ \text{W/m}^2\cdot\text{K} \quad \underline{\underline{\dot{m} = 0.0146\ \text{kg/s}}}$$

21.13 NEGLECT ΔT ACROSS TUBE

$$h_i = \frac{k}{D_i}\,0.023\,Re_D^{0.8}\,Pr^{1/3}$$

$$h_o = 0.725\left\{\frac{\rho_L g \Delta\rho k^3\left[h_{fg} + \frac{3}{8}c_{pL}\Delta T\right]}{\mu D\,\Delta T}\right\}^{1/4}$$

FOR h_i: $T_{IN} = 20\,C$, $T_W = ?$, $T_b = \frac{20 + T_{OUT}}{2}$

FOR h_o: PROPERTIES EVALUATED AT $T_f = \frac{100 + T_w}{2}$

21.13 CONT.

$$q = \frac{\Delta T_{OVERALL}}{\Sigma R} = \frac{\Delta T_i}{R_i} = \frac{\Delta T_o}{R_o}$$

A MESSY TRIAL & ERROR PROBLEM
– AFTER QUITE A BIT OF WORK –
ASSUMING $T_{i\,OUT} = 36\,C = 309\,K$
" $T_{W\,AVG} = 58\,C = 331\,K$
GIVING $T_{b,AVG} = 28\,C = 301\,K$

$$Re_i = \frac{DV\rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(4000)}{\pi(0.0165)(863\times10^{-6})\times(3600)}$$

$= 99000$

$Pr = 5.95 \quad k = 0.611$

$$h_i = \frac{0.611}{0.0165}(0.023)(99000)^{0.8}(5.95)^{0.4}$$

$= 17200\ W/m^2\cdot K$

$$h_o = 0.725\left\{\frac{971.8(9.81)(971.8)(0.673)^3}{(352\times10^{-6})(0.019)(42)}\times\left[2.25\times10^6 + \frac{3}{8}(4194)(42)\right]\right\}^{1/4}$$

$= 8960\ W/m^2\cdot K$

$$R_i = \frac{1}{17200(\pi)(0.0165)(2)} = 5.61\times10^{-4}$$

$$R_o = \frac{1}{8690(\pi)(0.019)(2)} = 9.35\times10^{-4}$$

$$\Sigma R = 14.96\times10^{-4}$$

$\Delta T_{TOTAL} = 72\,K \quad \Delta T_i \cong 27K \quad \Delta T_o \cong 45K$

$$q = \frac{27}{R_i} = \frac{45}{R_o} = 48000\ W$$

21.13 CONT.

$$q = \dot{m}c_p\Delta T = \frac{4000}{3600}(4180)\Delta T$$

$\Delta T \cong 10.4\,K$

$\Rightarrow T_{W\,OUT} \cong 303.5\,K \quad T_{b,AVG} \cong 298\,K$

~ CLOSE TO ORIGINAL ASSUMPTION –

FINALLY: $h_{H_2O} = \underline{17200\ W/m^2\cdot K}$ (a)

$h_{CONDENS} = \underline{8690}$ " (b)

$T_{W,OUT} = \underline{303.5\ K}$ (c)

$\dot{M}_{COND} = \dfrac{48000}{2.25\times10^6} = \underline{0.0213\ kg/s}$ (d)

21.14 FLOW RATE $= \dfrac{0.042}{0.586} = 0.0717\ m^3/s$ PER M

ALLOWABLE WIDTH $= \dfrac{0.0717}{(15)(1)} = 0.00478\ m$
$= 0.478\ m$

$0.478 + 2\delta = 1\ cm$

$\delta = 0.261\ cm = 0.00261\ m$

FILM MODEL:

$$\delta^4 = \left[\frac{4k\mu x\Delta T}{\rho_L g\Delta\rho\left(h_{fg} + \frac{3}{8}c_{pL}\Delta T\right)}\right]$$

$$= \frac{4(0.392)(0.195\times10^{-3})(59)\ x}{(59.5)(32.2)(59.5)(993.5)(3600)}$$

~ {ALL ENGLISH UNITS}

$\delta^4 = 4.425\times10^{-14}\ x$

$x = \underline{121,500\ FT}$

$= \underline{37000\ m}$

21.15 $\frac{q}{A} = k_L \frac{\Delta T}{y} = \rho h_{fg} \frac{dy}{dt}$

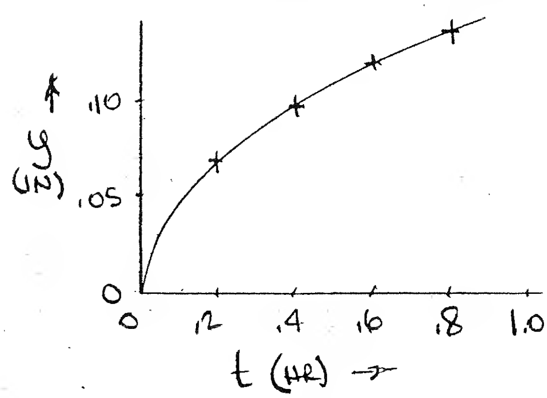
{At a given depth, y}

$$\int_0^y y\,dy = \frac{k_L \Delta T}{\rho h_{fg}} \int_0^t dt$$

$$\frac{y^2}{2} = \frac{k_L \Delta T}{\rho h_{fg}} t$$

$$y^2 = \frac{2(0.392)(12)}{60.1\,(970)} t = 1.614 \times 10^{-4} t$$

t, Hours	y ft $\times 10^2$	y inches
0	0	0
0.2	0.586	0.0681
0.4	0.804	0.0964
0.6	1.0	0.12
0.8	1.14	0.137
1.0	1.27	0.152



21.16 Horizontal Cylinder:

$$h_H = 0.725 \left\{ \frac{\rho_L g \Delta\rho k^3 \left[h_{fg} + \frac{3}{8} c_{pL} \Delta T\right]}{\mu D \Delta T} \right\}^{1/4}$$

$$= 0.725 \left\{ \frac{61.3(32.2)(61.3)(0.0562)^3(3600)(997)}{0.29 \times 10^{-3}(0.0656)(45)} \right\}^{1/4}$$

$$= 397 \ \text{Btu/hr ft}^2\text{F} = 2250 \ \text{W/m}^2\cdot\text{K}$$

$$h_V = h_H \left[1.3 \left(\frac{D}{L}\right)^{1/4}\right] = 1000 \ \text{W/m}^2\cdot\text{K}$$

21.16 cont. $\dot{m}_{cond} = \frac{h A \Delta T}{h_{fg}}$

$$= \frac{2250(\pi)(0.02)(1.5)(25)}{2.32 \times 10^6}$$

$= 2.29 \times 10^{-3}$ kg/s Horizontal

$= 1.02 \times 10^{-3}$ " Vertical

21.17 $h_{AVG} = \bar{h}\left(\frac{1}{8}\right)^{1/4} = \frac{h}{1.681}$

Horizontal Tube Case {See Prob 21.16}

$h_{HORIZ} = 2250$ W/m$^2\cdot$K

$h_{AVG} = 2250/1.681 = 1341$ W/m$^2\cdot$K

$q = h_{AVG} A \Delta T$

$= 1341(8)(\pi)(0.02)(1.5)(25)$

$= 25.3$ kW

21.18 Single Horizontal Tube:

$$h = 0.725 \left\{ \frac{\rho_L g \Delta\rho k^3 \left(h_{fg} + \frac{3}{8} c_{pL} \Delta T\right)}{D \mu \Delta T} \right\}^{1/4}$$

$$= 0.0725 \left\{ \frac{(60.1)(32.2)(60.1)(0.392)^3(1015)(100)}{5/96\,(0.206 \times 10^{-3})(100)} \right\}^{1/4}$$

$= 1600$ Btu/hr ft^2 F

21.19 $h_{AVG} = h\left(\frac{1}{8}\right)^{1/4} = h/1.681$

$h_{HORIZ} = 2250$ W/m$^2\cdot$K {From Prob 21.16}

For Bank: $h_{AVG} = \frac{2250}{1.68} = 1341$ W/m$^2\cdot$K

21.19 CONT. For n TUBES –

$$q = h_{AVG,n}\, n A_{TUBE} \Delta T$$
$$= h_{AVG,n-1} (n-1) A_{TUBE} \Delta T$$
$$+ h_n A_{TUBE} \Delta T$$

$$\bar{h}_n n \cancel{A \Delta T} = \bar{h}_{n-1} (n-1) \cancel{A \Delta T} + h_n \cancel{A \Delta T}$$

n^{th} TUBE: $h_n = n\bar{h}_n - (n-1)\bar{h}_{n-1}$

TOP TUBE: $h_1 = \underline{2250 \ W/m^2 \cdot K}$

3^{RD} TUBE: $\bar{h}_2 = 1890 \ W/m^2 \cdot K$

$\bar{h}_3 = 1710$ "

$h_3 = 3(1710) - 2(1890) = \underline{\underline{1350 \ W/m^2 \cdot K}}$

8^{th} TUBE: $\bar{h}_8 = 1341 \ W/m^2 \cdot K$

$\bar{h}_7 = 1383$ "

$h_8 = 8(1341) - 7(1383) = \underline{\underline{1047 \ W/m^2 \cdot K}}$

21.20 $\dfrac{4A\Gamma_c}{P\mu_f} = Re_c = 2000$

$$\frac{4A\Gamma_c}{P\mu_f} = 4\frac{\cancel{A}}{P}\frac{\dot{m}}{\cancel{A}}\frac{1}{\mu} = \frac{4}{\cancel{P}\mu}\frac{h\cancel{P}L\Delta T}{h_{fg}}$$

$$L = \frac{\mu h_{fg}}{4h\Delta T}(2000)$$

$$= \frac{(0.0206\times10^{-3})(970)(2000)L^{1/4}(3600)}{4(100)\left[2250(1.3)(0.02)^{1/4}\right]}$$

$L^{3/4} = 3.27$ $\quad \underline{\underline{L = 4.85 \text{ FT}}}$

21.21

$$h_L = 0.943\left[\frac{\rho_L g \Delta\rho\left(h_{fg} + \frac{3}{8}c_{pL}\Delta T\right)}{L\mu\Delta T}\right]^{1/4}$$

$$= 0.943\left[\frac{37.2(32.2)(37.2)(0.294)^3(505)}{2(14\times10^{-5})(25)}\right]^{1/4}$$

$= 694$ BTU/HR FT^2 F

$q = hA\Delta T = 694(2)(25)$

$= \underline{\underline{34{,}700 \text{ BTU/HR}}}$ PER FOOT OF WIDTH

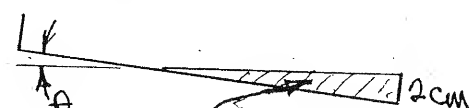
21.22 $q = \dfrac{k\Delta T}{y} = \rho h_{fg}\dfrac{dy}{dt}$ {FOR THICKNESS y}

$$\int_0^t dt = \frac{\rho h_{fg}}{k\Delta T}\int_0^y y\,dy = \frac{\rho h_{fg}}{k\Delta T}\frac{y^2}{2}$$

$$t = \frac{60.2(972)(0.02/0.3048)^2}{0.390(39.6)(2)}$$

$= \underline{\underline{16.3 \text{ HOURS}}}$ – IF PAN IS HORIZONTAL

FOR PAN INCLINED:



PER UNIT DEPTH: $Vol = \dfrac{1}{2}\dfrac{0.02}{\tan\theta}(0.02)$

$Vol = \dfrac{2\times10^{-4} L}{\tan\theta}$ ~ m^3

IF $\theta = 10°$ $\quad$ VOL $= 1.34\times10^{-3} \ m^3$

$30°$ $\quad$ " $= 3.46\times10^{-4}$ "

LENGTH OF SURFACE EXPOSED:

IF $\theta = 10°$ LENGTH – 40 TO 28.7 cm

$\theta = 30°$ " 40 TO 36.5 "

21.22 CONT.

ASSUME: ACCUMULATION OF CONDENSATE DUE PRINCIPALLY TO CONDENSATION ON EXPOSED SURFACE

$$h = 0.943\left[\frac{\rho_L g \sin\theta\, k^3 \Delta\rho\,(h_{fg} + \frac{3}{8} c_p \Delta T)}{L \mu \Delta T}\right]^{1/4}$$

$$= \frac{0.943}{(L/\sin\theta)^{1/4}}\left[\frac{60(32.2)(0.392)^3(60)(995)(3600)}{0.201\times10^{-3}(40)}\right]$$

$$= 1252\left(\frac{L}{\sin\theta}\right)^{-1/4}$$

@ $\theta = 10^\circ$ $L_{AVG} = \frac{40 + 287}{2} = 338\text{ cm} = 1.109\text{ FT}$

30° " $= \frac{40 + 365}{2} = 383$ " $= 1.257$ "

$h_{10} = 788\ \text{Btu/HR FT}^2\,\text{F}$ $h_{30} = 994\ \frac{\text{Btu}}{\text{HR FT}^2\,\text{F}}$

$$\dot{m} = \frac{hA\Delta T}{h_{fg}} = \frac{h L_{AVG}(40)}{980}$$

$\dot{m}_{10} = 35.7\ \text{Lbm/HR} = 0.572\ \text{FT}^3/\text{HR}$

$\dot{m}_{30} = 51.0$ " $= 0.817$ "

$$t = V/\dot{V}$$

$$t_{10} = \frac{1.34\times10^{-3}/(0.3048)^2}{0.572} = 0.0252\ \text{HR}$$

$= 1.51$ MIN

$\cong \underline{91\ \text{S}}$

$t_{30} = 0.00456\ \text{HR} = 0.274$ MIN

$\cong \underline{16.4\ \text{S}}$

21.23

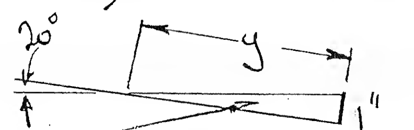
$$h = 0.943\left[\frac{\rho_L g \sin\theta\, k^3 \Delta\rho\,(h_{fg} + \frac{3}{8} c_p \Delta T)}{L \mu \Delta T}\right]^{1/4}$$

$$= 0.943\left[\frac{(0.601)(32.2)(\sin 20)(0.392)^3(60.1)}{10.625/12\,(0.206\times10^{-3})(30)} \times (981)(3600)\right]^{1/4}$$

$= 1050\ \text{Btu/HR FT}^2\,\text{F}$

$$\dot{m} = \frac{hA\Delta T}{h_{fg}}$$

$$= \frac{1050(1)(30)}{970} = 32.5\ \text{Lbm/HR}$$



$$\text{VOL} = \frac{1}{2}\left(\frac{1}{12}\right)\left(\frac{1}{\tan 20}\right)\left(\frac{1}{12}\right)$$

$= 0.00955\ \text{FT}^3$

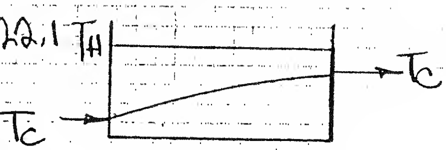
$$t = \frac{V}{\dot{V}} = \frac{0.00955}{32.5/62.4}$$

$= 0.0183$ HOUR

$= \underline{1.10\ \text{MIN}}$.

CHAPTER 22

22.1



$$q = \dot{m} c_p \Delta T \big|_{H_2O} = UA\,\Delta T_{LM}$$

$$UA = \text{CONST.} = q/\Delta T_{LM}$$

$$U = \frac{1}{\frac{1}{h_i} + \cancel{\frac{A_i}{2\pi k} \ln r_o/r_i} + \cancel{\frac{A_i}{A_o h_o}}} \simeq h_i$$

NEGL. NEGL.

$$\Rightarrow h_i A_i = \text{CONST.}$$

USING DITTUS-BOELTER CORRELATION

$$h_i = \frac{k}{D}(\text{CONST})\, Re^{0.8} Pr^{0.4} = K \frac{(\rho v)^{0.8}}{D}$$

$$= \frac{K\left(\frac{DQ}{\pi D^2/4}\right)^{0.8}}{D} = (\text{CONST.})\, D^{-1.8}$$

$$\Rightarrow hA = \text{CONST} = A(\text{CONST})\, D^{-1.8}$$

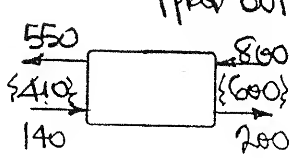
$\boxed{\text{AS DIAMETER INCREASES THE REQUIRED AREA INCREASES AS } D^{1.8}}$

22.2 $q = \dot{m} c_p \Delta T_W = \dot{m} c_p \Delta T_{PROD}$

$$10^5(1)(60) = 10^5(0.24)\,\Delta T_{PROD}$$

$$\Delta T_{PROD} = 250\text{ F} = 800 - T_{P\,OUT}$$

$$T_{PROD\ OUT} = \underline{\underline{550\text{ F}}}$$



$$\Delta T_{LM} = \frac{600-410}{\ln 600/410} = 500\text{ F}$$

$$A = \frac{q}{U \Delta T_{LM}} = \frac{10^5(1)(60)}{12(500)} = \underline{\underline{1000\text{ ft}^2}}$$

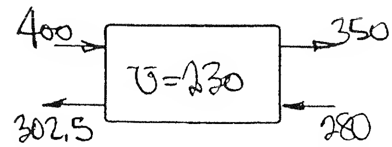
22.3 OIL: $T_{IN} = 400\text{ K}$ $T_{OUT} = 350\text{ K}$

$\dot{m} = 2\text{ kg/s}$ $c_p = 1880\text{ J/kg}\cdot\text{K}$

$$q = \dot{m} c_p \Delta T = 2(1880)(50) = 188000\text{ W}$$

$$\Delta T_W = \frac{q}{\dot{m} c_p} = \frac{188000}{2(4187)} = 22.5\text{ K}$$

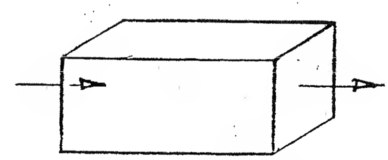
$T_{W\,IN} = 280\text{ K}$ $T_{W\,OUT} = 302.5\text{ K}$



$$\Delta T_{LM} = \frac{97.5-70}{\ln 97.5/70} = 83\text{ K}$$

$$A = \frac{q}{U \Delta T_{LM}} = \frac{188000}{230(83)} = \underline{\underline{9.85\text{ m}^2}}$$

22.4



$$D_{EQUIV} = \frac{4(0.1)(0.2)}{2(0.1+0.2)} = 0.0667\text{ M}$$

$T_{b\,AVG} = 295\text{ K}$ $T_f = 345\text{ K}$

$$Re = 0.0667 \frac{\rho v}{\mu} = 0.0667 \frac{\dot{m}}{A\mu}$$

$$q = hA\,\Delta T_{LM}$$

$$\Delta T_{LM} = \frac{105-95}{\ln 105/95} = 99.9\text{ K}$$

22.4 CONTINUED

ASSUMING TURBULENT FLOW:

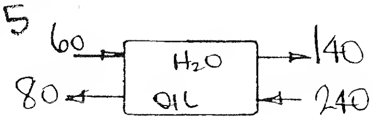
$$\dot{m} c_p \Delta T = \rho U c_p \left[0.023\, Re^{-0.2} Pr^{-2/3}\right] A_s \Delta T_{LM}$$

$$\dot{m}(10) = \frac{\dot{m}}{A}\left[0.023\left(\frac{0.0667}{0.205\times10^{-5}}\frac{\dot{m}}{A}\right)^{-0.2} \times (0.698)^{-2/3}\right](2)(0.3)(2.5)99.9$$

SOLVING FOR $\dot{m}$: $\quad \underline{\dot{m} = 105 \text{ kg/s}}$

$$q = \dot{m} c_p \Delta T = 105(1009)(10) = \underline{\underline{1060 \text{ kW}}}$$

22.5



$U = 50$ Btu/HR FT^{2}F

$c_{p\,OIL} = 0.45$ Btu/LB$_m$F

$\dot{m}_w = 150$ LBm/MIN

a) $q = 150(1)(80) = 12000$ Btu/MIN

$= UA\,\Delta T_{LM}$

$$\Delta T_{LM} = \frac{100-20}{\ln\frac{100}{20}} = 49.7 \text{ F}$$

$$A = \frac{q}{U\Delta T_{LM}} = \frac{12000(60)}{50(49.7)} = 290 \text{ FT}^2$$

b) WATER IN SHELL; OIL ~ 2 PASSES

$$Y = \frac{80-240}{60-240} = 0.889$$

$$Z = \frac{60-140}{80-240} = 0.5$$

$A = \infty$ ~ CAN'T BE DONE

c) $q = \dot{m} c_p \Delta T|_w = \dot{m} c_p \Delta T|_o$

$$C_o = \frac{12000(60)}{160} = 75(60)$$

$= 4500$ Btu/HR·F

22.5 CONTINUED

$C_o = C_{MIN}$

$$\frac{UA}{C_{MIN}} = \frac{50(290)}{4500} = 3.22$$

$$\frac{C_{MIN}}{C_{MAX}} = 0.625$$

Fig 22.12 ~ $\varepsilon \cong 0.86$

$q = \varepsilon C_{min}(180) = 7200\,\Delta T$

$$\Delta T_w = \frac{0.86(4500)(180)}{7200} \cong 97 \text{ F}$$

$T_{w\ EXIT} = \underline{\underline{157 \text{ F}}}$

22.6 $\Delta T_w = 340 - 255 = 85$ K

$\Delta T_o = 350 - 305 = 45$ K

$q = \varepsilon C_{MIN}(350-255) = C_w(85)$

$\varepsilon = 85/95 = \underline{\underline{0.895}}$

22.7 WATER $T_{IN} = 50$ F

$\dot{m} = 400$ LBm/HR

$c_p = 1$ Btu/LBm F

OIL: $T_{IN} = 250$ F $\quad c_p = 0.45$ Btu/LBmF

$\dot{m} = ?$

$U = 60$ Btu/HRFT^{2}F $\quad A = 18$ FT^2

$T_{w\ OUT,MAX} = 212$ F $\quad T_{o\ OUT,MAX} = 160$ F

$$q = \dot{m}_w c_{pw} \Delta T_w = \dot{m}_o c_{po} \Delta T_o$$

$$= 400(1)(T_w - 50) = \dot{m}_o(0.45)(250 - T_o)$$

$$= \varepsilon C_{MIN}(200)$$

22.7 CONTINUED -

FOR $T_{W\,OUT} = 212$ $\Delta T_W = 162$

$$q = 400(162) = 64800$$

$$= C_o \Delta T_o = \mathcal{E} C_{MIN}(200)$$

$\left\{ q = \dot{m}_o c_{po} \Delta T_o, \therefore \text{ MAX } \dot{m}_o \text{ WILL BE ASSOCIATED WITH MINIMUM } \Delta T_o \right\}$

IF $T_{o\,OUT\,MAX} = 160 \sim \Delta T_{o\,MIN} = 90$

FOR H_2O AS MINIMUM FLUID:

$$C_{MIN} = 400$$

$$400(162) = \mathcal{E}(400)(200)$$

$$\mathcal{E} = 0.81$$

$$NTU = UA/C_{MIN} = \frac{60(18)}{400} = 2.7$$

$\{$FIG 22.12 a $C_{MIN}/C_{MAX} \cong 0.65\}$

$$C_{MAX} = \dot{m} c_{po} = \frac{400}{0.65} = 615$$

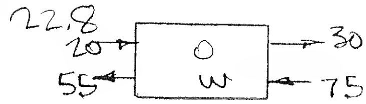
$$\Delta T_o = \frac{q}{C_o} = \frac{64800}{615} = 105.3 \text{ F}$$

$T_{o\,OUT} = 144.7$ - OK

FINALLY: $\dot{m}_{MAX} c_p = 615$

$$\dot{m}_{MAX} = \frac{615}{0.45} = \underline{\underline{1370 \text{ LBm/HR}}}$$

22.8



$\dot{m}_o = 12$ kg/s

$c_{po} = 2.2$ KJ/kg.K

$c_{pw} = 4.18$ "

$U = 1080$ W/m^2.K

$\left\{ \text{PROBLEM STATEMENT USES } C_H \text{ AS } C_{pw} \text{ \& } C_C \text{ AS } C_{po} \right\}$

$$q = UAF\Delta T_{LM}$$

$$= \dot{m}_o c_{po} \Delta T_o = \dot{m}_w c_{pw} \Delta T_w$$

$$= (12)(2200)(10) = \dot{m}_w (4180)(20)$$

$$\rightarrow \dot{m}_w = 3.16 \text{ kg/s}$$

$$\Delta T_{LM} = \frac{45-35}{\ln 45/35} = 39.8$$

TO FIND F: $Y = 10/55 = 0.182$

$Z = 20/10 = 2$

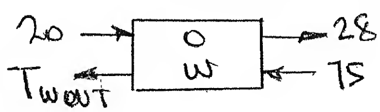
FIG 22.9 a: $F \cong 1$

$$F \Delta T_{LM} = \underline{\underline{39.8 \text{ C}}}$$

$$A = \frac{q}{UF\Delta T_{LM}} = \frac{(12)(2200)(10)}{1080(39.8)}$$

$$= \underline{\underline{6.14 \text{ m}^2}}$$

22.9 SAME EXCHANGER & ENTRANCE CONDITIONS AS PROB 22.8 -



$\left(\text{PROB STATEMENT SHOULD SAY } T_{o\,OUT} = 28°C \right)$

$$q = \dot{m}_o c_{po} \Delta T_o = 12(2200)(8)$$

$$= \dot{m}_w c_{pw} \Delta T_w = 3.16(4180)\Delta T_w$$

$$\Delta T_w \cong 16°C \quad T_{W\,OUT} \cong 59\ °C$$

22.9 CONTINUED –

$$\Delta T_{LM} = \frac{47-39}{\ln 47/39} = 42.9\,°C$$

To find F: $Y = 8/55 = 0.145$

$Z = 16/8 = 2$

Fig 22.9a – $F \cong 1$

$$U = \frac{q}{A F \Delta T_{LM}} = \frac{(12)(2200)(8)}{6.14(42.9)}$$

$= 802\ W/m^2\cdot K$

$$UA\Big|_0 = \frac{1}{\Sigma R_0} \qquad UA\Big|_1 = \frac{1}{\Sigma R_1}$$

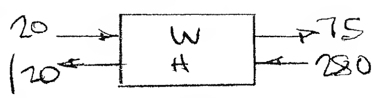
$$\Sigma R_0 = \frac{1}{(1080)(6.14)} = 1.508 \times 10^{-4}$$

$$\Sigma R_1 = \frac{1}{(802)(6.14)} = 2.031 \times 10^{-4}$$

Fouling Resistance

$= \Sigma R_1 - \Sigma R_0 = \underline{\underline{0.523 \times 10^{-4}\ K/W}}$

22.10.



$\dot m_W = 2.7\ kg/s \quad U = 160\ W/m^2\cdot K$

$q = \dot m_W c_{pW} \Delta T_W = (2.7)(4200)(55)$

$= \dot m_H (1200)(160)$

$\dot m_H = \underline{\underline{3.25\ kg/s}}$ a)

$q = U A F \Delta T_{LM}$

$$\Delta T_{LM} = \frac{205-100}{\ln 2.05} = 146.3\,°C$$

22.10 CONTINUED –

To find F: $Y = 55/260 = 0.211$

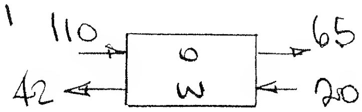
$Z = 160/55 = 2.91$

Fig 22.10a $F \cong 0.96$

$$A = \frac{q}{U F \Delta T_{LM}} = \frac{(2.7)(4200)(55)}{(160)(0.96)(146.3)}$$

$= \underline{\underline{27.8\ m^2}}$ b)

22.11



$\dot m_o = 11\ kg/s$

$c_{po} = 2.2\ kJ/kg\cdot K$

$U = 1200\ W/m^2\cdot K$

$$\Delta T_{LM} = \frac{68-35}{\ln 68/35} = 49.7\,°C$$

$q = \dot m_W c_{pW} \Delta T_W = \dot m_W (4200)(22)$

$= \dot m_o c_{po} \Delta T_o = (11)(2200)(45)$

$\dot m_W = \underline{\underline{11.79\ kg/s}}$ a)

To find F: $Y = \frac{110-65}{110-20} = 0.5$

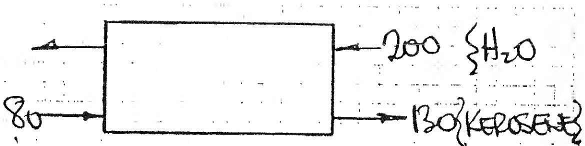
$Z = \frac{42-20}{110-65} = 0.489$

Fig 22.9a ~ $F \cong 1$

$$A = \frac{q}{U F \Delta T_{LM}} = \frac{11(2200)(45)}{(1200)(1)(49.7)}$$

$= \underline{\underline{18.26\ m^2}}$ b)

22.12



$\dot{m}_K = 2500\ \text{lbm/hr} \quad \dot{m}_W = 900\ \text{lbm/hr}$

$U = 260\ \text{Btu/hr-ft}^2\text{-F}$

$$\Delta T_W = \frac{2500(130-80)(0.51)}{900(1)} = 70.8$$

$$T_{W\,OUT} = 129\,F \quad \Delta T_{LM} = \frac{70-49}{\ln 70/49} = 58.9\,F$$

$$AF = \frac{q}{U\,\Delta T_{LM}} = \frac{2500(50)(0.51)}{58.9(260)} = 4.16$$

FIGURE 22.9 a

$$\left.\begin{aligned} Y &= \frac{130-80}{200-80} = \frac{50}{120} = 0.416 \\ Z &= \frac{200-129}{130-80} = \frac{71}{50} = 1.4 \end{aligned}\right\} F \cong 0.83$$

$$A = 4.16/0.83 = \underline{\underline{5.01\ \text{FT}^2}}$$

22.13 INPUT DATA - SEE PROB 22.3

$U = 230\ \text{W/m}^2\cdot\text{K} \quad T_{OIL\,IN} = 400\,K$

$A = 9.85\ \text{m}^2 \quad T_{W\,IN} = 280\,K$

(a) CROSSFLOW: $C_o = \dot{m}c_p = 3760\ \text{J/kg·s}$

$C_W = 8374$ "

$$\left.\begin{aligned} \frac{UA}{C_{MIN}} &= \frac{230(9.85)}{3760} = 0.603 \\ C_{MIN}/C_{MAX} &= 0.45 \end{aligned}\right\} \varepsilon = 0.43$$

$$q = \varepsilon C_{MIN}(400-280) = 194000\ W$$

$$T_{W\,OUT} = 280 + \frac{194000}{8374} = 303\,K$$

$$T_{O\,OUT} = 400 - \frac{194000}{3760} = \underline{\underline{348\,K}}$$

22.13 CONTINUED -

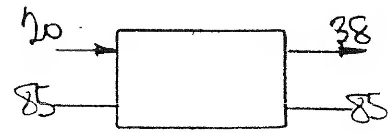
(b) SHELL AND TUBE

SAME DATA - $\varepsilon = 0.40$

$$q = 0.4(3760)(120) = 180500\ W$$

$$T_{W\,OUT} = 301.5\,K \quad \underline{\underline{T_{O\,OUT} = 352\,K}}$$

22.14



$$q = \dot{m}c_{pW}\Delta T_W = C_W \Delta T_W$$

$$C_W = \rho A V c_p = \frac{2\times10^8}{18} = 1.11\times10^7$$

$$UA/C_{MIN} = \frac{4600\,A}{1.11\times10^7} = 4.14\times10^{-4} A$$

$$A = n\pi DL = n\pi\left(\frac{1.37}{12}\right)L = 0.359\,nL$$

$$\frac{UA}{C_{MIN}} = 1.485\times10^{-4}\,nL$$

NEGLECTING TUBE RESIST

$$U = \frac{1}{\frac{1}{h_i}+\frac{1}{h_o}} \qquad \frac{1}{h_i} = \frac{1}{4600} - \frac{1}{10600}$$

$$h_i = 8130 \quad Nu_i = \frac{8130(1.37/12)}{0.615} = 1509$$

USING DITTUS-BOELTER EQN.

$$Nu = 1509 = 0.023\left[\frac{\rho V(1.37/12)}{825\times10^{-6}}\right]^{0.8}(5.65)^{0.4}$$

$$\rho V = \frac{q}{A\,c_p\Delta T} = \frac{2\times10^8}{n\frac{\pi D^2}{4}(4179)(18)}$$

$= 3192 \quad n = 81$ TUBES

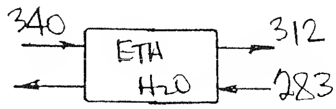
22.14 CONTINUED –

$$q = \varepsilon C_{MIN}(65) \quad \varepsilon = 0{,}277$$

FIG 22,2 c $\dfrac{UA}{C_{MIN}} \cong 0{,}38$

$$L = \frac{0{,}38}{(1{,}48 \times 10^{-4})(81)} = \underline{\underline{31{,}7\ m}}$$

22.15



$$\dot{m}_{ETH} = 6{,}93\ kg/s$$

$$q = \dot{m}c_p\Delta T\big|_{ETH} = 6{,}93(3810)(28)$$

$$= \dot{m}c_p\Delta T\big|_W = 6{,}30\,(4182)\Delta T$$

$$\Delta T_W = 28{,}1$$

(a) COUNTERFLOW: $\Delta T_{LM} \cong 29\ F$

$$A = \frac{q}{U\Delta T_{LM}} = \frac{6{,}93(3810)(28)}{568\,(29)} = \underline{\underline{44{,}9\ m^2}}$$

(b) PARALLEL FLOW:

$$\Delta T_{LM} = \frac{57 - 0{,}9}{\ln 57/0{,}9} = 13{,}52$$

$$A = \frac{q}{568\,(13{,}52)} = \underline{\underline{96{,}3\ m^2}}$$

c) CROSSFLOW:

$$C_{MIXED} = \dot{m}c_{pW} = 26350$$

$$C_{UNMIXED} = \dot{m}c_{pE} = 26403$$

$$Y = \frac{312-340}{283-340} = 0{,}491 \quad Z = \frac{28{,}1}{28} \cong 1$$

$$F \cong 0{,}85 \qquad A = \frac{44{,}9}{0{,}85} = \underline{\underline{52{,}8\ m^2}}$$

22.16

$$q = C_A\Delta T_A = C_W\Delta T_W = 95000\ \frac{BTU}{HR}$$

$$C_W = \frac{95000}{35} = 2720\ BTU/HR$$

$$C_A = \frac{95000}{80} = 1188\ \text{"} \quad \sim C_{MIN}$$

$$\frac{C_{MIN}}{C_{MAX}} = \frac{1188}{2720} = 0{,}437 \quad \varepsilon = \frac{q}{C_{MIN}(110)} \cong 0{,}73$$

a) COUNTERFLOW: $\dfrac{UA}{C_{MIN}} \cong 1{,}65$

$$V = \frac{A}{130} = \frac{1{,}65(1188)}{30(130)} = \underline{\underline{0{,}502\ FT^3}}$$

b) CROSSFLOW – AIR MIXED

$$UA/C_{MIN} \cong 2$$

$$V = \frac{A}{100} = \frac{2(1188)}{40(100)} = \underline{\underline{0{,}593\ FT^3}}$$

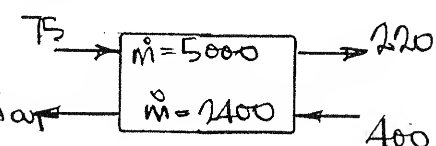
c) CROSSFLOW – BOTH MIXED

$$UA/C_{MIN} \cong 1{,}75$$

$$V = \frac{A}{90} = \frac{1{,}75(1188)}{50(90)} = \underline{\underline{0{,}462\ FT^3}}$$

CONFIGURATION (c) IS MOST COMPACT

22.17



$$U = 300\ W/m^2\,°K = 528\ \frac{BTU}{HR\,FT^2\,F}$$

$$q = 5000(1)(145) = 2400(1)\Delta T_H$$

$$\Delta T_H = 302 \qquad T_{H\,OUT} = 98\ F$$

$$\Delta T_{LM} = \frac{180 - 23}{\ln 180/23} = 76{,}3\ F$$

22.17 CONTINUED -

FOR COOL FLUID IN TUBES:

$$\left.\begin{aligned} Y &= \frac{220-75}{400-75} = 0.446 \\ Z &= 302/145 = 2.08 \end{aligned}\right\} F \cong ?$$

HOT FLUID IN TUBES:

$$\left.\begin{aligned} Y &= \frac{-302}{-325} = 0.929 \\ Z &= \frac{145}{302} = 0.480 \end{aligned}\right\} F \cong ?$$

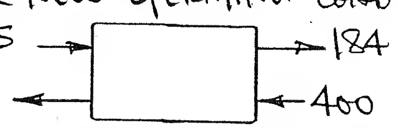
BOTH ARE OFF THE CHARTS

NEITHER IS POSSIBLE ~
CAN'T USE THIS CONFIGURATION

22.18 . IF COUNTERFLOW:

$$AU_{OLD} = \frac{1}{\left[\frac{1}{A_ih_i} + R_t + \frac{1}{A_oh_o}\right]} = 36.7(300)$$

FOR NEW OPERATING CONDITIONS:



$$q = 5000(1)(184-75) = 2400(1)\Delta T_H$$
$$= 545000 \quad \Delta T_H = 227 \quad T_{HOUT} = 173$$

$$\Delta T_{LM} = \frac{216-98}{\ln 216/98} = 149.3$$

$$AU_{NEW} = \frac{q}{\Delta T_{LM}} = \frac{545000}{149.3} = 3650$$

$$= \frac{1}{\left[\frac{1}{A_ih_i} + R_t + \frac{1}{A_oh_o}\right] + R_F}$$

$$AU_{NEW} = \frac{1}{1/36.7(300) + R_F}$$

22.18 CONTINUED -

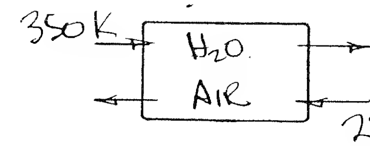
$$\frac{1}{36.7(300)} + R_F = \frac{1}{3650}$$

$$\underline{\underline{R_F = 1.83 \times 10^{-4} \ K/W}}$$

OR $AR_F = 36.7R$

$$= \underline{\underline{6.72 \times 10^{-3} \ m^2 \cdot K/W}}$$

22.19



$h_W = 470 \ W/m^2 K \quad \dot{m}_W = 10 \ kg/s$

$h_A = 210 \quad " \quad \dot{m}_A = 16 \quad "$

$$q = (10 \ kg/s)(4181 \ J/kg \cdot K)(350 - T_{W\,OUT})$$
$$= (16 \ kg/s)(1007 \ J/kg \cdot K)(T_{A\,OUT} - 288)$$

$C_W = 41810 \quad C_A = 16112 = C_{MIN}$

$C_{MIN}/C_{MAX} = 0.385$

$$A = \pi DL(50) = \pi(0.026)(6.7)(50)$$
$$= 27.36 \ m^2$$

{ASSUMES TOTAL LENGTH OF EACH TUBE IS 6.7 m}

$$U = \frac{1}{\frac{1}{h_i} + \underset{NEG}{\cancel{R_{COND}}} + \frac{1}{h_o}} = \frac{1}{\frac{1}{470} + \frac{1}{210}} = 145$$

$$UA/C_{MIN} = \frac{145(27.36)}{16112} = 0.246$$

$$\underline{\underline{\varepsilon \cong 0.20}}$$

22.19 CONTINUED –

$$q = \varepsilon C_{MIN}(T_{Wi} - T_{Ai})$$

$$= 0.2(16112)(62) = \underline{\underline{199800 \text{ W}}}$$

$$T_{W\,OUT} = \underline{\underline{345.2\text{K}}} \quad T_{A\,OUT} = \underline{\underline{300.4\text{K}}}$$

FOR FOULING RESISTANCE = 0.0021

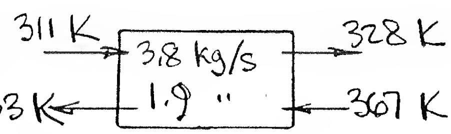
$$U = \frac{1}{\frac{1}{470} + \frac{1}{210} + 0.0021} = 111.2$$

$$UA/C_{MIN} = 0.188 \quad \underline{\varepsilon \cong 0.10}$$

$$q = 0.1(16112)(62) = \underline{\underline{99900 \text{ W}}}$$

$$T_{W\,OUT} = \underline{\underline{347.6}} \quad T_{A\,OUT} = \underline{\underline{294.2}}$$

22.20



$$U = 1420 \text{ W/m}^2\cdot\text{K}$$

TUBES: $ID = 0.01905$ m

$v = 0.366$ m/s

$L_{MAX} = 2.44$ m

$$q = \dot{m} c_p \Delta T_{SHELL} = \dot{m} c_p \Delta T_{TUBES}$$

$$\Delta T_S = \frac{3.8(17)}{1.9} = 34$$

$$C_{MIN} = 1.9(4180) = 7942 \text{ W/K}$$

$$\rho = 983 \text{ kg/m}^3$$

$$\dot{m} = \rho A V = 3.8 = n(983)\left(\frac{\pi}{4}\right)(0.01905)(0.366)$$

$$n = \underline{\underline{37 \text{ TUBES}}}$$

22.20 CONTINUED –

$$q = \varepsilon C_{MIN}(367 - 311) \quad \frac{C_{MIN}}{C_{MAX}} = 0.5$$

$$\varepsilon = \frac{C_{MIN}(34)}{C_{MIN}(56)} = 0.607$$

$$UA/C_{MIN} \cong 1.3 \quad \{\text{FIG. } 22.12\text{ c}\}$$

$$A = \frac{7942(1.3)}{1420} = 7.27 \text{ m}^2$$

$$= \underset{74}{\cancel{n}}\,\pi(0.01905)L$$

$$L = 1.64 \text{ m}$$

$$\boxed{\begin{array}{c}\text{2 TUBE PASSES WILL WORK} \\ \text{37 TUBES PER PASS} \\ L = 1.64 \text{ m PER PASS}\end{array}}$$

22.21 $NTU = 1.25$

$$C_{MIN}/C_{MAX} = 0 \quad \varepsilon \cong 0.72$$

$$q = \varepsilon C_{MIN}(T_{WIN} - T_{CIN})$$

$$= 0.72(0.07)(4.18)(93)$$

$$= 19.59 \text{ kW}$$

$$= C_W \Delta T_W = 4.18(0.07)\Delta T_W$$

$$\Delta T_W = \frac{0.72(0.07)(4.18)(93)}{4.18(0.07)} = 67 \text{ K}$$

$$T_{W\,OUT} = 280 + 67 = \underline{347 \text{ K}}$$

22.21 CONTINUED -

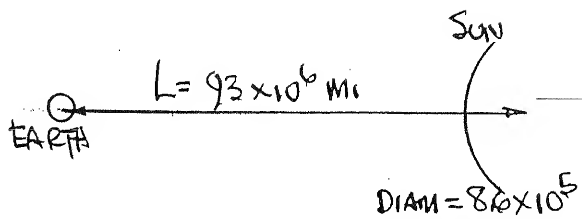
STEAM CONDENSATION RATE:

$$\dot{m}_{COND} = q/h_{fg} = \frac{19.59}{2256}$$

$$= \underline{\underline{8.68 \times 10^{-3}\ kg/s}}$$

CHAPTER 23

23.1



RADIANT EMISSION FROM SUN = $A_S E_{bS}$

ALL PASSES THROUGH A SPHERICAL SURFACE OF RADIUS, L.

AT THE EARTH

$$\frac{q}{A} = \frac{\pi D_S^2 E_{bS}}{4\pi L^2} = \left(\frac{D}{2L}\right)^2 E_{bS}$$

FLUX AT EARTH = 360 + 90 = 450 $\frac{\text{BTU}}{\text{HR FT}^2}$

$$450 = \left[\frac{86 \times 10^5}{2(93 \times 10^6)}\right]^2 \sigma T_S^4$$

$$T_S = \underline{\underline{10530\ R}}$$

23.2

$0 < \lambda < 0.35\mu$ $\quad \tau = 0$

$0.35 < \lambda < 2.7\mu$ $\quad \tau = 0.92$

$2.7 < \lambda$ $\quad \tau = 0$

FOR $T = 5800$ K

$\lambda_1 T = 2030$ $\quad F = 0.072$

$\lambda_2 T = 15660$ $\quad F = 0.972$

$\Delta F = 0.90$

PER CENT TX = $0.90(0.92) = 0.828$

$\underline{\underline{\cong 83\%}}$

FOR $T = 300$ K:

$$\left.\begin{array}{ll} \lambda_1 T = 105 & F \cong 0 \\ \lambda_2 T = 810 & F \cong 0 \end{array}\right\} \Delta F \cong 0$$

PERCENT TX = $\underline{\underline{0}}$

23.3

FROM WIEN'S DISPLACEMENT LAW:

$$\lambda_{MAX} T = 5215.6\ \mu R$$

$$\lambda_{MAX} = \frac{5215.6}{4000} = \underline{\underline{1.304\ \mu}}$$

FRACTION IN VISIBLE BAND:

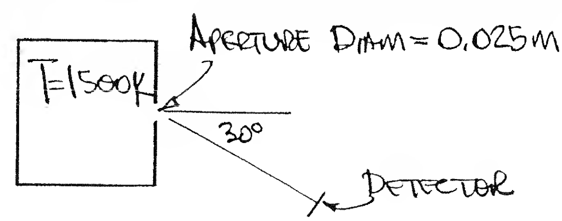
$$= \frac{\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda}{\int_0^\infty E_{b\lambda}\, d\lambda} = \frac{1}{\sigma T^4}\left(E_b\Big|_0^{\lambda_2 T} - E_b\Big|_0^{\lambda_1 T}\right)$$

{TABLE 23.1} $\lambda_1 T = 667\ \mu K$ $\quad F \cong 0$

$\lambda_2 T = 1667$ " $\quad F = 0.0256$

OR $\underline{\underline{2.56\%}}$

23.4



(a) $$I = \frac{q}{A_A \cos\theta_A\, \omega}$$

$$I = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi} = 9.137 \times 10^4\ \text{W/m}^2\cdot\text{Sr}$$

$$\omega = \frac{A_D}{r^2} = \frac{0.001\ m^2}{1\ m^2} = 0.001\ \text{Sr}$$

$$q = I A_A \cos\theta_A\, \omega$$

$$= (9.137 \times 10^4)\left(\frac{\pi}{4}\right)(0.025)^2 \cos 30°$$
$$\times (0.001)$$

$$= \underline{\underline{3.88 \times 10^{-2}\ W}}$$

23.4 CONTINUED -

(b) WITH WINDOW

$$q = \tau I A_A \cos\theta_A \omega$$

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

$$= \frac{\int_0^\infty \tau_\lambda E_{\lambda,b} d\lambda}{\int_0^\infty E_{\lambda,b} d\lambda}$$

$$= 0.8 \int_0^2 \frac{E_{\lambda b}}{E_b} d\lambda + 0 \int_2^\infty \sim$$

$$= 0.8\ F(0-2\mu m)$$

$$\lambda T = 2(1500) = 3000$$

$$F_{0-2\mu} = 0.273$$

$$\tau = 0.8(0.273) = 0.218$$

$$q = 0.218(3.88 \times 10^{-2})$$

$$= \underline{\underline{8.47 \times 10^{-3}\ W}}$$

23.5 $\tau_{SOLAR} = \tau_\lambda F_{\lambda_1 - \lambda_2}$

$$= \tau_\lambda (F_{0-\lambda_2} - F_{0-\lambda_1})$$

FOR SOLAR IRRADIATION:

PLAIN GLASS:

$\lambda_1 T = 0.3(5800) = 1740 \qquad F = 0.033$

$\lambda_2 T = 2.5(5800) = 14500 \qquad F = 0.966$

23.5 CONTINUED -

$$\tau_s = 0.9(0.966 - 0.033) = \underline{\underline{0.84}}$$

TINTED GLASS:

$\lambda_1 T = 0.5(5800) = 2900 \qquad F = 0.25$

$\lambda_2 T = 1.5(5800) = 8700 \qquad F = 0.881$

$$\tau = 0.9(0.881 - 0.25) = \underline{\underline{0.568}}$$

IN THE VISIBLE RANGE:

$\lambda_1 = 0.38\ \mu m \qquad \lambda_2 = 0.76\ \mu m$

{FOR TINTED GLASS $\lambda_1 = 0.5\ \mu$}

$\lambda_1 = 0.38 \qquad F_{0-\lambda_1 T} = 0.1017$

$\lambda_1 = 0.5 \qquad F_{0-\lambda_1 T} = 0.250$

$\lambda_2 = 0.76 \qquad F_{0-\lambda_2 T} = 0.550$

PLAIN GLASS: $\tau = 0.9(0.550 - 0.1017)$

$$= \underline{\underline{0.404}}$$

TINTED GLASS: $\tau = 0.9(0.550 - 0.250)$

$$= \underline{\underline{0.27}}$$

23.6 $\lambda_1 = 0.8\ \mu m$, $\lambda_2 = 5\ \mu m$ $\quad F_{\lambda_1 T - \lambda_2 T} = F_{0-\lambda_2 T} - F_{0-\lambda_1 T}$

T, K	$F_{0-\lambda_1 T}$	$F_{0-\lambda_2 T}$	$F_{\lambda_1 T - \lambda_2 T}$
500	~0	0.1613	0.1613
2000	0.0197	0.9142	0.8945
3000	0.1402	0.9689	0.8287
4500	0.4036	0.9896	0.586

23.7 $T = 5800\ K$

FOR $\lambda_1 = 0.4\ \mu m$ $\lambda_1 T = 2320$

$\lambda_2 = 0.7\ \mu m$ $\lambda_2 T = 4060$

$F_{0-\lambda_1 T} = 0.1220$ $F_{0-\lambda_2 T} = 0.4916$

FRACTION IN VISIBLE RANGE $\Big\} = \underline{\underline{0.3696}}$

IN UV RANGE: $0.01 < \lambda < 0.4$

$F_{0-\lambda_1 T} = 0$ $F_{0-\lambda_2 T} \cong 0.12$

FRACTION IN UV RANGE $\Big\} \cong \underline{\underline{0.12}}$

IN IR RANGE $0.4 < \lambda < 10^2$

$F_{0-\lambda_1 T} = 0.12$ $F_{0-\lambda_2 T} = 100$

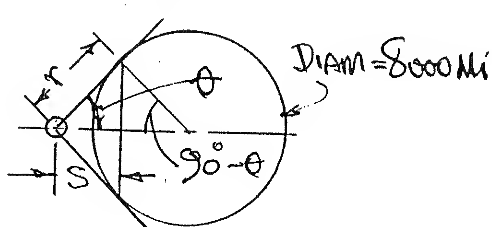
FRACTION IN IR RANGE $\Big\} \cong \underline{\underline{0.88}}$

WIEN'S LAW –

$\lambda_{MAX} T = 2897.6\ \mu m \cdot K$

$\lambda_{MAX} \cong \underline{\underline{0.500\ \mu m}}$

23.8



$\theta = \sin^{-1} \frac{4000}{4500} = 62.7° = 1.096\ RAD$

$S = 4500 - 4000 \cos(90 - \theta) = 945\ mi$

$r = S/\cos\theta = 2060\ mi$

23.8 CONTINUED –

AREA SUBTENDED BY EARTH

$$\Delta A = \int_0^{\theta} 2\pi r^2 \sin\theta\, d\theta = 2\pi r^2 (1 - \cos\theta)$$

$$= 2\pi r^2 (0.541)\ mi^2$$

$$F_{S-E} = \frac{\Delta A}{4\pi r^2} = 0.271$$

$$F_{S-SPACE} = 0.729$$

INCIDENT SOLAR ENERGY = $450\ \frac{Btu}{HR\text{-}FT^2}$

{FROM PROB 23.1}

$$q_{SUN-SAT} = 450\left(\frac{\pi}{4}\right)\left(\frac{50}{12}\right)^2 = 6150\ \frac{Btu}{HR}$$

$$q_{\substack{ABSORBED \\ BY\ SAT}} = 0.3(6150) = 1845\ ''$$

$$q_{REFLECTED} = 4305\ Btu/HR$$

$$q_{E-SAT} = \epsilon A_S F_{S-E} \sigma T_E^4$$

$$= 0.195\left[(\pi)\left(\frac{50}{12}\right)^2\right](0.271)(0.1714)(5.1)^4$$

$$= 164\ Btu/HR$$

$$q_{ABSORBED} = 0.05(164) = 8.2\ Btu/HR$$

$$q_{REFLECTED} = 155.8\ Btu/HR$$

$$q_{\substack{EMITTED \\ BY\ SAT.}} = 0.05(0.1714)\left(\frac{T_S}{100}\right)^4 \pi\left(\frac{50}{12}\right)^2$$

$$= 0.467\left(\frac{T_S}{100}\right)^4$$

ENERGY BALANCE:

$$6150 + 164 = 4305 + 155.8 + 0.467\left(\frac{T_S}{100}\right)^4$$

$$T_S = \underline{\underline{794\ R}} = \underline{\underline{334\ F}}$$

23.9

$$\frac{q}{A}\Big|_{NET} = \frac{q}{A}\Big|_{IN} - \frac{q}{A}\Big|_{OUT}$$

$$= 1000 - h(T_s - T_\infty) - \epsilon\sigma\left[\left(\frac{T_s}{100}\right)^4 - \left(\frac{T_\infty}{100}\right)^4\right]$$

$$= 1000 - 12(30-20) - 5.676(0.3)(106)$$

$$= \underline{\underline{862\ W/m^2}}$$

23.10

ENERGY BALANCE FOR COLLECTOR:

$q_{IN} = 800\,A$ W

$q_{OUT} = q_{RAD} + q_{CONV}$

$= \sigma A\left(T^4 - T_\infty^4\right) + hA\left(T - T_\infty\right)$

EQUATING:

$$800 = \sigma\left(T^4 - T_\infty^4\right) + h\left(T - T_\infty\right)$$

$$= 5.676\left(\frac{T}{100}\right)^4 - 5.676(3.03)^4$$

$$+ 35T - 10605$$

$$\left(\frac{T}{100}\right)^4 + 6.17T = 2094$$

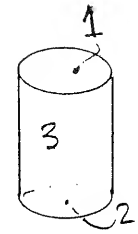
BY TRIAL & ERROR: $T = \underline{\underline{322\ K}}$

$$q_{RAD} = \sigma A\left(T^4 - T_\infty^4\right)$$

$$= 5.676(60)\left[3.22^4 - 3.03^4\right]$$

$$= \underline{\underline{7910\ W}}$$

23.11



$F_{12} = 0.12$ {FIG 23.14}

$F_{13} = 0.88$

$$F_{31} = F_{32} = \frac{A_1 F_{13}}{A_3}$$

$$= \frac{\pi D^2}{4}\,\frac{0.88}{\pi D L} = 0.17$$

$F_{3\text{-}SURR} = 0.34$

$$q_{3\text{-}SURR} = \pi(0.075)(0.1)(0.34)(5.676)\left(7^4 - 3.1^4\right)$$

$$= \underline{\underline{105\ W}}$$

23.12 ENTIRE HOLE INTERIOR IS SURF 2
OPENING (SURROUNDINGS) IS " 1

$F_{12} = 1 \qquad A_1 F_{12} = \frac{\pi D^2}{4}(1)$

$$q_{21} = A_2 F_{21}\,\epsilon\sigma\left(T_2^4 - T_1^4\right) = A_1 F_{12}\,\epsilon\sigma\left(T_2^4 - T_1^4\right)$$

$$= \frac{\pi}{4}(0.075)^2(1)(1)(5.676)\left(7^4 - 3.1^4\right)$$

$$= \underline{\underline{57.9\ W}}$$

23.13

$$\frac{q}{A} = \frac{1200\ W}{5(0.49\ m^2)} = 490\ W/m^2$$

$$= \epsilon\sigma\left[\left(\frac{T}{100}\right)^4 - 2.8^4\right]$$

$$490 = 0.7(5.676)\left[\left(\frac{T}{100}\right)^4 - 2.8^4\right]$$

$$T = \underline{\underline{369\ K}}$$

23.14 WITH NO INTERVENING PLATE:

$$q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

$$q/A = 5.676 \left[9^4 - 5.8^4 \right] = \underline{\underline{30.8 \text{ kW/m}^2}}$$

WITH INTERVENING PLATE PRESENT:

$$E_{b1} \xrightarrow{R_1} J_3 \xrightarrow{R_2} E_{b2}$$

PER UNIT AREA:

$$q/A = \frac{E_{b1} - E_{b2}}{1/F_{13} + 1/F_{23}} = \frac{E_{b1} - E_{b2}}{2}$$

$$= \underline{\underline{15.4 \text{ kW/m}^2}}$$

$$q/A = (E_{b1} - J_3) \cancel{F_{13}}^{1} = (J_3 - E_{b2}) \cancel{F_{32}}^{1}$$

$$J_3 = \sigma T_1^4 - q/A = 15.4 \text{ kW/m}^2$$

$$= \sigma T_3^4$$

$$T_3^4 = \frac{15.4 \times 10^3}{\sigma} \quad T_3 = \underline{\underline{722 \text{ K}}}$$

{EMISSIVITY OF INTERVENING PLATE HAS NO EFFECT}

23.15 FILAMENT AT 2910 K

$q = 100$ W

$$\lambda_{MAX} = \frac{2897.6}{2910} = \underline{\underline{0.999 \ \mu m}} \quad a)$$

VISIBLE RANGE: $0.38 < \lambda < 0.76$

$\lambda T_1 = 0.38(2910) = 1102 \quad F_{0-\lambda T} = 0.0009$

$\lambda T_2 = 0.76(2910) = 2204 \quad F_{0-\lambda T} = 0.1017$

FRACTION IN V.R. $= \underline{\underline{0.1008}}$ b)

23.16 FOR SURROUNDINGS AT 0 K:

$$E_b = \sigma T^4 = (5.676)(20)^4 = 9.08 \times 10^5 \text{ W/m}^2$$

$$100 \text{ W} = 9.08 \times 10^5 A$$

$$A = 1.109 \times 10^{-4} \text{ m}^2 = \pi D^2/4$$

$$\underline{\underline{D = 0.01188 \text{ m} = 1.188 \text{ cm}}} \quad a)$$

IN VISIBLE RANGE: $0.4 < \lambda < 0.7 \ \mu m$

$\lambda T|_1 = 2000(0.4) = 800 \quad F \cong 0$

$\lambda T|_2 = 2000(0.7) = 1400 \quad F = 0.0078$

FRACTION $= \underline{\underline{0.0078}}$ b)

IN UV RANGE – $0 < \lambda < 0.4$

$\lambda T|_1 = 0$

$\lambda T|_2 \cong 0$ FRACTION $= \underline{\underline{0}}$ c)

IN IR RANGE – $0.4 < \lambda < 100$

$\lambda T|_1 = 0.0078$

$\lambda T|_2 \cong 1.0$ FRACTION $= \underline{\underline{0.992}}$ d)

23.17 $q = 8$ W THROUGH HOLE WITH $D = 0.0025 \text{ m}^2$

$$E_b = \frac{8}{0.0025} = 3200 \text{ W/m}^2 = \sigma T^4$$

$$\underline{\underline{T = 487 \text{ K}}}$$

23.18 $\lambda_{max} T = 2897.6 \ \mu m \cdot K$

	T	λ_{MAX}
SUN	5790 K	1.998 μm ⟸
L. BULB	2910 K	1.004 "
SURFACE	1550 K	0.535 "
SKIN	308 K	0.1063 "

23.19 $T = 1500K$ PEEPHOLE $D = 10$ cm

$\tau = 0.78$ FOR $0 < \lambda < 3.2\ \mu m$

$0.08 \qquad 3.2 < \lambda < \infty$

$q_{MAX} = 5.676(15)^4\left(\frac{\pi}{4}\right)(0.01)^2$

$= 22.57$ W

FOR $\lambda T_1 = 0 \qquad f_{0-\lambda T_1} = 0$

$\lambda T_2 = 4800 \quad F_{0-\lambda T_2} = 0.6075$

$\lambda T_3 = \infty \quad F_{0-\lambda T_3} = 1$

TOTAL HT LOSS

$= 22.57\left[0.78(0.6075) + 0.08(0.3925)\right]$

$= \underline{\underline{11.40 \text{ W}}}$

23.20 $E_{b1} \xrightarrow{R_1} J_1 \xrightarrow{R_2} J_2 \xrightarrow{R_3} E_{b2}$

1 IS INNER CYLINDER

2 " OUTER "

$E_{b1} = \sigma(77)^4 = 2.0\ W/m^2$

$E_{b2} = \sigma(300)^4 = 460$ "

$R_1 = \frac{\rho_1}{A_1\epsilon_1} = \frac{0.8}{\pi(0.02)(1)(0.2)} = 63.7\ m^{-2}$

$R_2 = \frac{1}{A_1F_{12}} = \frac{1}{\pi(0.02)(1)} = 15.9$ "

$R_3 = \frac{\rho_2}{A_2\epsilon_2} = \frac{0.95}{\pi(0.05)(1)(0.05)} = 121$ "

$\Sigma R = 201\ m^{-2}$

$q = \frac{E_{b2} - E_{b1}}{\Sigma R} = \frac{460 - 2}{201} = \underline{\underline{2.28\ W/m}}$

23.20 CONT. – WITH RADIATION SHIELD

$E_{b1} \xrightarrow{R_1} \xrightarrow{R_2} \xrightarrow{R_4} \xrightarrow{R_3} E_{b2}$

$R_1 = 63.7 \qquad R_3 = 121$

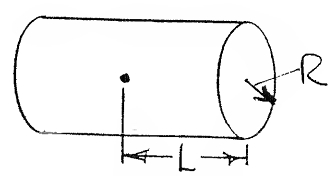
$R_2 = 15.9$

$R_4 = \frac{1}{A_3F_{32}} = \frac{1}{\frac{\pi}{4}(0.035)(1)} = 9.09$

$\Sigma R = 209.7$

$q = \frac{460 - 2}{209.7} = \underline{\underline{2.18\ W/m}}$

23.21 ASSUMING THERMOCOUPLE AT GEOMETRIC CENTER OF DUCT



SOLID ANGLE OF DUCT OPENING

$\Omega \cong \frac{\pi R^2}{\pi R_0^2} = \frac{\text{DUCT AREA}}{\text{HEMISPHERE SURFACE}}$

$= \frac{(1.5/12)^2}{1^2} = 0.0156$

{THERMOCOUPLE SEES DUCT, PRIMARILY}

FOR THERMOCOUPLE: $q_{RAD} = q_{CONV}$

$A\mathcal{F}_{cw}(E_{bc} - E_{bw}) = hA(T_G - T_c)$

$E_{bc} \xrightarrow{R_1} \xrightarrow{R_2} \xrightarrow{R_3} E_{bw}$

23.21 CONT.

$$A_c \mathcal{F}_{cw} = \frac{1}{\frac{\rho_c}{A_c\epsilon_c}+\frac{1}{A_cF_{cw}}+\frac{\rho_w}{A_w\epsilon_w}}$$

$$\mathcal{F}_{cw} = \frac{1}{\rho_c/\epsilon_c + 1/F_{cw} + \frac{\rho_w A_c}{A_w\epsilon_w}}$$

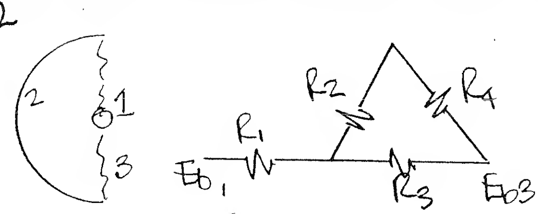
$F_{cw} \cong 1 \quad A_c/A_w \simeq 0$

$$\therefore \quad \mathcal{F}_{cw} = \frac{1}{\frac{1-\epsilon_c}{\epsilon_c}+1} \cong \epsilon_c = 0.6$$

$$30(T_G - T_c) = \epsilon_c(0.1714)\left[\left(\frac{T_c}{100}\right)^4 - \left(\frac{T_w}{100}\right)^4\right]$$

$$\underline{\underline{T_c = 816\ F}}$$

23.22



$$R_1 = \frac{\rho_1}{A_1\epsilon_1} = \frac{0.2}{\pi/6(0.8)} = \frac{1.5}{\pi}$$

$$R_2 = \frac{1}{A_1F_{12}} = \frac{1}{\pi/6(0.5)} = \frac{12}{\pi}$$

$$R_3 = \frac{1}{A_1F_{13}} = \frac{1}{\pi/6(0.5)} = \frac{12}{\pi}$$

$$R_4 = \frac{1}{A_2f_{23}} = \frac{1}{A_3f_{32}} = \frac{1}{1.5-0.167} = 0.75$$

$$R_{EQUIV} = \frac{1}{\frac{1}{R_3}+\frac{1}{R_2+R_4}} = 2.08$$

23.22 CONT.

$$\Sigma R = 1.5/\pi + 2.08 = 2.557$$

$$q = \frac{0.1714(24.6^4 - 5.3^4)}{2.557} = 24{,}500\ \frac{Btu}{hr\ ft} \quad (a)$$

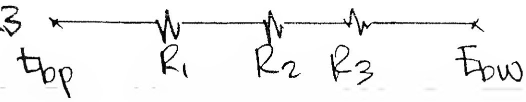
WITH NO REFLECTOR:

$$q = \epsilon A(E_{b1} - E_{b3})$$

$$= 0.8\left(\frac{\pi}{6}\right)(0.1714)(24.6^4 - 5.3^4)$$

$$= \underline{\underline{25{,}500\ Btu/hr\ ft}} \quad (b)$$

23.23



$$R_1 = \frac{\rho_p}{A_p\epsilon_p} = \frac{0.3}{\pi(1/4)(1)(0.7)} = 0.546$$

$$R_2 = \frac{1}{A_pf_{pw}} = \frac{1}{\pi(1/4)(1)} = 1.273$$

$$R_3 = \rho_w/A_w\epsilon_w = \frac{0.2}{(A_w)\epsilon_w} \cong \text{VERY SMALL}$$

$$q = \frac{0.1714(6.65^4 - 5.3^4)}{1.819}$$

$= \underline{\underline{110\ Btu/hr}}$ PER FOOT {RADIANT LOSS}

CONVECTION: $q = hA\Delta T$

$$h = \frac{k}{D}\left\{0.60 + \frac{0.387\,Ra^{1/6}}{\left[1+\left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2$$

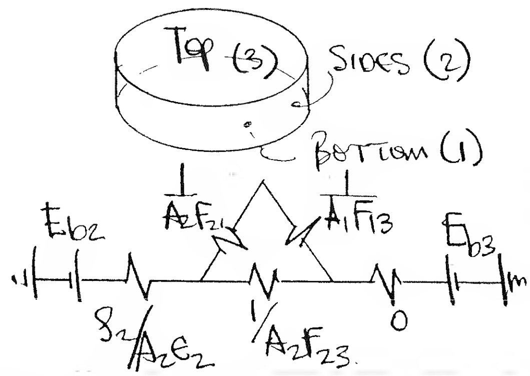
23.23 CONTINUED –

For $T_f = 137F$

$h = 1.21\ \text{Btu/hr ft}^2\ F$

$q_{conv} = 1.21(\pi)(1/4)(135) = \underline{\underline{128\ \frac{Btu}{hr}\ \text{Per ft}}}$

23.24



{Fig 23.14} $F_{13} = 0.38 \quad F_{12} = 0.62$

$A_1F_{12} = A_2F_{21} \qquad F_{21} = F_{23} = A_1F_{12}/A_2$

$$\frac{1}{A_2F_{21}} = \frac{1}{A_1F_{12}} = \frac{1}{\pi(6)^2(0.62)}$$

$$\frac{1}{A_1F_{13}} = \frac{1}{\pi(6)^2(0.38)}$$

$$\frac{1}{A_2F_{23}} = \frac{1}{A_3F_{32}} = \frac{1}{A_1F_{12}} = \frac{1}{A_2F_{21}} = \frac{1}{\pi(6)^2(0.62)}$$

$$\frac{1}{R_{equiv}} = A_2F_{23} + \frac{1}{\frac{1}{A_2F_{21}} + \frac{1}{A_1F_{13}}}$$

$$= \pi(6)^2(0.62) + \frac{1}{\frac{1}{\pi(6)^2(0.62)} + \frac{1}{\pi(6)^2(0.38)}}$$

$$= \pi(6)^2(0.856)$$

23.24 CONTINUED –

$$\frac{\rho_2}{A_2\epsilon_2} = \frac{0.2}{\pi(6)(12)(0.8)} = \frac{1}{\pi(6)^2(8)}$$

$$\Sigma R = \frac{1}{\pi(6)^2(0.856)} + \frac{1}{\pi(6)^2(8)}$$

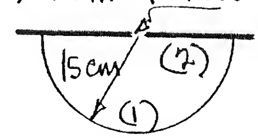
$$= \frac{1.29}{\pi(6)^2}$$

$$q = \frac{\sigma(T_2^4 - T_1^4)}{\Sigma R}$$

$$= \frac{0.1714(\pi)(6)^2}{1.29}(10^4 - 5^4)$$

$$= \underline{\underline{140{,}900\ \text{Btu/hr}}}$$

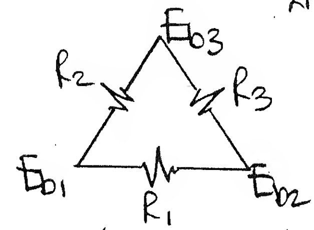
23.25 (3) Diam of Hole = 5 cm



$F_{11} + F_{12} + F_{13} = 1 \qquad F_{21} = F_{31} = 1$

$$F_{12} = F_{21}\frac{A_2}{A_1} = \frac{\pi(0.15^2 - 0.025^2)}{2\pi(0.15^2)} = 0.486$$

$$F_{13} = F_{31}\frac{A_3}{A_1} = \frac{\pi(0.25^2)}{2\pi(0.15)^2} = 0.0139$$



Black Case

$$R_1 = \frac{1}{A_1F_{12}} = \frac{1}{2\pi(0.15)^2(0.486)} = 14.55$$

$$R_2 = \frac{1}{A_1F_{13}} = \frac{1}{2\pi(0.15)^2(0.0139)} = 509$$

$R_3 = \infty$

23.25 CONTINUED -

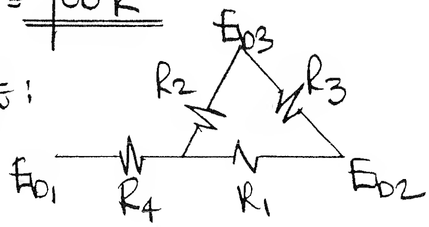
$$q_{1\text{-}3} = A_1 F_{13} E_{b1} = A_3 F_{31} E_{b3}$$

$$= \frac{\pi (0.05)^2}{4} (1)(5.676)(7^4)$$

$$= \underline{\underline{26.7 \text{ W}}}$$

$$T_2 = T_1 = \underline{\underline{700 \text{ K}}}$$

GRAY CASE:

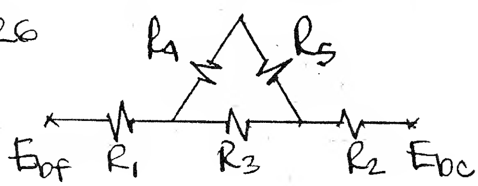


$$R_4 = \frac{\delta_1}{A_1 \epsilon_1} = \frac{0.3}{2\pi(0.15^2)(0.7)} = 3.03$$

$$q = \frac{E_{b1} - 0}{\Sigma R} = \frac{\sigma T_1^4}{512.03} = \underline{\underline{26.7 \text{ W}}}$$

$$T_2 = T_1 = \underline{\underline{700 \text{ F}}}$$

23.26



{WALLS ASSUMED TO BE AT A UNIFORM TEMPERATURE}

$$R_1 = \frac{0.2}{12(20)(0.8)} = 0.00104$$

$$R_2 = \text{SAME} = 0.00104$$

$$R_3 = \frac{1}{A_F F_{F\text{-}C}} = \frac{1}{(12)(20)(0.45)} = 0.00903$$

$$R_4 = \frac{1}{A_F F_{F\text{-}W}} = \frac{1}{(12)(20)(0.55)} = 0.0076$$

$$R_5 = R_4 = 0.0076$$

23.26 CONTINUED

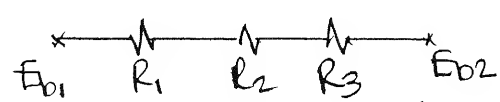
$$R_{EQUIV} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4 + R_5}} = 0.0058$$

$$\Sigma R = R_1 + R_2 + R_{EQUIV} = 0.00785$$

$$q = \frac{\sigma (T_F^4 - T_C^4)}{\Sigma R} = \frac{0.1714 (5.45^4 - 5.25^4)}{0.00785}$$

$$= \underline{\underline{2680 \text{ Btu/hr}}}$$

23.27



{EQUIVALENT CIRCUIT}

$$R_1 = \frac{\delta_1}{A_1 \epsilon_1} \qquad R_2 = \frac{1}{A_1 F_{12}} = \frac{1}{A_2 F_{21}} \qquad R_3 = \frac{\delta_2}{A_2 \epsilon_2}$$

$$T_1 = 300 \text{ K} \qquad T_2 = 78 \text{ K}$$

$$A_1 = \pi D_1^2 = \pi (1.3)^2 = 1.69\pi \text{ m}^2$$

$$A_2 = \pi D_2^2 = \pi (1)^2 = \pi \text{ m}^2$$

$$R_1 = \frac{0.8}{(1.69\pi)(0.2)} = \frac{2.37}{\pi} \text{ m}^{-1}$$

$$R_2 = \frac{1}{\pi (1)^2 (1)} = \frac{1}{\pi} \text{ m}^{-1}$$

$$R_3 = \frac{0.8}{\pi (0.2)} = \frac{4}{\pi} \text{ m}^{-1}$$

$$\Sigma R = \frac{7.37}{\pi} = 2.35 \text{ m}^{-1}$$

23.27 CONTINUED -

$$q = \frac{E_{b1}-E_{b2}}{\Sigma R} = \frac{\sigma(T_1^4-T_2^4)}{\Sigma R}$$

$$= \frac{5.676(3^4-0.78^4)}{2.35} = 194.8\,W$$

BOIL-OFF RATE: $\dot{m} = q/h_{fg}$

$$\dot{m} = \frac{194.8}{2\times10^5} = 9.74\times10^{-4}\ kg/s$$

$$= \underline{\underline{3.51\ kg/HR}}$$

23.28 OPENING DIAM = 5 mm

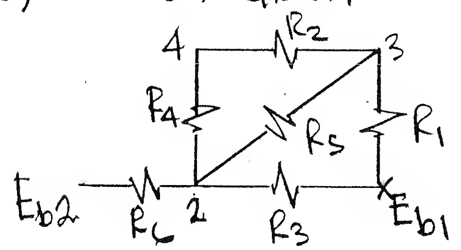
(a) EQUIV. SURFACE (1) SEES INTERIOR AS A SINGLE SURFACE

$$q = \frac{E_{b2}-E_{b1}}{R} = \frac{\sigma T_2^4 - 0}{1/A_1}$$

$$= \frac{\pi}{4}(5)^2(5.676)(6)^4\left(10^{-6}\frac{m^2}{mm^2}\right)$$

$$= \underline{\underline{0.144\ W}}$$

b) ANALOG CIRCUIT:



$R_1 = \frac{1}{A_1F_{13}} = 0.060\ mm^{-2}$ $\qquad F_{12} \cong 0.15$

$R_2 = \frac{1}{A_3F_{34}} = 1.61\times10^{-3}$ " $\qquad F_{13} = 0.85$

$R_3 = \frac{1}{A_1F_{12}} = 0.339$ " $\qquad F_{2-(1+4)} \cong 0.1$

$\qquad F_{23} = 0.9$

$R_4 = \frac{1}{A_2F_{24}} = 0.0148$ " $\qquad F_{21} = 4.167\times10^{-3}$

23.28 CONTINUED -

$R_5 = \frac{1}{A_2F_{23}} = 1.57\times10^{-3}$ " $\qquad F_{24} = 0.0958$

$R_6 = \frac{\delta_2}{A_2\epsilon_2} = 9.43\times10^{-4}$ " $\qquad F_{42} = 0.0985$

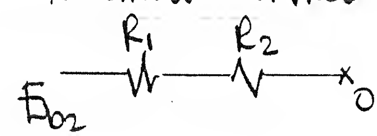
$\qquad F_{43} = 0.9015$

$$\frac{1}{R_{23\,EQUIV}} = \frac{1}{R_5} + \frac{1}{R_4+R_2} \qquad R_{23\,EQ} = 1.433\times10^{-3}$$

$$\frac{1}{R_{21\,EQUIV}} = \frac{1}{R_3} + \frac{1}{R_5+R_1} \qquad R_{21\,EQ} = 0.052$$

$$q = \frac{\sigma T^4}{R_6 + R_{21\,EQIV}} = \underline{\underline{0.139\ W}}$$

(c) ALL INTERIOR MAY BE CONSIDERED A SINGLE SURFACE -



$$R_2 = \frac{1}{A_1F_{12}} = \frac{1}{A_1} = 0.0509$$

$$R_1 = \frac{\delta}{A\epsilon} = \frac{0.4}{A_1(0.6)} = 1.291\times10^{-4}$$

$$q = \frac{\sigma T^4}{\Sigma R} = \underline{\underline{0.144\ W}}$$

(SLIGHTLY LESS THAN IN PART (a))

23.29 PROBLEM STATEMENT ASKS FOR RADIANT ENERGY REACHING TANK BOTTOM - i.e. THE IRRADIATION

a) $q_{TOTAL} = q_{FROM\ HTR} + q_{FROM\ SPACE}$

23.29 CONTINUED

$$q_{HTR} = A_1 F_{12} E_{b1}$$

$$\{F_{12} \cong 0.39 - \text{FIG } 23.15\}$$

$$= \frac{\pi}{4}(0.2)^2(0.39)\sigma T_1^4$$

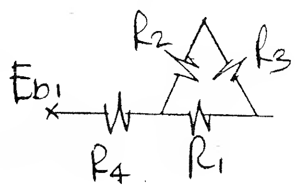
$$= 1826 \text{ W}$$

$$q_{SPACE} = \frac{\pi}{4}(0.2)^2(0.61)\sigma T_2^4$$

$$= 8.8 \text{ W}$$

$$q_{TOTAL} \cong \underline{\underline{1835 \text{ W}}}$$

b)



$$R_1 = \frac{1}{A_1 F_{12}} = \frac{1}{\frac{\pi}{4}(0.2)^2(0.39)} = 81.62$$

$$R_2 = \frac{1}{A_1 F_{13}} = \frac{1}{\frac{\pi}{4}(0.2)^2(0.61)} = 52.18$$

$$R_3 = \frac{1}{A_2 F_{23}} = R_2 = 52.18$$

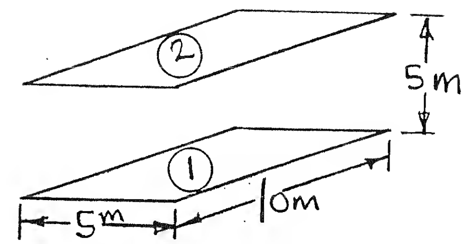
$$R_4 = \frac{\delta_1}{A_1 \epsilon_1} = \frac{0.4}{\frac{\pi}{4}(0.2)^2(0.6)} = 21.22$$

$$\frac{1}{R_{12\,EQUIV}} = \frac{1}{R_1} + \frac{1}{R_2 + R_3} \qquad R_{12\,EQUIV} = 45.8$$

$$\Sigma R = R_4 + R_{12\,EQUIV} \cong 67$$

$$q_{\substack{REACHING \\ SURF}} = \frac{\sigma T^4}{67} = \underline{\underline{2224 \text{ W}}}$$

23.30

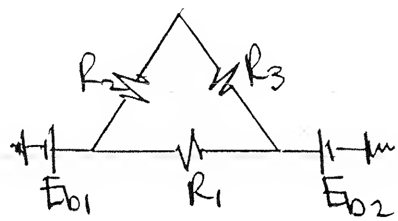


SURROUNDINGS ARE CONSIDERED AN EQUIVALENT SURFACE (3) AT 0K

$T_1 = 100\text{K}$ $\quad F_{12} \cong 0.28$ {FIG 23.14}

$T_2 = 200\text{K}$ $\quad F_{13} = 0.72$

$T_3 = 0\text{K}$ $\quad F_{23} = 0.72$



$$R_1 = \frac{1}{50(0.28)} = 0.0714$$

$$R_2 = R_3 = \frac{1}{50(0.72)} = 0.0278$$

(a)

$$q_{12} = \frac{E_{b1} - E_{b2}}{R_1} = \frac{\sigma(T_1^4 - T_2^4)}{R_1} = \underline{\underline{-1192 \text{ W}}}$$

(b)

$$q_1 = q_{12} + q_{13}$$

$$q_{12} = -1192$$

$$q_{13} = \frac{E_{b1} - 0}{R_2} = 204 \text{ W}$$

$$q_1 = \underline{\underline{-988 \text{ W}}}$$

23.30 CONTINUED-

$$q_2 = q_{21} + q_{23}$$

$$q_{21} = 1192$$

$$q_{23} = \frac{E_{b2} - 0}{R_3} = 3270 \text{ W}$$

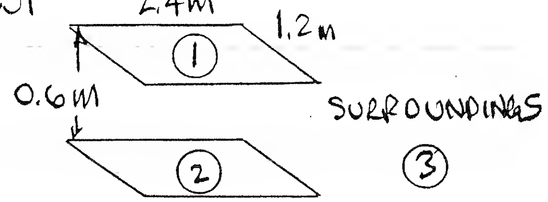
$$q_2 = \underline{\underline{4462 \text{ W}}}$$

c) TO SURROUNDINGS

$$q_{13} = \underline{\underline{204 \text{ W}}}$$

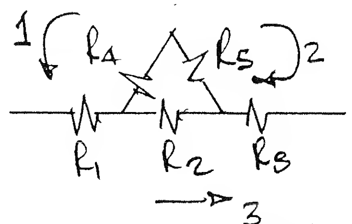
$$q_{23} = \underline{\underline{3270 \text{ W}}}$$

23.31



$\epsilon_1 = 0.6$ $T_1 = 1000 \text{ K}$ $A_1 = 2.88 \text{ m}^2$

$\epsilon_2 = 0.9$ $T_2 = 400 \text{ K}$ $A_2 =$ "



$\{F_{12} \cong 0.5$ FIG 23.14$\}$

{NOTE 3 "LOOPS"}

$R_1 = \frac{\delta_1}{A_1\epsilon_1} = 0.231$ $R_2 = \frac{1}{A_1F_{12}} = 0.694$

$R_3 = \frac{\delta_2}{A_2\epsilon_2} = 0.039$ $R_4 = R_5 = \frac{1}{A_1F_{13}} = 0.694$

23.31 CONTINUED-

WRITING EQNS FOR LOOPS AS SHOWN:

$$E_{b1} - 0 = (I_1 + I_3)R_1 + I_1R_4$$

$$E_{b2} - 0 = (I_2 - I_3)R_3 + I_2R_5$$

$$E_{b1} - E_{b2} = (I_1 + I_3)R_1 + I_3R_2 + (I_3 - I_2)R_3$$

SUBSTITUTING VALUES & SOLVING SIMULTANEOUS EQNS.

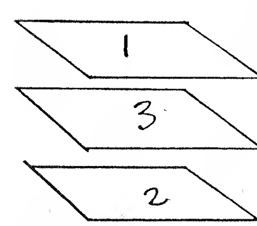
$I_1 = 59550$ $I_2 = 4695$ $I_3 = 42970$

$$q_{1NET} = I_1 + I_3 = \underline{\underline{102.5 \text{ kW}}}$$

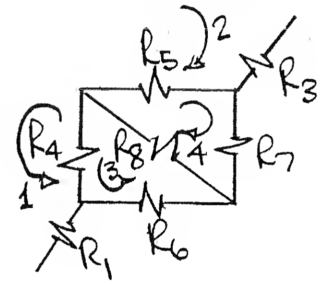
$$q_{12} = I_3 = \underline{\underline{42.97 \text{ kW}}}$$

{THESE RESULTS PRESUME NO HT TX FROM OTHER SIDES OF PLATES}

23.32



$T_1 = 1000 \text{ K}$
$A_1 = 2.88 \text{ m}^2$
$\epsilon_1 = 0.6$
$T_3 = ?$ $A = 2.88$
$\epsilon = 0.8$
$T_2 = 420 \text{ K}$
$A = 2.88$ $\epsilon = 0.9$



$R_1 = 0.231$

$R_3 = 0.694$

$R_4 = 1.157$

$R_5 = 1.157$

$R_6 = 0.496$

$R_7 = 0.496$

$R_8 = 0.579$

23.32 CONTINUED-

EQUATIONS FOR LOOPS SHOWN:

$$E_{b1}-0 = I_1R_1 + (I_1-I_3)R_4$$

$$E_{b2}-0 = I_2R_3 + (I_2-I_4)R_5$$

$$0 = (I_3-I_1)R_4 + I_3R_6 + (I_3+I_4)R_8$$

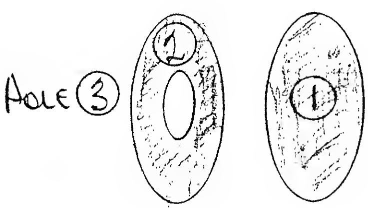
$$0 = (I_3+I_4)R_8 + I_4R_7 + (I_4-I_2)R_5$$

SOLVING: $I_1 = 62100$ $I_2 = 25500$

$I_3 = 16940$ $I_4 = 25680$

$$q_1 = \underline{\underline{62.1\ kW}}$$

23.33



FROM FIGS 23.14 & 23.15

$$F_{(2+3)-1} = F_{1-(2+3)} \cong 0.04$$

$$F_{31} \cong 0.04$$

$$F_{13} = 0.04\ \frac{(2.5)^2}{(4)^2} = 0.0156$$

$$F_{12} = 0.04 - 0.0156 = 0.0244$$

FOR BLACK DISKS:

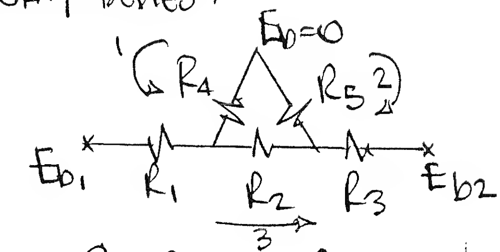
$$q_{12} = A_1F_{12}\sigma(T_1^4 - T_2^4)$$

$$= \frac{\pi}{4}\left(\frac{4}{12}\right)^2(0.0244)(0.1714) \times (9.6^4 - 6.7^4)$$

$$= \underline{\underline{2.36\ \text{Btu/hr}}} \qquad (a)$$

23.33 CONTINUED-

FOR GRAY BODIES:



$\epsilon_1 = 0.6$ $\epsilon_2 = 0.3$

$$R_1 = \frac{\rho_1}{A_1\epsilon_1} = \frac{0.4}{\frac{\pi}{4}\left(\frac{4}{12}\right)^2(0.6)} = 76.4$$

$$R_2 = \frac{1}{A_1F_{12}} = 470$$

$$R_3 = \frac{\rho_2}{A_2\epsilon_2} = 65$$

$$R_4 = \frac{1}{A_1F_{1-S}} = 11.74$$

$$R_5 = \frac{1}{A_2F_{2-S}} = 19.6$$

FOR LOOPS SHOWN:

$$E_{b1}-0 = I_1R_1 + I_1R_4 + I_3R_1$$

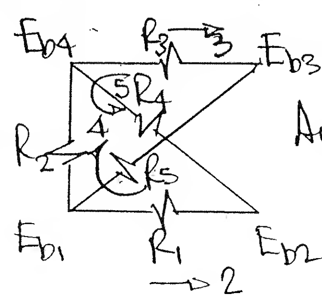
$$E_{b2}-0 = I_2R_3 + I_2R_5 - I_3R_3$$

$$E_{b1}-E_{b2} = (I_1+I_3)R_1 + I_3R_2 + (I_3-I_2)R_3$$

SOLVING SIMULTANEOUSLY:

$$q_{1-2} = I_3 = \underline{\underline{1.67\ \text{Btu/hr}}} \qquad (b)$$

23.34



ANALOG CIRCUIT

23.34 CONTINUED-

$R_1 = 470$ $E_{b1} = 1460$
$R_2 = 11.94$ $E_{b2} = 345$
$R_3 = 30.6$ $E_{b3} = 0$
$R_4 = 735$ 4 IS ADIABATIC
$R_5 = 19.6$

FOR BLACK SURFACES:

$$q_{12} = \frac{E_{b1} - E_{b2}}{R_{EQUIV,12}}$$

$$R_{EQUIV,12} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_4}} = 288$$

$$q_{12} = \frac{1460 - 345}{288} = \underline{\underline{3.87 \frac{\text{BTU}}{\text{HR}}}}$$

$$q_{\text{LOST THROUGH HOLE}} = q_{13} = \frac{E_{b1} - 0}{R_{EQUIV}}$$

$$R_{EQUIV} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_1 + R_2}} = 18.83$$

$$q_{LOST} = \frac{1460}{18.83} = \underline{\underline{77.5 \frac{\text{BTU}}{\text{HR}}}}$$

FOR GRAY SURFACES: $\epsilon_1 = 0.6$ $\epsilon_2 = 0.3$

ADDITIONAL RESISTANCES R_A, R_B

$$R_A = \frac{\delta_1}{A_1 \epsilon_1} = \frac{0.4}{\frac{\pi}{4}\left(\frac{4}{12}\right)^2 0.6} = 7.64$$

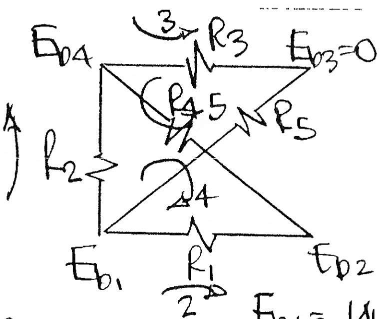
$$R_B = \frac{0.3}{\frac{\pi}{4}\left[\left(\frac{4}{12}\right)^2 - \left(\frac{2.5}{12}\right)^2\right] 0.7} = 8.06$$

23.34 CONTINUED-

$$q_{12} = \frac{1460 - 345}{288 + 7.64 + 8.06} = \underline{\underline{3.67 \frac{\text{BTU}}{\text{HR}}}}$$

$$q_{LOST} = \frac{1460}{18.83 + 7.64} = \underline{\underline{55.2 \frac{\text{BTU}}{\text{HR}}}}$$

23.35



$R_1 = 470$ $R_4 = 735$ $E_{b1} = 1460$
$R_2 = 11.94$ $R_5 = 19.6$ $E_{b2} = 345$
$R_3 = 30.6$ $E_{b3} = 0$
$E_{b4} = 738$

WRITING LOOP EQNS:

1: $E_{b1} - E_{b4} = R_2(I_1 + I_4 + I_5)$
2: $E_{b4} - 0 = R_3(I_3 - I_5)$
3: $E_{b1} - E_{b2} = R_1(I_2 - I_4)$
4: $0 = R_1(I_4 - I_2) + R_2(I_4 + I_1 - I_5) + I_4 R_4$
5: $0 = R_3(I_5 - I_3) + R_2(I_5 - I_1 - I_4) + I_5 R_5$

SOLVING:

$I_1 = 134.8$ $I_3 = 98.7$ $I_5 = 74.6$
$I_2 = 2.84$ $I_4 = 0.47$

$$q_{12} = I_2 = \underline{2.84 \text{ BTU/HR}}$$

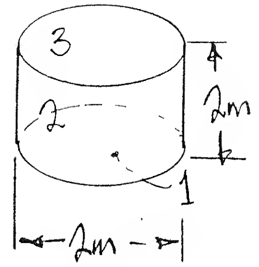
$$q_{LOST} = I_3 + I_5 = \underline{\underline{173.3 \text{ BTU/HR}}}$$

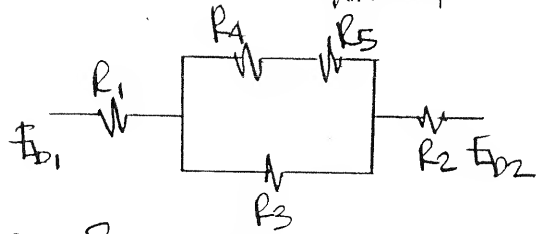
23.36

{FIG 23.14}

$F_{13} = 0.18$

$F_{12} = 0.82$





$$R_1 = \frac{\delta_1}{A_1 \epsilon_1} = \frac{0.69}{\frac{\pi}{4}(2)^2(0.31)} = 0.708\ m^{-2}$$

$$R_2 = \frac{\delta_2}{A_2 \epsilon_2} = 0.996\ m^{-2}$$

$$R_3 = \frac{1}{A_1 F_{12}} = 0.388\ m^{-2}$$

$$R_4 = \frac{1}{A_1 F_{13}} = 1.77\ m^{-2}$$

$$R_5 = 0.388\ m^{-2}$$

$$R_{EQUIV} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4 + R_5}} = \frac{1}{\frac{1}{0.388} + \frac{1}{1.77 + 0.388}}$$

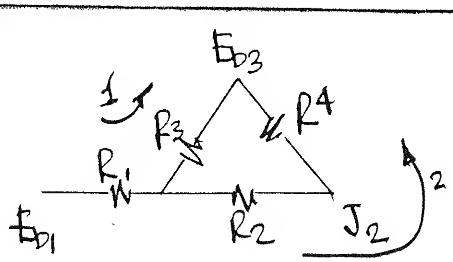
$$= 0.329$$

$$\Sigma R = 0.708 + 0.329 + 0.996 = 2.03\ m^{-2}$$

$$q = \frac{E_{b1} - E_{b2}}{\Sigma R} = \frac{5.676\left(7.55^4 - 3.95^4\right)}{2.03}$$

$$= \underline{\underline{8400\ W}} = \underline{\underline{8.4\ kW}}$$

23.37



{SURFACE 3 IS SURROUNDINGS}

23.37 CONTINUED.

$$A_1 = A_2 = \frac{\pi}{4}(0.15)^2 = 0.0177\ m^2$$

$$R_1 = \frac{\delta_1}{A_1 \epsilon_1} = \frac{0.2}{A(0.8)} = 14.12$$

$$\{\text{FIG } 23.14 \quad F_{12} \cong 0.37\} \Rightarrow F_{13} = 0.63$$

$$R_2 = \frac{1}{A_1 F_{12}} = 153$$

$$R_3 = \frac{1}{A_1 F_{13}} = 89.7$$

$$R_4 = \frac{1}{A_2 F_{23}} = 89.7$$

$$E_{b3} = \sigma(3.5)^4 = 852\ W/m^2$$

LOOP EQNS:

$$E_{b1} - E_{b3} = (I_1 + I_2)R_1 + I_1 R_3$$

$$E_{b1} - E_{b3} = (I_1 + I_2)R_1 + I_2(R_2 + R_4)$$

SOLVING: $I_1 = 2.706\, I_2$

$$I_1 + I_2 = 300$$

$$\therefore I_2 \cong 81 \qquad I_1 = 219$$

$$J_2 = E_{b2} + I_2 R_4$$

$$= 57.9 + 81(89.7) = 7319$$

$$J_1 = J_2 + I_2 R_2$$

$$J_1 = 7319 + 81(153) = 19710$$

$$E_{b1} = J_1 + 300(R_1)$$

$$= 19710 + 300(14.12)$$

$$= 23950$$

23.37 CONTINUED–

FINALLY: $T_1 = \left(\frac{E_{b1}}{\sigma}\right)^{1/4} = \underline{\underline{806\ K}}$ (a)

$T_2 = \left(\frac{J_2}{\sigma}\right)^{1/4} = \underline{\underline{599\ K}}$ (b)

$q_{\text{TO SURR}} = I_1 + I_2 = \underline{\underline{300\ W}}$ (c)

$q_{1-2} = I_2 = \underline{\underline{81\ W}}$ (d)

23.37 ALTERNATE SOLUTION USING EQNS 23.37 & 23.38

APPLYING THEM TO EACH NODE:

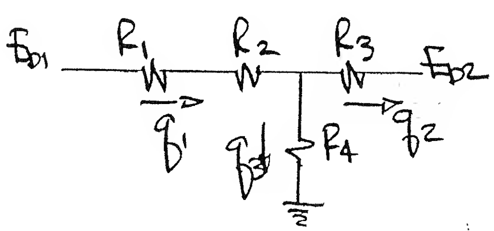
$$\frac{300}{A_1} = J_1 - F_{12}J_2 - F_{13}J_3$$

$$0 = J_2 - F_{21}J_1 - F_{23}J_3$$

$$E_{b3} = J_3$$

SOLVING THESE EQNS SIMULTANEOUSLY GIVES SAME RESULTS AS ABOVE

23.38 TEST SPECIMEN IS 1
TUBE IS 2
VIEWING PORT W



$R_1 = \frac{\delta_1}{A_1\epsilon_1} = \frac{0.2}{0.833(0.8)} = 0.30$

$R_2 = \frac{1}{A_1F_{12}} = \frac{1}{A_1} = 1.133$ $\{F_{12} \cong 1\}$

23.38 CONTINUED-

$R_3 = \frac{\delta_2}{A_2\epsilon_2} = \frac{0.77}{340(0.23)} = 9.85\times10^{-4}$

$R_4 = \frac{1}{A_W F_W}$

$A_1 = 0.883\ \text{IN}^2$ $A_W = 0.049\ \text{IN}^2$

$A_2 = \frac{\pi}{4}(16) + 8\pi + 4\pi(24) = 340\ \text{IN}^2$

$$q_1 = q_2 + q_3$$

$$q_2 = \frac{A_2\epsilon_2}{\delta_2}(J_2 - E_{b2})$$

$$q_3 = A_W J_2$$

$$q_1 = \frac{E_{b1}-J_1}{R_1+R_2} = A_1\epsilon_1(E_{b1}-J_1)$$

J_2 BECOMES:

$$J_2 = \frac{E_{b1} + \frac{A_1}{A_2}\frac{\epsilon_2}{\delta_2\epsilon_1}E_{b2}}{1+\frac{A_2}{A_1}\frac{\epsilon_2}{\delta_2\epsilon_1}+\frac{A_W}{A_1\epsilon_1}}$$

$E_{b1} = 131{,}500\ \frac{\text{BTU}}{\text{HR FT}^2}$ $E_{b2} = 16$

$\Rightarrow J_2 = 344\ \text{BTU/HR FT}^2$

$q_1 = \frac{0.883}{144}(0.2)(131{,}500 - 344)$

$= \underline{\underline{161\ \frac{\text{BTU}}{\text{HR}}}}$ (b)

~ FROM SPECIMEN

$q_3 = \frac{0.049}{144}(344) = \underline{\underline{0.177\ \frac{\text{BTU}}{\text{HR}}}}$ (c)

~ LOSS THROUGH WINDOW

$F_{1W} \cong \frac{A_W F_{W1}}{A_1} = \frac{\pi/4\,(1/16)}{2\pi\,(144)} \cong \underline{5\times10^{-5}}$ (a)

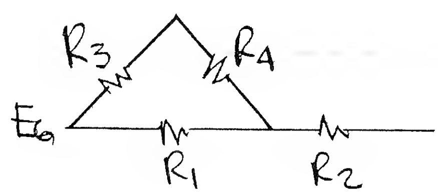
23.39 $q_{\text{GAS-WALL DIRECT}}$
$$= A_1 F_{1G} \alpha_G \sigma (T_G^4 - T_1^4)$$

$q_{\text{GAS TO RERADIATING WALLS}}$
$$= A_2 F_{2G} \alpha_G \sigma (T_G - T_2^4)$$

$q_{\text{RERADIATING WALLS TO (1)}}$
$$= A_1 F_{12} \tau_G \sigma (T_2^4 - T_1^4)$$

$$q_{G2} = q_{21} = q_R = \frac{\sigma (T_G^4 - T_1^4)}{\dfrac{1}{A_1 F_{12} \tau_G} + \dfrac{1}{A_2 F_{2G} \alpha_G}}$$

$$q_{\text{TOTAL TO 1}} = q_{G1} + q_R$$



$$L = \frac{3.4(0.2)(0.2)(1)}{4(0.2)(1)} = 0.17 \text{ m}$$

$p = 1$ ATM $\qquad \alpha_G = 0.22$

$p_L = 0.558$ ATM-FT $\qquad \tau_G = 0.78$

$$R_1 = \frac{1}{0.2(1)(0.22)} = 22.7$$

$$R_2 = \frac{0.2}{0.2(1)(0.8)} = 1.25$$

$$R_3 = \frac{1}{3(0.2)(1)(1)(0.22)} = 7.58$$

$$R_4 = \frac{1}{0.2(1)(1)(0.78)} = 6.41$$

$$R_{\text{EQUIV}} = \frac{1}{1/R_1 + \frac{1}{R_3 + R_4}} = 8.66$$

23.39 CONTINUED-

$$\sum R = 8.66 + 1.25 = 9.91$$

$$q = \frac{5.676\,(6^4 - 4.2^4)}{9.91} = \underline{\underline{564 \text{ W}}}$$
PER M

23.40 $$q_{NET} = \sigma A (\epsilon_G T_G^4 - \alpha_G T_W^4)$$

$$A = 4\pi r^2 = \pi (3\text{m})^2 = 28.27$$

$T_G = 1000$ K $\qquad T_W = 600$ K

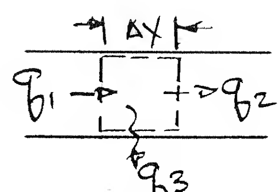
$L = 2/3\, D = 2$ m

$pL = 0.15(5)(6.56) = 4.92$ ATM-FT

$\alpha_G = 0.18 \qquad \epsilon_G = 0.22$

$$q_{NET} = 5.676(9\pi)\left[0.22(10^4) - 0.18(6^4)\right]$$
$$= \underline{\underline{316 \text{ kW}}}$$

23.41



$$q_2 - q_1 = A \rho v c_p \left(T|_{x+\Delta x} - T|_x\right)$$
$$q_3 = h P \Delta x\, (T - T_W)$$
$$+ A_W F_{WG} \sigma \epsilon_W \left[\epsilon_G T_G^4 - \alpha_G T_W^4\right]$$

$$q_1 = q_2 + q_3$$

$$A v \rho c_p \frac{T_{x+\Delta x} - T_x}{\Delta x} + hP(T - T_W)$$
$$+ P F_{WG} \sigma \epsilon_W (\epsilon_G T_G^4 - \alpha_G T_W^4) = 0$$

23.41 CONTINUED –

IN LIMIT AS $\Delta x \to 0$:

$$A\rho v c_p \frac{dT}{dx} + hP(T-T_w) + P\epsilon_w\sigma(\epsilon_G T_G^4 - \alpha_G T_w^4) = 0$$

$$P_c = 0.20 \qquad L = \frac{3.4\,WD^2}{4WD} = 0.425$$

$$P_cL = 0.085 \sim \text{@ } 2000\,F \quad \epsilon_G \cong 0.035$$
$$\text{@ } 1000 \cdot \quad \epsilon_G \cong 0.065$$

$$T_w = 1260\,R \qquad \alpha_G = 0.071$$

$$\frac{dT}{dx} = \left[-\frac{hP}{\rho v c_p}(T-T_w) - \frac{P\epsilon_w\sigma}{\rho v c_p}(\epsilon_G T_G^4 - \alpha_G T_w^4)\right]\frac{1}{A}$$

$$= \left\{-\frac{1.5(2)(T-1260)}{0.4(0.28)} - \frac{2(0.9)(0.1714)}{(0.4)(0.28)} \times \left[\epsilon_G\left(\frac{T_G}{100}\right)^4 - 0.071(12.6)^4\right]\right\}\frac{4}{3600}$$

$$\frac{dT}{dx} = \frac{1}{900}\left\{-26.8(T-1260) - 2.75\left[\epsilon_G\left(\frac{T_G}{100}\right)^4 - 1772\right]\right\}$$

$$dx = -\left[\frac{1}{-26.8(T-1260) - 2.75\,\epsilon_G\left(\frac{T_G}{100}\right)^4 + 4870}\right] \times 900\,dT$$

BY GRAPHICAL INTEGRATION

$$x = \int_{1000}^{2000} [\;]\,900\,dT = \underline{\underline{35.2\,FT}} \quad (a)$$

$$q_{TOTAL} = \rho A v c_p \Delta T$$
$$= 0.4\left(\tfrac{1}{4}\right)(0.28)(1000)$$
$$= 28 \text{ Btu/s}$$

23.41 CONTINUED –

$$q_{CONV} = hP(x)(T-800)$$
$$= \frac{1.5(2)(x)(T-800)}{3600} = x\,\frac{T-800}{1200}$$

T_{AVG}	x	$q_{CONV,\,i}$
1900	3.2	2.93
1700	4.0	3.0
1500	5.4	3.15
1300	8.0	3.33
1100	14.6	3.65

$$q_{CONV} = \sum q_i = 16.06 \text{ Btu/s}$$

$$q_{RAD} = 28 - 16.06 = 11.94 \ \frac{\text{Btu}}{\text{s}}$$

$$\text{RADIANT FRACTION} = \frac{11.94}{28} = 0.43 \quad (b)$$

FOR v DOUBLED:

$$dx = \left[\frac{900}{-13.4(T-1260) - 1.375\left[\epsilon_G\left(\frac{T}{100}\right)^4 - 1772\right]}\right]dT$$

INTEGRATE GRAPHICALLY UNTIL $x = 35.2$

AT THIS LOCATION $\underline{\underline{T = 1265\,F}}$

CHAPTER 24

24.1 BASIS 1 gmole LNG

	gmol	MW	g	WT FRACTION
CH_4	0.935	16	14.96	0.871
C_2H_6	0.046	30	1.38	0.080
C_3H_8	0.012	44	0.528	0.031
CO_2	0.007	44	0.308	0.018
			17.176	1.00

WT FRACTION ETHANE - $\underline{0.080}$ a)

AVG. M.WT - $\underline{17.176\ g/mol}$ b)

DENSITY:

$$\rho = \frac{PM}{RT} = \frac{1.4\times10^5\,(17.176)}{8.314(207)}$$

$$= 1397\ g/m^3 = \underline{1.397\ kg/m^3} \quad c)$$

$$P_{CH_4} = y_{CH_4}P = (0.935)(1.4\times10^5)$$

$$= \underline{131\ kPa} \quad d)$$

MASS FRACTION CO_2

$$= \frac{0.308}{17.176} = \underline{0.0179} \quad e)$$

24.2 BASIS - 1 kg MOLE

	kg MOLE	M.W.	kg	WT FRACTION
$SiCl_4$	0.40	32.12	12.85	0.914
H_2	0.60	2.02	1.21	0.086
			14.06	1.0

↑ a

24.2 CONTINUED -

$$\underline{M.WT. = 14.06\ kg/kg\ mole} \quad b)$$

$$C_{A,\,SiCl_2} = y_A C$$

$$P = \frac{60}{760}(1.013\times10^5) = 7.99\times10^3\ Pa$$

$$C = \frac{P}{RT} = \frac{7.99\times10^3}{8.314(900)} = 1.068\ mole/m^3$$

$$C_A = (0.40)(1.068) = \underline{0.427\ mole/m^3} \quad c)$$

24.3 BASIS 1 g MOLE

	y	MOLE	M.WT.	g
O_2	0.21	0.21	32	6.72
N_2	0.79	0.79	28	22.12
		1.0		28.84

MOLE FRACTION OF O_2 = $\underline{0.21}$ a)

VOLUME " " " = $\underline{0.21}$ b)

WT OF MIXTURE = $\underline{28.84\ g}$ c)

$$Vol/mole = \frac{RT}{P} = \frac{8.314(400)}{1.013\times10^5}$$

$$= 0.0328\ m^3/mole$$

$$\rho_{O_2} = 6.72/0.0328 = \underline{204.9\ g/m^3} \quad d)$$

$$\rho_{N_2} = 22.12/0.0328 = \underline{674.4\ "} \quad e)$$

$$\rho_{mix} = \underline{879\ "} \quad f)$$

M.WT OF MIXTURE = $\underline{28.84}$ g)

24.4

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az}+N_{Bz})$$

$$N_{Bz} = -cD_{BA}\frac{dy_B}{dz} + y_B(N_{Az}+N_{Bz})$$

ADDING:

$$N_{Az}+N_{Bz} = -cD_{AB}\frac{dy_A}{dz} - cD_{BA}\frac{dy_B}{dz}$$
$$+ \underbrace{(y_A+y_B)}_{1}(N_{Az}+N_{Bz})$$

$$cD_{AB}\frac{dy_A}{dz} + cD_{BA}\frac{dy_B}{dz} = 0$$

$$y_A + y_B = 1$$

$$\therefore \frac{dy_A}{dz} + \frac{dy_B}{dz} = 0 \quad \therefore \frac{dy_A}{dz} = -\frac{dy_B}{dz}$$

GIVING $\underline{D_{AB} = D_{BA}}$

IN HIRSCHFELDER EQN:

$$\sigma_{AB}^2,\ \Omega\ ,\ \left[\frac{1}{M_A}+\frac{1}{M_B}\right]^{1/2}$$

WILL BE THE SAME FOR D_{AB}, D_{BA}

$\therefore$ AGREEMENT — Q.E.D.

24.5

$$\vec{N}_A = -cD_{AB}\nabla y_A + y_A(\vec{N}_A+\vec{N}_B)$$

c = CONST. ; MULTIPLY BY M_A

$$\vec{N}_A M_A = -D_{AB}M_A\nabla c_A + y_A M_A(\vec{N}_A+\vec{N}_B)$$

$$\omega_A = \frac{y_A M_A}{x_A M_A + x_B M_B} = \frac{y_A M_A}{M_{AVG}}$$

$$\vec{n}_A = -D_{AB}\nabla \rho_A + \omega_A(\vec{n}_A+\vec{n}_B)$$

$$\therefore \underline{\vec{n}_A = -D_{AB}\rho\nabla\omega_A + \omega_A(\vec{n}_A+\vec{n}_B)}$$

(a)

24.5 CONTINUED

$$\vec{N}_A = -D_{AB}\nabla c_A + c_A\left[\frac{c_A\vec{v}_A + c_B\vec{v}_B}{c}\right]$$

$$c_A\vec{v}_A = -D_{AB}\nabla c_A + c_A\vec{V}$$

$$c_A(\vec{v}-\vec{V}) = -D_{AB}\nabla c_A$$

$$\therefore \underline{\vec{J}_A = -D_{AB}\nabla c_A} \qquad b)$$

24.6

$$\vec{N}_A + \vec{N}_B = \left[c_A\vec{v}_A + c_B\vec{v}_B\right]\frac{c}{c}$$

$$= \underline{c\vec{V}} \qquad a)$$

$$n_A + n_B = \left(\rho_A\vec{v}_A + \rho_B\vec{v}_B\right)\frac{\rho}{\rho}$$

$$= \underline{\rho\vec{v}} \qquad b)$$

$$\vec{J}_A + \vec{J}_B = -D_{AB}\rho\nabla\omega_A - D_{AB}\rho\nabla\omega_B$$

AS $\omega_A + \omega_B = 1$; $\nabla\omega_A + \nabla\omega_B = 0$

$$\therefore \underline{\vec{j}_A + \vec{j}_B = 0} \qquad c)$$

24.7

$$-\frac{dc_A}{dz} = \beta_{AB}\frac{\rho_A}{M_A}\frac{\rho_B}{M_B}(v_{Az}-v_{Bz})$$
$$+ \beta_{AC}\frac{\rho_A}{M_A}\frac{\rho_C}{M_C}(v_{Az}-v_{Cz}) + \cdots$$

AS $c = \frac{\rho_i}{M_i} = \frac{p_i}{RT}$

$$-\frac{1}{RT}\frac{dp_A}{dz} = \beta_{AB}\left[\frac{p_B}{RT}c_A v_{Az} - \frac{p_A}{RT}c_B v_{Bz}\right]$$
$$+ \beta_{AC}\left[\frac{p_C}{RT}c_A v_{Az} - \frac{p_A}{RT}c_C v_{Cz}\right] + \cdots$$

24.7 CONTINUED -

$$-\frac{1}{RT}\frac{dp_A}{dz} = \frac{\beta_{AB}}{RT}\left[p_B N_{Az} - p_A N_{Bz}\right] + \frac{\beta_{AC}}{RT}\left[p_C N_{Az} - p_A N_{Cz}\right] + \cdots$$

$$-\frac{dp_A}{dz} = \left[\beta_{AB} p_B + \beta_{AC} p_C + \cdots\right] N_{Az} - p_A\left[\beta_{AB} N_{Bz} + \beta_{AC} N_{Cz} + \cdots\right]$$

LET $D_{Ai} = \frac{RT}{\beta_{Ai} P}$ OR $\beta_{Ai} = \frac{RT}{D_{Ai} P}$

$$-\frac{dp_A}{dz} = \left\{\frac{RT}{P}\left[\frac{p_B}{D_{AB}} + \frac{p_C}{D_{AC}} + \cdots\right] N_{Az} - \frac{p_A RT}{P}\left[\frac{N_{Bz}}{D_{AB}} + \frac{N_{Cz}}{D_{AC}} + \cdots\right]\right.$$

FOR A DIFFUSING THROUGH NON-DIFFUSING B, C, D, ...

$$N_{Bz} = N_{Cz} = N_{Dz} = \cdots = 0$$

GIVING
$$-\frac{dp_A}{dz} = \frac{RT}{P}\left(\frac{p_B}{D_{AB}} + \frac{p_C}{D_{AC}} + \cdots\right) N_{Az}$$

$$\text{\&} \quad \frac{P}{RT N_{Az}}\left(-\frac{dp_A}{dz}\right) = \frac{p_B}{D_{AB}} + \frac{p_C}{D_{AC}} + \cdots \qquad (1)$$

NOW - CONSIDER A BINARY CASE WITH $N_{Bz} = 0$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A N_{Az}$$

24.7 CONTINUED -

$$N_{Az} = -\frac{P}{RT}\frac{D_{AB}}{1-y_A}\frac{dy_A}{dz}$$

$$= -\frac{P}{RT}\frac{D_{AB}}{P-p_A}\frac{dp_A}{dz}$$

$$\text{OR} \quad \frac{P-p_A}{D_{AB}} = \frac{P}{RT}\left[\frac{-dp_A/dz}{N_{Az}}\right] \qquad (2)$$

COMBINING (1) & (2)

$$\frac{P-p_A}{D_{A-mix}} = \frac{p_B}{D_{AB}} + \frac{p_C}{D_{AC}} + \frac{p_D}{D_{AD}} + \cdots$$

$$\text{\&} \quad D_{A-mix} = \frac{P-p_A}{\frac{p_B}{D_{AB}} + \frac{p_C}{D_{AC}} + \frac{p_D}{D_{AD}} + \cdots}$$

DIVIDING NUMERATOR & DENOMINATOR BY P WE GET

$$D_{A-mix} = \frac{1-y_A}{y_B/D_{AB} + y_C/D_{AC} + y_D/D_{AD} + \cdots}$$

DIVIDING NUMERATOR & DENOMINATOR BY $1-y_A$ & DESIGNATING $y_i' = y_i/1-y_A$

WE HAVE, FINALLY,

$$D_{A-mix} = \frac{1}{y_B'/D_{AB} + y_C'/D_{AC} + y_D'/D_{AD} + \cdots}$$

24.8 CO_2 IN AIR @ 310 K, 1.5×10^5 Pa

APPENDIX J: $D_{AB}P = 1.378\ m^2/s\ Pa$

$$D_{AB}@T_2P_2 = D_{AB}\Big|_{T,P}\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)^{3/2}\frac{\Omega_{D\,T_1}}{\Omega_{D\,T_2}}$$

CO_2: $\epsilon_i/k = 190$

AIR $\epsilon_i/k = 97$

$$\epsilon_{AB}/k = \sqrt{190(97)} = 135.76$$

T_1: $\frac{Tk}{\epsilon_{AB}} = \frac{273}{135.76} = 2.011$ $\Omega_D = 1.673$

T_2: $\frac{Tk}{\epsilon_{AB}} = \frac{310}{135.76} = 2.283$ $\Omega_D = 1.028$

$$D_{AB} = \frac{1.378}{1.5\times10^5}\left(\frac{310}{273}\right)^{3/2}\frac{1.673}{1.028}$$

$$= \underline{1.16\times10^{-5}\ m^2/s} \quad (a)$$

ETHANOL IN AIR @ 325 K 2×10^5 Pa

SAME PROCEDURE AS ABOVE -

$$D_{AB}P\Big|_{298} = 1.337\ m^2/s\ Pa$$

$\epsilon_{AB}/k = 194.7$ $\Omega_{D\,T_1} = 1.188$ $\Omega_{D\,T_2} = 1.148$

$$D_{AB} = \frac{1.377}{2\times10^5}\left(\frac{325}{298}\right)^{3/2}\left(\frac{1.188}{1.148}\right)$$

$$= \underline{7.88\times10^{-6}\ m^2/s} \quad (b)$$

24.8 CONTINUED -

CO IN AIR @ 310 K, 1.5×10^5 Pa

MUST USE HIRSCHFELDER EQN

$$D_{AB} = \frac{0.001858\,T^{3/2}\left[1/M_A + 1/M_B\right]^{1/2}}{P\,\sigma_{AB}^2\,\Omega_D}$$

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.265$

$P = 1.4807$ ATM

$\sigma_{AB}^2 = 12.985$ $\epsilon_{AB}/k = 103.29$

$Tk/\epsilon_{AB} = 3.0$ $\Omega_D = 0.949$

SUBSTITUTING & SOLVING:

$$\underline{D_{AB} = 1.47\times10^{-5}\ m^2/s} \quad (c)$$

CCl_4 IN AIR @ 298 K, 1.913×10^5 Pa

AGAIN - HIRSHFELDER EQN - SEE PART (c)

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.202$

$P = 1.888$ ATM

$\sigma_{AB}^2 = 22.553$ $\epsilon_{AB}/k = 178.1$

$Tk/\epsilon_{AB} = 1.67$ $\Omega_D = 1.148$

SUBSTITUTING & SOLVING:

$$\underline{D_{AB} = 3.95\times10^{-6}\ m^2/s} \quad (d)$$

24.9 n Butane - i Butane @ 673 K, 2.0 atm

Use Hirshfelder Eqn - See Prob 24.8

Values: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.1857$

$\sigma_{AB}^2 = 26.718 \quad \epsilon_{AB}/k = 358.2$

$Tk/\epsilon_{AB} = 1.88 \quad \Omega_D = 1.098$

Substituting & Solving:

$\underline{D_{AB} = 1.03 \times 10^{-5}\ m^2/s}$

Fuller-Schettler-Giddings

$$D_{AB} = \frac{10^{-3} T^{1.75} \left(1/M_A + 1/M_B\right)^{1/2}}{P\left[(\Sigma v_A)^{1/3} + (\Sigma v_B)^{1/3}\right]^2}$$

$\Sigma v_A = \Sigma v_B = \left[4(14.8) + 10(3.7)\right]$
$= 96.2$

Substituting Values & Solving

$\underline{D_{AB} = 9.9 \times 10^{-6}\ m^2/s}$

24.10 CH_4 in Air, 373 K, 1.5×10^5 Pa

Hirshfelder Eqn - See Prob 24.8

Values: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.311$

$\sigma_{AB}^2 = 13.834 \quad \epsilon_{AB}/k = 115.07$

$Tk/\epsilon_{AB} = 3.24 \quad \Omega_D = 0.930$

Substituting & Solving:

$\underline{D_{AB} = 2.19 \times 10^{-5}\ m^2/s}$ (a)

24.10 Continued -

Wilke Eqn: $$D_{A-mix} = \frac{1}{\frac{0.21}{D_{A-O_2}} + \frac{0.79}{D_{A-N_2}}}$$

$A = CH_4$

Must use Hirshfelder Eqn for D_{Ai} - See Prob 24.8 for Eqn.

For D_{A-O_2}:

Values: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.306$

$\sigma_{AB}^2 = 13.159 \quad \epsilon_{AB}/k = 124.19$

$Tk/\epsilon_{AB} = 3.0 \quad \Omega_D = 0.949$

Substituting $D_{A-O_2} = 2.22 \times 10^{-5}\ m^2/s$

For D_{A-N_2}

Values: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.313$

$\sigma_{AB}^2 = 14.074 \quad \epsilon_{AB}/k = 111.76$

$\frac{Tk}{\epsilon_{AB}} = 3.337 \quad \Omega_D = 0.923$

Substituting: $D_{A-N_2} = 2.19 \times 10^{-5}\ m^2/s$

For Mixture: (Wilke Eqn)

$$D_{A-AIR} = \frac{1 \quad (10^{-5})}{0.21/2.22 + 0.79/2.19}$$

$$= \underline{2.19 \times 10^{-5}\ m^2/s} \qquad (b)$$

24.11 NH_3-AIR 373 K 1.013×10^5 Pa

BROKAW METHOD: $\mu_{P,NH_3} = 1.46$ DEBYE

$$\delta_{NH_3} = \frac{1.94\times10^3\,(1.46)^2}{25\,(239.7)} = 0.690$$

$\delta_{AIR} = 0 \quad \therefore \quad \delta_{AB} = 0$

VALUES: $\epsilon_{AB}/K = 210.75$

$$T^* = \frac{kT}{\epsilon_{AB}} = 1.77$$

EQN 24-46

$$\Omega_{D_0} = \frac{1.06036}{(T^*)^B} + \frac{0.19300}{\exp(DT^*)} + \frac{1.03587}{\exp(FT^*)} + \frac{1.03587}{\exp(HT^*)} = 1.1238$$

$\delta_{NH_3} = 2.900 \quad \sigma_{AB}^2 = 10.49$

SUBSTITUTING INTO H.E. & SOLVING:

$$\underline{D_{AB} = 3.47\times10^{-5}\ m^2/s}$$

FROM APPENDIX: @ 273 K

$D_{AB} = 2.006\ m^2/s\ Pa$

$T^* = Tk/\epsilon_{AB} = 1.295$

$$\Omega_{D_{T1}} = \frac{1.06036}{1.295} + \cdots \left\{ \begin{matrix}\text{SEE PART 1} \\ \text{FOR FORM}\end{matrix} \right\}$$

$= 1.272$

$$D_{AB} = \frac{(2.006)}{1.015\times10^5}\left(\frac{373}{273}\right)^{3/2}\left(\frac{1.272}{1.1238}\right)$$

$$= \underline{3.57\times10^{-5}\ m^2/s}$$

24.12 $SiCl_4$ IN H_2 1073 K, 1.5×10^5 Pa

HIRSCHFELDER EQN - SEE PROB 24.8

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.7084$

$$P = \frac{1.5\times10^5}{1.015\times10^5} = 1.478\ atm$$

$\sigma_{AB}^2 = 16.193 \qquad \epsilon_{AB}/K = 109.19$

$Tk/\epsilon_{AB} = 9.83 \qquad \Omega_D = 0.7446$

SUBSTITUTING INTO H.E. & SOLVING:

$$\underline{D_{AB} = 2.596\times10^{-4}\ m^2/s} \quad (a)$$

FOR $SiCl_4$ in HCl - SAME P & T

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.182$

$\sigma_{AB}^2 = 17.58 \qquad \epsilon_{AB}/K = 359$

$Tk/\epsilon_{AB} = 2.99 \qquad \Omega_D = 0.9586$

SUBSTITUTING INTO H.E. & SOLVING:

$$D_{AC} = 4.77\times10^{-5}\ m^2/s$$

FOR MIXTURE - $y_{SiCl_4} = 0.40 \quad y_{H_2} = 0.40$

$y_{HCl} = 0.20$

$$y'_{H_2} = \frac{0.4}{0.6} = 0.667, \quad y'_{HCl} = \frac{0.2}{0.6} = 0.333$$

$$D_{SiCl_4\text{-}mix} = \frac{1\times10^{-5}}{\frac{0.667}{25.96} + \frac{0.333}{4.777}}$$

$$= \underline{10.482\times10^{-5}\ m^2/s} \quad (b)$$

24.13 H_2S IN MIXTURE 350K, 1 ATM

$A = H_2S$ $B = N_2$ $C = SO_2$

FOR A INTO B: USE H.E. (PROB. 24.8)

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.255$

$\sigma_{AB}^2 = 14.27$ $\epsilon_{AB}/k = 162.2$

$kT/\epsilon_{AB} = 2.158$ $\Omega_D = 1.048$

SUBSTITUTING & SOLVING:

$D_{AB} = 2.07 \times 10^{-5}\ m^2/s$

FOR A INTO C: -- SAME PROCEDURE

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.212$

$\sigma_{AB}^2 = 16.66$ $\epsilon_{AB}/k = 269.1$

$kT/\epsilon_{AB} = 1.30$ $\Omega_D = 1.273$

$D_{AC} = 1.20 \times 10^{-5}\ m^2/s$

MIXTURE: $y_A = 0.03$ $y_B = 0.92$ $y_C = 0.05$

$y'_B = 0.948$ $y'_C = 0.0515$

$$D_{H_2S-MIX} = \frac{1\times10^5}{\frac{0.948}{2.07} + \frac{0.0515}{1.20}} = \underline{2.00 \times 10^{-5}\ m^2/s}$$

24.14 $D_{AB} = \dfrac{kT}{6\pi r \mu_B}$ ~ $r = \dfrac{kT}{6\pi D_{AB}\mu_B}$

GIVEN $D_{AB} = 5.94 \times 10^{-11}\ m^2/s$

$T = 293\,K$ $\mu_B = 993 \times 10^{-6}\ Pa\cdot s$

SUBSTITUTING: $\underline{r = 3.637\ nm}$

24.15 O_2 IN C_2H_5OH 293 K

FOR C_2H_5OH – $\mu = 1.25\ cp$ $M_B = 46$ $\phi_B = 1.5$

$V_{O_2} = 25.6$

$$D_{AB_L} = \frac{T}{\mu_B}\frac{(7.4\times10^{-8})(\phi_B M_B)^{1/2}}{V_A^{0.6}} \quad (a)$$

SUBSTITUTING VALUES – $D_{AB} = \underline{2.06 \times 10^{-9}\ m^2/s}$

CH_3OH IN H_2O, 288K

$\mu_B = 1.14\ cp$ $M_B = 18$ $\phi_B = 2.26$

$V_{CH_3OH} = 14.8 + 4(3.7) + 7.4 = 37$

SUBSTITUTION INTO EQ (24-52) {SEE PART A}

$D_{AB} = \underline{1.336 \times 10^{-9}\ m^2/s}$ (b)

H_2O IN CH_3OH 288 K

$\mu_B = 0.62\ cp$ $M_B = 32$ $\phi_B = 1.9$

$V_A = 18.9$

~ SUBSTITUTING INTO EQN (24-52)

$D_{AB} = \underline{4.59 \times 10^{-9}\ m^2/s}$ (c)

C_3H_7OH IN H_2O 288 K

$\mu_B = 1.14\ cp$ $M_B = 18$ $\phi_B = 2.26$

~ SUBSTITUTION INTO EQN (24-52)

$\underline{D_{AB} = 7.37 \times 10^{-10}\ m^2/s}$ (d)

FROM TEXT – APPENDIX J

$\underline{D_{AB} = 7.7 \times 10^{-10}\ m^2/s}$

24.16 Cl_2 IN H_2O 289 K

$\mu_B = 1.13$ cp $M_B = 18$ $\phi_B = 2.26$
$V_A = 48.4$

SUBSTITUTION INTO EQN (24-52)

$\underline{D_{AB} = 1.177 \times 10^{-9}\ m^2/s}$

USING EQN (24-53)

$D_{AB} = (13.26 \times 10^{-5})\,\mu_B^{-1.14}\,V_A^{-0.589}$

$= \underline{1.174 \times 10^{-9}\ m^2/s}$

APPENDIX J: $D_{AB} = 1.26 \times 10^{-9}\ m^2/s$

24.17 C_6H_6 IN C_2H_5OH 288 K

$\mu_B = 1.3$ cp $M_B = 46$ $\phi_B = 1.5$
$V_A = 96$

SUBSTITUTING INTO EQN (24-52)

$\underline{D_{AB} = 8.81 \times 10^{-10}\ m^2/s}$

C_2H_5OH INTO C_6H_6 -

$\mu_B = 0.75$ cp $M_B = 78$ $\phi_B = 1.0$
$V_A = 59.2$

SUBSTITUTING INTO EQ. (24-52)

$\underline{D_{AB} = 2.17 \times 10^{-9}\ m^2/s}$

24.18 O_2 IN $H_2O(l)$ 288 K

EQN (24-52) -- $\mu_B = 1.14$ cp
$M_B = 18$ $\phi_B = 2.26$ $V_A = 25.6$

SUBSTITUTING: $\underline{D_{AB} = 1.70 \times 10^{-9}\ m^2/s}$

EQN. (24-53) $\underline{D_{AB} = 1.69 \times 10^{-9}\ m^2/s}$

24.19 P IN Si(s)

@ 1316 K $D_{AB} = 1 \times 10^{-17}\ m^2/s$
1408 K $D_{AB} = 1 \times 10^{-16}\ m^2/s$

$D_i = D_o e^{-Q/RT}$

$\ln D_i = \ln D_o - Q/RT$

SUBSTITUTING: $Q/R = 4.645 \times 10^4$
$D_o = 213.31$

@ 1373 K $\ln D_i = -38.47$

$\underline{D_{AB} = 4.32 \times 10^{-17}\ m^2/s}$

24.20 C IN FCC Fe 1000 K

$D_o = 2.5 \times 10^{-6}\ m^2/s$ $Q = 144.2$ kJ/mol

$D_i = D_o e^{-Q/RT} = \underline{7.34 \times 10^{-10}\ m^2/s}$

C IN BCC Fe

$D_o = 2.0 \times 10^{-6}\ m^2/s$ $Q = 84.1$ kJ/mol

$D_i = D_o e^{-Q/RT} = \underline{8.09 \times 10^{-9}\ m^2/s}$

24.21 EFFECTIVE DIFFUSION OF H_2 IN N_2 373 K, 1 ATM

STRAIGHT PORES ($D = 100\,\text{Å}$) IN PARALLEL

$d_p = 1\times10^{-8}\,m$

EQN (24-58) $D_{KA} = 4850\, d_p \sqrt{T/M_A}$

$= 4850(10^{-8})\left[\frac{373}{2.015}\right]^{1/2}$

$= 6.6\times10^{-8}\,m^2/s$

AT 288 K $D_{AB} = 0.743\times10^{-6}\,m^2/s$

AT 373 K $D_{AB} = 1.095\times10^{-6}\,m^2/s$

ASSUMING DILUTE N_2

$$D_{EFFECTIVE} = \frac{1\times10^{-6}}{1/1.095 + 1/0.066}$$

$= 0.062\times10^{-6}\,m^2/s$ (a)

RANDOM PORES – VOID FRACTION = 0.4

$D'_{EFF} = \varepsilon^2 D_E = (0.4)^2(0.062\times10^{-6})$

$= 9.92\times10^{-7}\,m^2/s$ (b)

RANDOM PORES $1000\,\text{Å}$ $\varepsilon = 0.4$

EQN. (24-58) $D_{KA} = 0.66\times10^{-6}\,m^2/s$

$D_{AE} = 0.383\times10^{-6}\,m^2/s$

$D'_{AE} = (0.4)^2(0.383\times10^{-6})$

$= 0.0614\times10^{-6}\,m^2/s$ (c)

24.21 CONTINUED · $d_p = 20{,}000\,\text{Å}$ PARALLEL

EQN (24-58) $D_{KA} = 1.3197\times10^{-5}\,m^2/s$

$$D_{AE} = \frac{1\times10^{-6}}{1/1.095 + 1/13.197}$$

$= 1.011\times10^{-6}\,m^2/s$ (d)

24.22 $A = CH_4$ ~ 20 MOL %

$B = H_2O$ ~ 80 " "

USE H.F. – EQN. (24-33)

VALUES $\left[1/M_A + 1/M_B\right]^{1/2} = 0.3436$

$\sigma_{AB}^2 = 10.468$ $\quad \epsilon_{AB}/\kappa = 220.4$

$\kappa T/\epsilon_{AB} = 2.60$ $\quad \Omega_D = 0.9878$

SUBSTITUTING: $D_{AB} = 1.694\times10^{-6}\,m^2/s$

$$D_{AE} = \frac{1}{1/D_{AB} + 1/D_{AK}}$$

$D_{AK} = 4850(2\times10^{-7}\,m)\sqrt{\frac{573}{16}}$

$= 0.580\times10^{-6}\,m^2/s$

SUBSTITUTING: $D_{AE} = 0.432\times10^{-6}\,m^2/s$

KNUDSEN DIFFUSION IS ~75% OF TOTAL

24.23 H_2O INTO CO 353 K 2 ATM

$A = H_2O \quad B = CO$

D_{AB} @ 273 K, 1 ATM $= 0.651 \times 10^{-4}\ m^2/s$

$$\text{AT}\left\{\begin{matrix}353\ K\\ 2\ ATM\end{matrix}\right\} D_{AB} = 0.651\left(\frac{353}{273}\right)^{3/2}\left(\frac{1}{2}\right)$$

$$= 0.479 \times 10^{-4}\ m^2/s$$

$$D'_{AE} = 0.036 \times 10^{-4}\ m^2/s$$

$$= (0.3)^2 D_{AE}$$

$$D_{AE} = 0.4\ m^2/s$$

$$0.4 = \frac{1 \times 10^{-4}}{1/0.479 + 1/D_{AK}}$$

$$D_{AK} = 2.425 \times 10^{-4}\ m^2/s$$

FROM EQN (24-58)

$$2.425 \times 10^{-4} = 4850\, d_p \left[\frac{353}{2.0158}\right]^{1/2}$$

$$\underline{\underline{d_p = 3.78 \times 10^{-7}\ m}}$$

24.24 O_2 INTO He ~ A INTO B

$d_p = 5 \times 10^{-6}\ m \quad P = 300\ Pa$

$T = 373\ K \quad M_A = 32 \quad M_B = 4$

$$C = \frac{P}{RT} = \frac{300}{8.314(373)} = 0.0967\ MOL/m^3$$

$$C_{CO_2} = 0.01(0.0967) = \underline{\underline{9.67 \times 10^{-4}\ MOL/m^3}}$$

(a)

24.24 CONTINUED -

$$\text{IN PORES} - D_{EFF} = \frac{1}{1/D_{AB} + 1/D_{AK}}$$

USE EQN (24-33) TO FIND D_{AB}:

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.530$

$\sigma_{AB}^2 = 9.027 \quad \epsilon_{AB}/\kappa = 33.98$

$\kappa T/\epsilon_{AB} = 10.98 \quad \Omega_D = 0.8161$

SUBSTITUTING: $D_{AB} = 0.0325\ m^2/s$

USE EQN (24-58) TO FIND D_{AK}

$$D_{AK} = 8.28 \times 10^{-4}\ m^2/s$$

$$D_{AE} = \frac{1 \times 10^{-4}}{1/325 + 1/8.28} = \underline{\underline{8.08 \times 10^{-4}\ m^2/s}}$$

24.25 C_6H_6 IN $H_2O(l)$

$d_p = 1.50 \times 10^{-7}\ m \quad \mu_B = 0.95\ cp$

$\varepsilon = 0.4 \quad \phi_B = 2.26 \quad M_B = 18$

$V_A = 96.38$

SUBSTITUTING INTO EQN (24-52)

$$D_{AB} = 0.955 \times 10^{-9}\ m^2/s$$

USE EQN (24-58) TO GET D_{AK}

$$D_{AK} = 0.142 \times 10^{-4}\ m^2/s$$

$$D_{AE} = \frac{1 \times 10^{-9}}{1/14200 + 1/0.955} \cong 0.955 \times 10^{-9}\ m^2/s$$

$$D'_{AE} = \varepsilon^2 D_{AE} = \underline{1.528 \times 10^{-10}\ m^2/s}$$

24.26 CO IN H_2

CO ~ A H_2 ~ B

$d_p = 1.5 \times 10^{-8}\ m$ $\varepsilon = 0.10$

$T = 673\ K$ $P = 5.0\ ATM$

APPENDIX J: $D_{AB} = 0.651 \times 10^{-4}$ @ 1 ATM

$\sim D_{AB} = 0.130 \times 10^{-4}\ m^2/s$ @ 5 ATM

$\epsilon_{AB}/k = 60.52$

@ 273 K – $kT/\epsilon_{AB} = 4.51$ $\Omega_D = 0.8606$

@ 673 K – $kT/\epsilon_{AB} = 11.12$ $\Omega_D = 0.7345$

$$D_{AB}\Big|_{673} = 0.130 \times 10^{-4}\left(\frac{673}{273}\right)^{3/2}\left(\frac{0.8606}{0.7345}\right)$$

$$= 0.5891 \times 10^{-4}\ m^2/s$$

OBTAIN D_{AK} FROM EQN (24-58)

$$D_{AK} = 4850\left(1.5\times10^{-8}\right)\sqrt{\frac{673}{28}}$$

$$= 0.0357 \times 10^{-4}\ m^2/s$$

$$D_{AE} = \frac{1\times10^{-4}}{1/0.5891 + 1/0.0357}$$

$$= 0.337 \times 10^{-4}\ m^2/s$$

$$D'_{AE} = (0.1)^2(0.337\times10^{-4})$$

$$= \underline{\underline{0.337 \times 10^{-6}\ m^2/s}}$$

$$K.D. = \frac{0.0357}{0.589 + 0.0357} \cong \underline{\underline{5.7\%}}$$

24.27 GLUCOSE (A) IN H_2O

$T = 303\ K$ $d_p = 3\times10^{-9}\ m$

$d_A = 0.86 \times 10^{-9}\ m$

$\mu_B = 825\ g/cm\cdot s$

USE STOKES-EINSTEIN EQN: (24-50)

$$D_{AB} = \frac{kT}{6\pi\mu_B r_A} = \frac{(1.38\times10^{-16})(303)}{6\pi(825)\left(\frac{0.86}{2}\times10^{-7}\right)}$$

$$= 6.25 \times 10^{-15}\ m^2/s$$

USE EQN (24-62) TO OBTAIN D_{AE}

$$\phi = \frac{8.6\times10^{-10}\ m}{30\times10^{-10}\ m} = 0.2867$$

$$F_1 = (1-\phi)^2 = 0.508$$

$$F_2 = 1 - 2.104(0.287) + 2.09(0.287)^3 - 0.95(0.287)^5 = 0.444$$

$$D_{AE} = D_{AB}F_1F_2 = (6.25\times10^{-15})(0.508)(0.444)$$

$$= \underline{\underline{1.41 \times 10^{-15}\ m^2/s}}$$

24.28 UREASE (A) INTO SUPPORT (B)

$D_{AB} = 3.46 \times 10^{-11}\ m^2/s$

$d_{MOLECULE} = 12.38\ nm$ $d_p = 100\ nm$

$$\phi_A = \frac{12.38}{100} = 0.1238$$

$$F_1(\phi) = (1-0.1238)^2 = 0.7677$$

$$F_2(\phi) = 1 - 2.104(0.1238) + 2.09(0.1238)^3 - 0.95(0.1238)^5$$

$$= 0.743$$

24.28 CONTINUED ~

$$D_{Ae} = (3.46\times10^{-7})(0.7677)(0.743)$$

$$= \underline{\underline{1.97\times10^{-11}\ m^2/s}}$$

24.29 RIBONUCLEASE (A)
INTO SUPPORT (B)

$$D_{Ae} = 5.0\times10^{-11}\ m^2/s$$

$$D_{AB} = 1.19\times10^{-10}\ m^2/s$$

$$d_m = 3.6\ nm$$

$$D_{Ae} = D_{AB} F_1(\phi) F_2(\phi)$$

$$F_1(\phi)F_2(\phi) = \frac{5.0\times10^{-11}}{1.19\times10^{-10}}$$

$$= 0.4202$$

TRIAL & ERROR ~

$$\phi \cong 0.183$$

$$F_1(\phi) = (1-0.183)^2 = 0.6675$$

$$F_2(\phi) = 1 - 2.104(0.183) + 2.09(0.183)^3 - 0.95(0.183)^5$$

$$= 0.6276$$

$$F_1F_2 \cong 0.4190$$

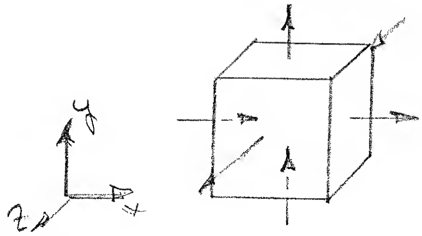
CLOSE ENOUGH ~

$$d_p = \frac{3.6\ nm}{0.183} = \underline{\underline{19.67\ nm}}$$

CHAPTER 25

25.1 CONSERVATION OF MASS:

$$\iint_{C.S.} \rho(\vec{v}\cdot\vec{n})\,dA + \frac{\partial}{\partial t}\iiint_{C.V.} \rho\, dV = 0$$



MASS FLOW:

$$N_{Ax}\Delta y\Delta z\big|_{x+\Delta x} - N_{Ax}\Delta y\Delta z\big|_{x}$$
$$+ N_{Ay}\Delta x\Delta z\big|_{y+\Delta y} - N_{Ay}\Delta x\Delta z\big|_{y}$$
$$+ N_{Az}\Delta x\Delta y\big|_{z+\Delta z} - N_{Az}\Delta x\Delta y\big|_{z}$$

ACCUMULATION: $\frac{\partial c_A}{\partial t}\Delta x\Delta y\Delta z$

PRODUCTION: $R_A\Delta x\Delta y\Delta z$

PROCEDURE:

1. RELATE ACCORDING TO BASIC EQN.
2. DIVIDE THROUGH BY $\Delta x\Delta y\Delta z$
3. CANCEL Δ TERMS WHERE APPLICABLE
4. TAKE LIMIT AS $\Delta x, \Delta y, \Delta z \to 0$

RESULT!
$$\nabla\cdot\vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

25.2 $\nabla\cdot\vec{n}_A + \frac{\partial \rho_A}{\partial t} - r_A = 0$

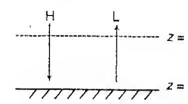
FOR ρ & D_{AB} CONSTANT

$$\vec{n}_A = -D_{AB}\nabla\rho_A + \rho_A\vec{v}$$
$$\nabla\cdot\vec{n}_A = -D_{AB}\nabla^2\rho_A + \nabla\cdot\rho_A\vec{v}$$

SUBSTITUTION YIELDS:

$$\frac{\partial\rho_A}{\partial t} - D_{AB}\nabla^2\rho_A + \nabla\cdot\rho_A\vec{v} = r_A$$

25.3



$H \to 2L$

$$\nabla\cdot\vec{N}_H + \cancel{\frac{\partial c_H}{\partial t}}^{\,0} = \cancel{R_H}^{\,0}$$

ONE-DIRECTIONAL, STEADY STATE, NO HOMOGENEOUS REACTION -

$$\frac{d}{dz}N_{Hz} = 0 \qquad (a)$$

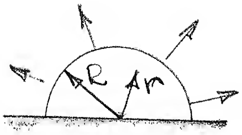
$$N_{Hz} = -CD_{HL}\frac{dy_H}{dz} + y_H(N_{Hz} + N_{Lz})$$

AS $N_{Lz} = -2N_{Hz}$

$$N_{Hz} = -CD_{HL}\frac{dy_H}{dz} + y_H(N_{Hz} - 2N_{Hz})$$

$$N_{Hz}(1 + y_H) = -CD_{HL}\frac{dy_H}{dz}$$

$$N_{Hz} = -\frac{CD_{HL}}{1+y_H}\frac{dy_H}{dz}$$

25.4 

1. T, P constant; C = constant
2. Steady state
3. No homogeneous reaction, $R_A = 0$
4. One directional diffusion
5. Concentration constant @ r = R
6. $N_{Air} = 0$

$$\nabla \cdot \vec{N}_A + \cancel{\frac{\partial c_A}{\partial t}}^{0} - \cancel{R_A}^{0} = 0$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2 N_{Ar}) = 0 \qquad r^2 N_{Ar} = \text{const.}$$

$$N_{Ar} = -c D_{AB}\frac{dy_A}{dr} + y_A(N_{Ar} + \cancel{N_{Br}}^{0})$$

$$\underline{\underline{N_{Ar} = -\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dr}}}$$

25.5 $O_2 \sim A \qquad H_2O \sim B$

$$\nabla \cdot \vec{N}_A + \frac{\partial c_A}{\partial t} - \cancel{R_A}^{0} = 0$$

For z-direction:

$$\frac{d}{dz}N_{Az} = 0 \qquad N_{Az} = \text{const}$$

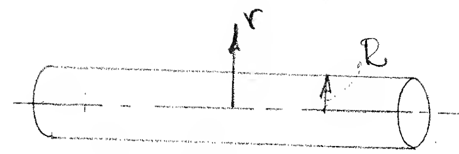
$$N_{Az} = -c D_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$$

Since dilute: $y_A \approx 0$, $C \cong$ const

$$\underline{\underline{\frac{\partial c_A}{\partial t} = D_{AB}\frac{\partial^2 c_A}{\partial z^2}}}$$

For $R_A = -kc_A$

$$\underline{\underline{\frac{\partial c_A}{\partial t} - D_{AB}\frac{\partial^2 c_A}{\partial z^2} + kc_A = 0}}$$

25.6 

(a)
1. Diffusion in r-direction only
2. No homogeneous reaction, $R_A = 0$
3. C_A @ r = R+10 is known & constant
4. C_A @ r = R is constant, $y_A = P_a/P$
5. Molecular diffusion only
6. Steady state

$$\nabla \cdot N_A + \cancel{\frac{\partial c_A}{\partial t}}^{0} = \cancel{R_A}^{0}$$

$$\sim \underline{\underline{\frac{1}{r}\frac{d}{dr}(rN_{Ar}) = 0}} \qquad (b)$$

$$\frac{d}{dr}(rN_{Ar}) = 0$$

$$\sim \underline{\underline{rN_{Ar} = \text{constant}}} \qquad (c)$$

$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar} + \cancel{N_{Br}}^{0})$$

$$= -cD_{AB}\frac{dy_A}{dr} + y_A N_{Ar}$$

$$\underline{\underline{N_{Ar} = -\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dr}}}$$

For dilute concentration: $y_A \sim 0$

$$\underline{\underline{N_{Ar} = -cD_{AB}\frac{dy_A}{dr}}}$$

25.7 $\nabla \cdot \vec{N}_A + \cancel{\frac{\partial c_A}{\partial t}} - \cancel{R_A} = 0$

Assumptions/conditions:

1. Steady state
2. No homogeneous reaction
3. Diffusion in x & y directions
4. $v_y = 0$
5. Constant c, D_{AB}
6. $v_x = ay$

25.7 CONTINUED-

$$\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Ay}}{\partial y} = 0$$

$$N_{Ax} = -D_{AB}\frac{\partial C_A}{\partial x} + a y C_A$$

$$N_{Ay} = -D_{AB}\frac{\partial C_A}{\partial y}$$

SUBSTITUTING:

$$-D_{AB}\frac{\partial^2 C_A}{\partial y^2} + a y \frac{\partial C_A}{\partial x} - D_{AB}\frac{\partial^2 C_A}{\partial y^2} = 0$$

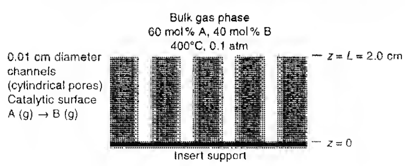
$$\underline{D_{AB}\left[\frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial y^2}\right] = a y \frac{\partial C_A}{\partial x}}$$

B.C. $C_A(0,y) = 0$

$C_A(x,0) = C_{AS}$

$C_A(x,\delta) = 0$

25.8



1. DIFFUSION IN r & z DIRECTIONS
2. STEADY STATE
3. NO HOMOGENEOUS REACTION

$$\nabla \cdot \vec{N}_A + \cancel{\frac{\partial C_A}{\partial t}} - \cancel{R_A} = 0$$

$$\underline{\frac{1}{r}\frac{\partial}{\partial r}(r N_{Ar}) + \frac{\partial}{\partial z} N_{Az} = 0}$$

IN BOTH DIRECTIONS:

$N_{Ar} = -N_{Br}$ $N_{Az} = -N_{Bz}$

~ EQUIMOLAR COUNTERDIFFUSION

25.8 CONTINUED-

$$\therefore N_{Ar} = -cD_{AB}\frac{\partial y_A}{\partial r} = -D_{AB}\frac{\partial C_A}{\partial r}$$

$$N_{Az} = -cD_{AB}\frac{\partial y_A}{\partial r} = -D_{AB}\frac{\partial C_A}{\partial r}$$

INTO MASS CONSERVATION EQN:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(-D_{AB}\, r\frac{\partial C_A}{\partial r}\right) + \frac{\partial}{\partial z}\left(-D_{AB}\frac{\partial C_A}{\partial z}\right) = 0$$

$$\underline{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right) + \frac{\partial^2 C_A}{\partial z^2} = 0}$$

B.C. $\frac{\partial C_A}{\partial r}(0,z) = 0$

$C_A(0.005\,cm, z) = 0$

$C_A(r, 2.0\,cm) = 0.6\,C$

25.9 $\nabla \cdot \vec{N}_A + \cancel{\frac{\partial C_A}{\partial t}} - R_A = 0$ {A is O_2}

STEADY STATE, $R_A = -m$

$$\nabla^2 N_A + m = 0$$

DIFFUSION IN r-DIRECTION ONLY

$$\underline{\frac{1}{r}\frac{\partial}{\partial r}(r N_{Ar}) + m = 0}$$

EQUIMOLAR COUNTERD.

$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr} + y_A\cancel{(N_{Ar} + N_{Br})}$$

$$\therefore N_{Ar} = -cD_{AB}\frac{dy_A}{dr} = -D_{AB}\frac{dc_A}{dr}$$

OR -- $$\underline{N_{Ar} = -\frac{D_{AB}}{RT}\frac{dp_A}{dr}}$$

25.10 $\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$

{A IS O_2}

STEADY STATE

NO HOMOGENEOUS REACTION

ONE-D (SPHERICAL) DIFFUSION

$$\underline{\frac{d}{dr}\left(r^2 N_{Ar}\right) = 0}$$

FOR $2C + O_2 \rightarrow 2CO$

$$2N_{Ar} = -N_{Br}$$

$$y_A\left(N_{Ar} + N_{Br}\right) = -y_A N_{Ar}$$

$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr} - y_A N_{Ar}$$

$$\underline{= -\frac{cD_{AB}}{1+y_A}\frac{dy_A}{dr}}$$

FOR $C + O_2 \rightarrow CO_2$ $N_{Ar} = -N_{Br}$

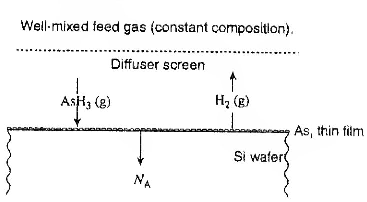
$$y_A\left(N_{Ar} + N_{Br}\right) = 0$$

$$\underline{N_{Ar} = -cD_{AB}\frac{dy_A}{dr}}$$

B.C. $y_A(R) = 0$

$y_A(\infty) = 0.21$

25.11



ASSUMPTIONS:

1. TEMP = CONST, D_{AB} & ρ_S CONSTANT
2. NO HOMOGENEOUS REACTION

25.11 CONTINUED:

3. SILICON TREATED AS SEMI-INFINITE
4. $C_A(z,0) = 0$
5. MOLECULAR DIFFUSION IN SOLID
6. ONE DIRECTIONAL (z) DIFFUSION

GENERAL MASS CONSERVATION EQN IS

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} = 0$$

$$\text{\& } N_A = -D_{AB}\frac{\partial C_A}{\partial z}$$

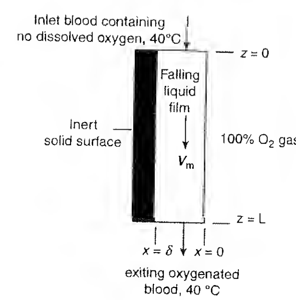
COMBINING: $\underline{\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}}$

B.C. $C_A(z,0) = 0$

$C_A(0,t) = C_{AS}$

$C_A(\infty,t) = 0$

25.12 A IS O_2



$$\nabla \cdot \vec{N}_A + \cancel{\frac{\partial C_A}{\partial t}} - \cancel{R_A} = 0$$

ST. ST. NO RX

$$\underline{\frac{\partial N_{Ax}}{\partial x} + \frac{\partial N_{Az}}{\partial z} = 0}$$

$$N_{Ax} = -D_{AB}\frac{\partial C_A}{\partial x} + \cancel{y_A}\left(N_{Ax} + N_{Bx}\right)$$

≈ 0 -- DILUTE CONCENTR.

$$N_{Ax} = -D_{AB}\frac{\partial C_A}{\partial x}$$

$$N_{Az} = \cancel{-D_{AB}\frac{\partial C_A}{\partial z}} + C_A v_m$$

BULK FLOW $\gg$ MOLECULAR FLOW IN z DIRECTION

25.12 CONTINUED –

SUBSTITUTING INTO MASS CONT. EQ.

$$\frac{\partial}{\partial x}\left(-D_{AB}\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial z}\left(C_A v_m\right)=0$$

$$-D_{AB}\frac{\partial^2 C_A}{\partial x^2}+v_m\frac{\partial C_A}{\partial z}=0$$

B.C. $C_A(x,0)=0$,

$C_A(0,z)=C_A$

$\frac{dC_A}{dx}(\delta,z)=0$

25.13 $\nabla\cdot N_A+\frac{\partial C_A}{\partial t}-R_A=0$

ONE-DIMENSIONAL (r) DIFFUSION IN SPHERICAL GEOMETRY

NO HOMOGENEOUS REACTION

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 N_{Ar}\right)+\frac{\partial C_A}{\partial t}=0$$

FICK'S LAW:

$$N_{Ar}=-D_{AB}\frac{\partial C_A}{\partial r}+\cancel{C_A v_r}$$

COMBINING:

$$-\frac{D_{AB}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right)+\frac{\partial C_A}{\partial t}=0$$

B.C. $C_A(R,t)=0$

$C_A(r,0)=C_{A0}$

$\frac{\partial C_A}{\partial r}(0,t)=0$

25.14 SPHERICAL GEOMETRY –

$$\nabla\cdot\vec{N}_A+\frac{\partial C_A}{\partial t}-\cancel{R_A}=0$$

NO HOMOG. RX

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 N_{Ar}\right)+\frac{\partial C_A}{\partial t}=0$$

FICK'S LAW:

$$N_{Ar}=-D_{A\,eff}\frac{\partial C_A}{\partial r}+\underbrace{\quad}_{\text{0 – NO BULK CONTRIB.}}$$

& COMBINING –

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_{A\,eff}\frac{\partial C_A}{\partial r}\right)+\frac{\partial C_A}{\partial t}=0$$

$$\frac{\partial C_A}{\partial t}=\frac{D_{A\,eff}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right)$$

$C_A(r\le R,0)=C_{A0}$

$C_A(R,t)=C_A^*$

$\frac{\partial C_A}{\partial r}(0,t)=0$

25.15 INTO AIR: {A – HERBICIDE}

$$\nabla\cdot\vec{N}_A+\underbrace{\cancel{\frac{\partial C_A}{\partial t}}}_{\text{0-ST.ST.}}-\underbrace{\cancel{R_A}}_{\text{No HOMOG. RX}}=0$$

ONE DIMENSIONAL (z) DIFFUSION

$$\frac{dN_A}{dz}=0$$

FICK'S LAW

$$N_{Az}=-CD_{AB}\frac{dy_A}{dz}+y_A\left(N_{Az}+\cancel{N_{Bz}}^{\,0}\right)$$

$$N_{Az}=-\frac{CD_{AB}}{1-y_A}\frac{dy_A}{dz} \qquad \text{(a)}$$

25.15 CONTINUED -

INTO SOIL -

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - \overset{\text{NO RX}}{\cancel{R_A}} = 0$$

NO BULK CONTR.

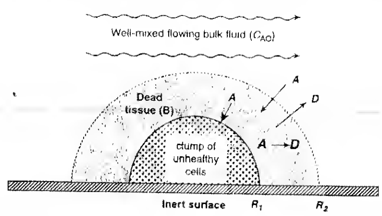
$$N_{Az} = -D_{AB}\frac{dC_A}{dz} + \cancel{}^{0}$$

$$\frac{\partial N_{Az}}{\partial z} + \frac{\partial C_A}{\partial t} = 0$$

COMBINING:

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

25.16



$$R_A = -kC_A$$

$$\nabla \cdot \vec{N}_A + \frac{\partial C_A}{\partial t} - R_A = 0$$

SPHERICAL GEOMETRY - STDY STATE

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 N_{Ar}\right) + kC_A = 0$$

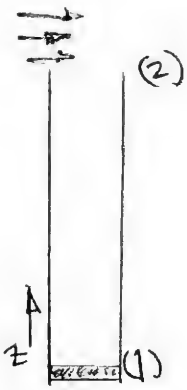
$$N_{Ar} = -D_{A\text{-mix}}\frac{dC_A}{dr} + \overset{\text{DILUTE}}{\sim\cancel{}0}$$

COMBINING:

$$\frac{-D_{A\text{-mix}}}{r^2}\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) + kC_A = 0$$

CHAPTER 26

26.1 ARNOLD CELL



$d = 1\ \text{cm}$
$T = 308\ \text{K}$
$P_A^0 = 0.195\ \text{ATM}$
$\rho_L = 0.85\ \text{g/cm}^2$
$M_A = 78$

$y_{A1} = 0.195$
$y_{A2} = 0$

EQN (26-19) APPLIES

$$D_{AB} = \frac{\rho_{AL} y_{BLM}/M_A}{c(y_{A1} - y_{A2})t}\left(\frac{z_{t1}^2 - z_{t0}^2}{2}\right)$$

$$y_{BLM} = \frac{1.0 - 0.805}{\ln 1/0.805} = 0.899$$

$$c = \frac{P}{RT} = \frac{1}{(82.06)(308)} = 3.956\times 10^{-5}\ \text{mol/cm}^2$$

$$t = 72\ h = 2.592 \times 10^5\ s$$

SUBSTITUTING —

$$\underline{\underline{D_{AB} = 9.6\times 10^{-6}\ m^2/s}}$$

FROM APPENDIX J.1.

AT 298 K: $D_{AB} = 9.62\times 10^{-6}\ m^2/s$

AT 308 K: $D_{AB} = (9.62\times 10^{-5})\left(\frac{308}{298}\right)^{3/2}$

$$= 1.01\times 10^{-5}\ m^2/s$$

— IN EXPERIMENT - EDDIES AT TOP OF CELL WOULD ALTER DIFFUSION MECHANISM

26.2 CYLINDRICAL GEOMETRY:

STDY STATE, NO HOMOG. REACTION -

$$\frac{1}{r}\frac{d}{dr}(r N_{Ar}) = 0$$

$$N_{Ar} = -c D_{AB}\frac{dy_A}{dr} + y_A \cancel{\ }$$ 0 - DILUTE

$$= -\frac{D_{AB}}{RT}\frac{dp_A}{dr}$$

$$r N_{Ar}\int_{r_i}^{r_o}\frac{dr}{r} = -\frac{D_{AB}}{RT}\int_{p_{Ai}}^{p_{Ao}} dp_A$$

$$\underline{r N_{Ar} \ln\frac{r_o}{r_i} = \frac{D_{AB}}{RT}(p_{Ai} - p_{Ao})}$$

TABLE J.3. AT 293 K $D_{AB} = 4.49\times 10^{-15}\ m^2/s$

SUBSTITUTING VALUES & SOLVING:

$$\underline{\underline{N_{Ar} = 3.92\times 10^{-11}\ \text{mol/m}^2\cdot s}}$$

TO GET CONCENTRATION PROFILE:

$$\frac{d}{dr}(r N_{Ar}) = \frac{d}{dr}\left(-r D_{AB}\frac{dc_A}{dr}\right) = 0$$

$$\frac{d}{dr}\left(r\frac{dc_A}{dr}\right) = 0$$

SOLVING: $$r\frac{dc_A}{dr} = C_1$$

$$c_A = C_1 \ln r + C_2$$

AT $r_i = 5\ mm$

$$c_{Ai} = \frac{p_{Ai}}{RT} = \frac{1.5\times 10^5}{8.314(293)} = 61.58\ \text{mol/m}^3$$

AT $r_o = 8\ mm$

$$c_{Ao} = \frac{1.0\times 10^5}{(8.314)(293)} = 41.05\ \text{mol/m}^3$$

26.2 CONTINUED -

UNITS: C_A, mol/m^3 r, mm

$@ r_1 \quad 61.58 = C_1 \ln 5 + C_2$

$@ r_0 \quad 41.05 = C_1 \ln 8 + C_2$

$C_1 = -43.64 \quad C_2 = 131.8$

$$\underline{C_A = -43.64 \ln r + 131.8}$$

26.3 ONE DIRECTIONAL
STEADY STATE
B INSOLUBLE IN A & STAGNANT

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + \cancel{N_{Bz}})$$

$$= \frac{cD_{AB}}{z_2 - z_1}\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right) \quad \{\text{EQ. (26-5)}\}$$

$y_A(3.0) = 1.0 \quad y_A(0.5) = \frac{163}{760} = 0.214$

$$c = \frac{P}{RT} = \frac{1.0}{(82.06)(303)} = 4.02\times10^{-5}\ \text{mol/cm}^3$$

APP. J: @ 298 K $- D_{AB} = 1.62\times10^{-4}$ m^2/s

AT 303 K $D_{AB} = (1.62\times10^{-4})\left(\frac{303}{298}\right)^{3/2}$

$= 1.66\times10^{-4}$ m^2/s

SUBSTITUTING VALUES & SOLVING:

$N_{Az} = 6.42\times10^{-5}$ mol/m$^2\cdot$s

$W_A = (6.42\times10^{-5})(32)(3600)(24)A$

$A = \pi/4 (1)^2 = 0.785$ m^2

$\underline{W = 139 \text{ g/DAY}}$ (a)

26.3 CONTINUED

IF TEMPERATURE IS 313 K:

$$D_{AB} = (1.62\times10^{-4})\left(\frac{313}{298}\right)^{3/2} = 1.74\times10^{-4}\ \text{m}^2/\text{s}$$

$y_{A1} = 265/760 = 0.349$

ALL OTHER VALUES REMAIN THE SAME -
SOLVING:

$$\underline{W_A = 260.6 \text{ g/DAY}}$$

26.4 C_2H_5OH (A) THROUGH STAGNANT H_2O (B)

ONE DIMENSIONAL, STEADY DIFFUSION
DILUTE CONCENTRATION: $y_A \sim$ SMALL

$$N_{Az} = -D_{AB}\frac{dc_A}{dz} = \frac{D_{AB}}{\delta}(c_{A1} - c_{A2})$$

TO EVALUATE D_{AB} - USE EQN. (24-53)

$V_A = 2(14.8) + 6(3.7) + 7.4 = 59.2$ cm^3/mol
$\mu_B = 1.45$ cp

$$D_{AB} = (13.26\times10^{-9})(1.45)^{-1.14}(59.2)^{-0.589}$$

$= 7.82\times10^{-10}$ m^2/s

$c_{A1} = 0.1$ mol/m^3 $c_{A2} = 0.02$ mol/m^3

SUBSTITUTING & SOLVING:

$$\underline{N_{Az} = 1.56\times10^{-12}\ \text{mol/m}^2\cdot\text{s}}$$

TO DETERMINE $c_A(z)$:

$\nabla\cdot\vec{N}_A = 0 \sim \frac{dN_A}{dz} = 0$

GIVING $\frac{d^2N_{Az}}{dz^2} = 0$

26.4 CONTINUED

$$C_A = C_1 z + C_2$$

B.C. $C_A(0) = 0.1$ mol/m^3

$C_A(0.004) = 0.02$ "

$$C_2 = 0.1 \qquad C_1 = \frac{(0.02-0.1)}{0.004} = -20$$

$$\underline{\underline{C_A = 0.1 - 20z}} \quad \left\{ \begin{array}{l} C_A,\ \text{mol/m}^3 \\ z,\ \text{m} \end{array} \right.$$

FOR C_2H_5OH (A) IN AIR (B) 283 K

$$C = \frac{P}{RT} = \frac{1.013\times10^5}{8.314(283)} = 43.15\ \text{mol/m}^3$$

$$y_{A1} = C_{A1}/C = \frac{0.1}{43.05} = 2.32\times10^{-3}$$

$$y_{A2} = \frac{0.02}{43.05} = 4.64\times10^{-4}$$

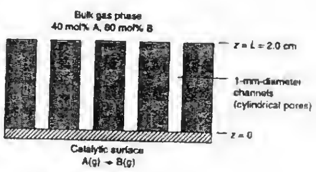
$$D_{AB} = (1.32\times10^{-5})\left(\frac{283}{298}\right)^{3/2} = 1.22\times10^{-5}\ \text{m}^2/\text{s}$$

SAME EQN FOR N_{Az} AS IN PART (a)

$$N_{Az} = \frac{(1.22\times10^{-5})(0.1-0.02)}{4\times10^{-3}}$$

$$= \underline{\underline{2.44\times10^{-4}\ \text{mol/m}^2\cdot\text{s}}}$$

26.5



$P = 2$ ATM

$T = 373$ K

$M_A = 58$

STEADY STATE, 1D DIFFUSION

26.5 CONTINUED -

$$\frac{dN_{Az}}{dz} = 0 \qquad \underline{\underline{N_{Az} = -N_{Bz} = \frac{c D_{AB}}{\delta}(y_{A1} - y_{A2})}}$$

$y_{A1}(0) = 0.4$

$y_{A2}(0.02\ \text{m}) = 0$

$$c = \frac{P}{RT} = \frac{2}{(82.06)(373)} = 6.53\times10^{-5}\ \text{mol/cm}^3$$

$$D_{AB} = 0.1\left(\frac{373}{298}\right)^{3/2}\left(\frac{1}{2}\right) = 0.07\ \text{cm}^2/\text{s}$$

SUBSTITUTING INTO N_{Az} EXPRESSION

$$N_{Bz} = -N_{Az} = \underline{\underline{1.829\times10^{-6}\ \text{g-mol/cm}^2\text{s}}}$$

$$W_A = N_{Az}\cdot S$$

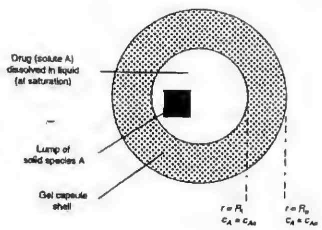
$$0.01\ \text{mol/min} = (1.829\times10^{-6})(60)\ S$$

S = SURFACE AREA = 91.12 cm^2

PER CHANNEL - $S' = \pi/4\,(0.1)^2 = 0.00785$

$$\text{NO. CHANNELS} = \frac{91.12}{0.00785} = \underline{\underline{11608}}$$

26.6 $D_{AB} = 1.5\times10^{-9}$ m^2/s



$C_{AS} = C_A(R_1) = 0.01$ mol/cm^2

$C_{A0} = C_A(R_0)$

STEADY STATE, NO HOMOG. RX

$$\nabla\cdot\vec{N}_A = 0 \qquad N_{Ar} = -D_{AB}\frac{dC_A}{dr}$$

$$\frac{d}{dr}\left(r^2 N_{Ar}\right) = 0 \sim r^2 N_{Ar} \text{ IS CONST.}$$

$$\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) = 0$$

26.6 CONTINUED -

$$r^2\frac{dc_A}{dr} = C_1 \qquad \frac{dc_A}{dr} = r^{-2}C_1$$

$$C_A = -\frac{C_1}{r} + C_2$$

USING B.C.

$$C_{A1} = 0.01 = -\frac{C_1}{0.2} + C_2$$

$$C_{A0} = -\frac{C_1}{0.35} + C_2$$

SUBTRACTING - $C_1 = \frac{C_{A0} - 0.01}{0.466}$

$$W = 4\pi r^2 N_{Az} = 4\pi(-D_{AB})C_1$$

$$= -\frac{4\pi D_{AB}}{0.466}(C_{A0} - 0.01)$$

$$= -\frac{4\pi\,(1.5\times10^{-5})}{0.466}(C_{A0} - 0.01)$$

$$= -4.045\times10^{-4}(C_{A0} - 0.01)\ \text{mol/s}$$

W_A IS MAX FOR $C_{A0} = 0$

$$= 4.045\times10^{-4}\ \text{mol/s}$$

$$= \underline{\underline{1.456\ \text{mol/H}}}$$

26.7 SPHERICAL GEOMETRY
STEADY STATE, NO HOMOG Rx
A INTO STAGNANT B

$$\nabla\cdot\vec{N}_A = 0$$

$$\frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$N_{Ar} = -CD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar} + \cancel{N_{Br}}^{\,0})$$

26.7 - CONTINUED -

$$\nabla\cdot\vec{N}_A = 0 \Rightarrow \frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$N_{Ar} = -CD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar} + \cancel{N_{Br}}^{\,0})$$

$$N_{Ar} = -\frac{CD_{AB}}{1-y_A}\frac{dy_A}{dr}$$

$$N_{Ar}(4\pi r^2)\int_R^{\infty}\frac{dr}{r^2} = -4\pi C D_{AB}\int_{y_{A0}}^{y_A=0}\frac{dy_A}{1-y_A}$$

$$W_A\left(-\frac{1}{r}\Big|_R^{\infty}\right) = -4\pi C D_{AB}\left(\ln\frac{1}{1-y_A}\Big|_{y_{A0}}^{0}\right)$$

$$W_A = 4\pi C D_{AB} R \ln\left(\frac{1}{1-y_{A0}}\right)$$

MASS BALANCE FOR A:

$$4\pi C D_{AB} R \ln\frac{1}{1-y_{A0}} = \frac{\rho}{M_A}\frac{dV}{dt}$$

$$\cancel{4\pi} C D_{AB} \cancel{R} \ln\frac{1}{1-y_{A0}} = \frac{\cancel{4\pi}\rho}{M_A} R^{\cancel{2}}\frac{dR}{dt}$$

SEPARATING VARIABLES & INTEGRATING:

$$CD_{AB}\ln\left(\frac{1}{1-y_{A0}}\right)t = \frac{\rho_s}{M_A}\left(\frac{R_1^2 - R_2^2}{2}\right)$$

VALUES: $D_{AB} = 8.19\times10^{-6}\ \text{m}^2/\text{s}$

$$C = \frac{P}{RT} = \frac{1.013\times10^5}{8.314(347)} = 35.11\ \text{gmol/m}^3$$

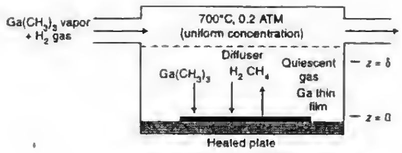
$$\rho_s = \frac{1.145}{128}\times(100)^3 = 8945\ \text{gmol/m}^3$$

SOLVING FOR t:

$$t = \frac{(8945)\left(\frac{10^{-4} - 0.0625\times10^{-4}}{2}\right)}{(35.11)(8.19\times10^{-6})\ln\left(\frac{1}{0.993}\right)}$$

$$= \underline{\underline{2.076\times10^5\ \text{s}}} = \underline{\underline{57\ \text{H}}}$$

26.8



PSEUDO STEADY STATE –
NO HOMOG. RX
ONE (Z) DIRECTIONAL DIFFUSION

$$\nabla \cdot \vec{N}_A = 0 \qquad \frac{dN_{Az}}{dz} = 0$$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz} + N_{Cz})$$

$$\left\{ \begin{array}{l} H_2: \quad N_{Bz} = 3/2\, N_{Az} \\ CH_4: \quad N_{Cz} = -3N_{Az} \end{array} \right\}$$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A N_{Az}\left(1 + 3/2 - 3\right)$$

$$= \frac{-cD_{AB}}{(1 + y_A/2)}\frac{dy_A}{dz}$$

$$N_{Az}\int_\delta^0 dz = -cD_{AB}\int_{y_{A0}}^{0}\frac{dy_A}{1 + y_A/2} \quad \text{(a)}$$

FOR DILUTE A ~ y_A SMALL

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz}$$

$$N_{Az}\int_\delta^0 dz = -cD_{AB}\int_{0.0002}^{0} dy_A$$

$$c = P/RT = \frac{0.20}{82.06(973)} = 2.5 \times 10^{-6}\ \text{g mol/cm}^3$$

$$D_{AB}\Big|_{T_2, P_2} = 2.0\ \text{cm}^2/\text{s}\left(\frac{1}{0.2}\right)\left(\frac{973}{1023}\right)^{3/2}$$

$$= 9.276\ \text{cm}^2/\text{s} \quad \text{(c)}$$

IN TERMS OF δ:

$$N_{Az} = \frac{cD_{AB}}{\delta}(0.0002) \quad \text{(b)}$$

26.9 $P = 303.9$ Pa $T = 873$ K

$y_A = y_{AS} = 0$ @ $z = 0$

$y_A = 0.2$ @ $z = \delta = 6$ cm

$M_A = 78$

FOR D_{AB} – HIRSHFELDER EQN.

VALUES: $\left[1/M_A + 1/M_B\right]^{1/2} = 0.716$

$\sigma_{AB}^2 = 12.418 \qquad \epsilon_{AB}/k = 69.13$

$\epsilon_{AB}/kT = 7.92 \qquad \Omega_D = 0.8556$

~SUBSTITUTING: $D_{AB} = 0.0221\ \text{m}^2/\text{s}$ (a)

PHYSICAL SITUATION IS EQUIVALENT TO CASE EXAMINED IN EXAMPLE 2, CH 25

$$N_{Az} = \frac{cD_{AB}}{\delta}\ln\left(\frac{1 + y_{A0}}{1 + y_{AS}}\right)$$

$$c = \frac{P}{RT} = \frac{3 \times 10^{-3}}{82.06(873)} = 4.188 \times 10^{-8}\ \text{mol/cm}^3$$

$$N_{Az} = \frac{(4.188 \times 10^{-8})(221\ \text{cm}^2/\text{s})}{6\ \text{cm}}\ln\left(\frac{1.2}{1}\right)$$

$$= 2.814 \times 10^{-7}\ \text{mol/cm}^2 \cdot \text{s}$$

$$W_A = N_{Az}A = N_{Az}(\pi/4)(D)^2$$

$$= (2.814 \times 10^{-7})(\pi/4)(15)^2(60)(78)$$

$$= 0.2327\ \text{g/m}$$

26.10 HEMISPHERICAL DROPLET ON A PLANE SURFACE

STEADY STATE, NO HOMOG. Rx · ONE D

$$\nabla \cdot \vec{N}_A = \frac{1}{r^2}\frac{d}{dr}\left(r^2 N_{Ar}\right) = 0$$

$r^2 N_{Ar} \sim$ CONSTANT

$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr} + y_A\left(N_{Ar} + \cancel{N_{Br}}^{\,0}\right)$$

$$N_{Ar} = -\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dr}$$

$$\underbrace{2\pi r^2 N_{Ar}}_{W_A}\int_R^{\infty} \frac{dr}{r^2} = 2\pi c D_{AB}\int_0^{y_{A1}} \frac{dy_A}{1-y_A}$$

@ $t=0$ $r = 0.005$ m

$$y_{A1} = \frac{31.824}{760} = 0.0419$$

$$W_A\left[-\frac{1}{r}\right]_R^{\infty} = 2\pi c D_{AB} \ln\left(\frac{1}{0.958}\right)$$

$$W_A = 2\pi c D_{AB} R \ln(1.0437) \quad (1)$$

FOR DROPLET ~ PSEUDO S.S.

$$W_A = -\frac{\rho_A}{M_A}\frac{dV}{dt}$$

$$= -\frac{1}{18}\left(2\pi R^2 \frac{dR}{dt}\right) \quad (2)$$

EQUATING (1) & (2) & INTEGRATING:

$$cD_{AB}\ln(1.0437)\,t = 0.0556\left(\frac{R_i^2 - R_f^2}{2}\right)$$

$$c = \frac{P}{RT} = \frac{1.03\times10^5}{8.314(303)} = 4.021\times10^{-7}\ \text{g mol/cm}^3$$

26.10 CONTINUED -

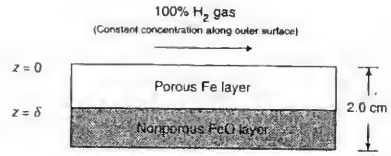
$$D_{AB} = 0.260\left(\frac{303}{298}\right)^{3/2} = 0.266\ \text{cm}^2/\text{s}$$

$R_i = 0.5$ cm $\quad R_o = 0$

SUBSTITUTE & SOLVE -

$$\underline{\underline{t = 1.517\times10^{6}\ \text{s}}} = \underline{\underline{421.4\ \text{h}}}$$

26.11



1 ATM, 400K

$D_{AB} = 1.7$ cm²/s

$\rho_{FeO} = 2.5$ g/cm³

$M_{FeO} = 71.85$

STEADY STATE
NO HOMOG Rx

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \sim N_{Az} \sim \text{CONST.}$$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az}+N_{Bz})$$

AS $N_{Bz} = -N_{Az}$; $y_A(N_{Az}+N_{Bz}) = 0$

$$N_{Az}\int_0^{\delta} dz = -cD_{AB}\int_{1.0}^{0} dy_A$$

$$N_{Az} = \frac{cD_{AB}}{\delta} \quad (a)$$

$$c = P/RT = \frac{1}{82.06(400)} = 3.047\times10^{-5}\ \text{gmol/cm}^3$$

$$\underline{N_{Az} = 5.18\times10^{-5}\ \text{gmol/cm}^2\cdot\text{s}} \quad (b)$$

FOR $0.1 < \delta < 0.2$

$$W = N_{Az}(1) = \rho\frac{d\delta}{dt}$$

$$\frac{M_B}{\rho_B}D_{AB}\,c\int_0^t dt = \int_{\delta_1}^{\delta_2} \delta\, d\delta$$

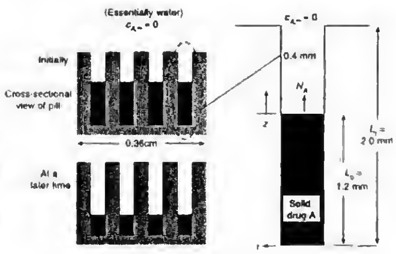
26.11 CONTINUED –

$$\frac{M_B D_{AB} C}{\rho_B} t = \frac{\delta_2^2 - \delta_1^2}{2}$$

SUBSTITUTING NUMERICAL VALUES:

$$\underline{\underline{t = 1007\,s = 16.78\ \text{min.}}}$$

26.12



$T = 310\,K$

$D_{AB} = 2 \times 10^{-5}\ cm^2/s$

$\rho_A = 1.10\ g/cm^3$

$M_A = 120$

$C_A^* = 2.0 \times 10^{-4}\ gmol/cm^3$

PSEUDO STDY STATE, NO HOMOG. RX

ONE-DIMENSIONAL – DILUTE SOLN

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \sim N_{Az}\ \text{CONST.}$$

$$N_{Az} = -D_{AB}\frac{dC_A}{dz}$$

$$N_{Az}\int_{z_1}^{z_2} dz = -D_{AB}\int_{C_A^*}^{0} dC_A$$

$$\underline{\underline{N_{Az} = \frac{D_{AB}C^*}{z_2 - z_1}}} \qquad (a)$$

FOR A PORE:

$$W_{Az} = N_{Az}A = \frac{D_{AB}C^* A}{z_2 - z_1}$$

$$W_A = \frac{(2 \times 10^{-5})(2.0 \times 10^{-4})(\pi/4)(0.04)^2}{0.2 - 0.12}$$

$$= 6.3 \times 10^{-11}\ gmol/s \ \text{PER PORE}$$

26.12 CONTINUED –

FOR 1 PILL ~ 16 PORES ~ (b)

$$W_A = 6.3 \times 10^{-11}(16) = \underline{\underline{1.008 \times 10^{-9}\ gmol/s}}$$

TIME TO DISSOLVE –

$$\frac{\rho_B}{M_B}\frac{d\delta}{dt} = \frac{D_{AB}C^*}{\delta}$$

$$\int_{0.08}^{0.2} \delta\, d\delta = \frac{D_{AB}C^* M_B}{\rho_B}\int_0^t dt$$

$$\left.\frac{\delta^2}{2}\right|_{0.09}^{0.2} = \frac{D_{AB}C^* M_B}{\rho_B} t$$

$$\underline{\underline{t = 3.65 \times 10^4\ s = 10.14\ h}}$$

26.13 FOR CONDITIONS DESCRIBED –

$$\nabla \cdot \vec{N}_A = \frac{dN_A}{dz} = 0 \qquad N_{Az} \sim \text{CONSTANT}$$

$$N_{Az} = -CD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$$

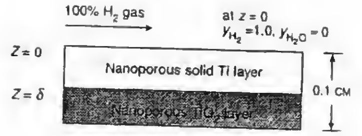
IN EACH REACTION – $N_{Bz} = -N_{Az}$

$$\therefore N_{Az} = -CD_{AB}\frac{dy_A}{dz}$$

$$N_{Az} = \frac{CD_{AB}(y_{A0} - 0)}{\delta}$$

ALL REACTIONS INVOLVE EQUIMOLAR DIFFUSION –

26.14



T = 900K P = 1 ATM

A - H_2 B - H_2O

FOR CONDITIONS STATED:

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \qquad N_{Az} \text{ CONST.}$$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} - N_{Bz})$$

SINCE $N_{Az} - N_{Bz} = 0$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz}$$

INTEGRATING: $N_{Az} = \dfrac{cD_{AB}y_{A0}}{\delta}$

$$c = \frac{P}{RT} = \frac{1}{(82.06)(900)} = 1.354 \times 10^{-5} \text{ mol/cm}^3$$

FOR $\delta = 0.05$ cm

$$N_{Az} = \frac{(0.031)(1.354\times10^{-5})}{0.05}$$

$$= \underline{8.39 \times 10^{-6} \text{ mol/cm}^2\cdot\text{s}} \quad (a)$$

BY STOICHIOMETRY:

$$\left\{\begin{matrix}\text{RATE OF}\\ \text{Ti DEPOSITED}\end{matrix}\right\} = \frac{1}{2}\left\{\begin{matrix}\text{RATE OF}\\ A_2 \text{ DIFFUSED}\end{matrix}\right\}$$

$$\frac{\rho_{Ti}}{M_{Ti}}\frac{d\delta}{dt} = \frac{1}{2}\frac{D_{AB}c_{A0}}{\delta}$$

$$\frac{\rho_T}{M_T}\int_0^{0.1}\delta\, d\delta = \frac{D_{AB}c_{A0}}{2}\int_0^t dt$$

$$\delta = \left[\frac{M_T}{\rho_T}D_{AB}c_{A0}\right]^{1/2} t^{1/2}$$

26.14 CONTINUED -

INSERTING VALUES - FOR $\delta = 0.1$ cm

$$\underline{t = 1293 \text{ s} = 0.359 \text{ h}} \quad (b)$$

@ $\delta = 0.05$ cm ; $N_A = 8.39 \times 10^{6}$ mol/cm$^2\cdot$s $= A$ (A CONSTANT)

$$A\int_0^z dz = -D_{AB}\int_{c_{A0}}^{c_A} dc_A$$

$$c_A - c_{A0} = -\left[A/D_{AB}\right]z$$

$$\underline{c_A = c_{A0} - \frac{A}{D_{AB}}z} \quad (c)$$

26.15 ACETONE (A) DIFFUSING IN AIR (B)

$$D_{AB}\big|_{298K} = 0.093 \text{ cm}^2/\text{s}$$

$$D_{AB}\big|_{323K} = 0.093\left(\frac{323}{298}\right)^{3/2} = 0.105 \text{ cm}^2/\text{s}$$

STEADY STATE - NO HOMOG. RX

$$\nabla \cdot \vec{N}_A = \frac{dN_{Az}}{dz} = 0 \qquad N_{Az} \text{ CONST.}$$

FOR T & P CONSTANT $N_{Az} = -N_{Bz}$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} = \frac{cD_{AB}}{z_2 - z_1}(y_{A1} - y_{A2})$$

$$c = \frac{P}{RT} = \frac{1 \text{ ATM}}{82.06(323)} = 3.77 \times 10^{-5} \text{ mol/cm}^3$$

$z_2 - z_1 = 500$ cm $y_{A1} - y_{A2} = 0.5$

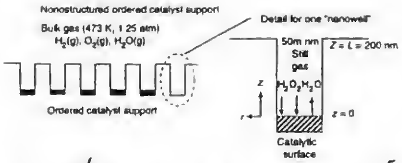
SUBSTITUTING: $N_{Az} = 3.96 \times 10^{-9}$ mol/cm$^2\cdot$s

$$W_A = N_{Az}(A)$$

$$= (3.96\times10^{-9})\left(\frac{\pi}{4}\right)(10 \text{ cm})^2$$

$$= \underline{3.11 \times 10^{-7} \text{ mol/s}}$$

26.16



$d = 5\times10^{-6}$ cm $\quad \Delta z = 2\times10^{-5}$ cm

$T = 473$ K $\quad P = 1.25$ ATM

ASSUMPTIONS – STEADY
NO HOMOGENEOUS Rx
ONE DIMENSIONAL

$$\nabla\cdot\vec{N}_A = \frac{d}{dz}N_{Az} = 0 \qquad N_{Az}\text{ CONST.}$$

$$N_{Az} = -cD_{A\text{-}MIX}\frac{dy_A}{dz} + y_A(N_{Az}+N_{Bz}+N_{Cz})$$

AT $z=0$ $\quad 2H_2(g) + O_2(g) - 2H_2O(g)$

$H_2(A)$, $O_2(B)$, $H_2O(C)$

$N_{Bz} = \frac{1}{2}N_{Az} \quad N_{Cz} = -N_{Az}$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + \frac{1}{2}y_A N_{Az}$$

$$N_{Az}\int_0^L dz = cD_{AB}\int_0^{0.01}\frac{dy_A}{1-y_A/2}$$

$$N_{Az}L = 2cD_{AMIX}\ln\frac{1-0.01/2}{1-0}$$

$$N_{Az} = \frac{2cD_{Am}}{L}(-0.0050)$$

$$c = \frac{P}{RT} = \frac{1.25}{82.06(473)} = 3.22\times10^{-5}\text{ mol}/m^3$$

$$D_{AB} = \frac{0.697}{1.25}\left(\frac{473}{273}\right)^{3/2} = 1.222\text{ cm}^2/s$$

$$D_{AC} = \frac{0.850}{1.25}\left(\frac{473}{373}\right)^{3/2} = 1.551 \text{ ''}$$

26.16 CONTINUED –

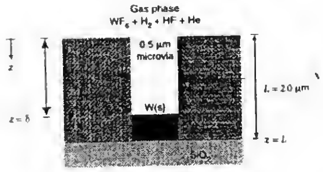
$$D_{H_2\text{-}MIX} = \frac{1}{y_B/D_{AB} + y_C/D_{AC}} = 1.274\text{ cm}^2/s$$

SUBSTITUTING NUMERICAL VALUES:

$$N_{Az} = -0.0205\text{ mol}/\text{cm}^2\cdot s$$

26.17 $\quad WF_6(A)$

KNUDSEN DIFFUSION



VERY DILUTE

$$\nabla\cdot\vec{N}_A = \frac{dN_{Az}}{dz} = 0 \qquad N_{Az}\text{ – CONSTANT}$$

$$N_{Az} = -D_{AB}\frac{dc_A}{dz} = \frac{D_{AB}c_{A0}}{\delta}$$

$$WF_6 + 3H_2 = W + 6HF$$

RATE OF FORMATION OF W

$$= N_{Az}(A) = \frac{D_{AB}c_{A0}}{\delta}(A) = \frac{\rho_W}{M_W}\frac{d(A\delta)}{dt}$$

$$\frac{\rho_W}{M_W}\frac{d\delta}{dt} = \frac{D_{AB}c_{A0}}{\delta}$$

$$\delta^2 = \frac{2D_{AB}M_W c_{A0}t}{\rho_W}$$

$$\delta = \left[2D_{AB}\frac{M_W}{\rho_W}c_{A0}\right]^{1/2}t^{1/2}$$

FOR KNUDSEN DIFFUSION – EQN (24-58)

$$D_{KA} = 4850\,d_p\sqrt{\frac{T}{M_{WF_6}}}$$

$d_p = 2.5\times10^{-5}$ cm, $T = 700$ K, $M = 298$

$$D_{KA} = 0.1858\text{ cm}^2/s$$

26.17 CONTINUED

$$C = \frac{P}{RT} = \frac{75\ Pa}{8.314(700)} = 0.0129\ mol/m^3$$

$$C_{A0} = y_{A0}C = 1.29 \times 10^{-3}\ mol/cm^3$$

SUBSTITUTING INTO EQN FOR $\delta(t)$

$$\underline{t = 8.80 \times 10^4\ s = 24.44\ h}$$

26.18 C_6H_6 (A) IN C_7H_8

$h_{fg}(A) = 30\ kJ/mol$

$h_{fg}(B) = 33$ "

$$\nabla \cdot \vec{N}_A = \frac{d N_{Az}}{dz} = 0 \qquad N_{Az}\ \text{CONST.}$$

$$N_{Az} = -CD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$$

$$N_{Az}(30) = N_{Bz}(33)$$

$$N_{Bz} = -0.909\ N_{Az}$$

$$N_{Az} = -CD_{AB}\frac{dy_A}{dz} + y_A N_{Az}(1 - 0.909)$$

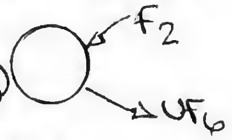
$$N_{Az}\int_0^{\delta} dz = -CD_{AB}\int_{y_{A\infty}}^{y_{As}} \frac{dy_A}{1 - 0.091 y_A}$$

$$N_{Az}\,\delta = \frac{CD_{AB}}{0.091}\ln\left[\frac{1 - 0.091 y_{As}}{1 - 0.091 y_{A\infty}}\right]$$

$$\underline{N_{Az} = \frac{CD_{AB}}{0.091\,\delta}\ln\left[\frac{1 - 0.091 y_{As}}{1 - 0.091 y_{A\infty}}\right]}$$

26.19 SPHERICAL GEOMETRY –

$U(s) + 3F_2(g) = UF_6(g)$



$T = 1000\ K \quad P = 1\ ATM$

$D_{AB} = 0.273\ cm^2/s \quad d = 0.4\ cm$

STEADY STATE, NO HOMOG. Rx

$$\nabla \cdot \vec{N}_A = \frac{1}{r^2}\frac{d}{dr} r^2 N_{Ar} = 0 \qquad r^2 N_{Ar} \sim \text{CONST.}$$

$$N_{Ar} = -CD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar} + N_{Br})$$

$$N_{Br} = -3N_{Ar} \Rightarrow N_{Ar} + N_{Br} = -2N_{Ar}$$

$$N_{Ar} = -\frac{CD_{AB}}{1 + 2y_A}\frac{dy_A}{dr}$$

$$\underbrace{4\pi r^2 N_{Ar}}_{W_A}\int_R^{\infty} \frac{dr}{r^2} = 4\pi CD_{AB}\int_{1.0}^{0} \frac{dy_A}{1 + 2y_A}$$

$$W_A\left(\frac{1}{R}\right) = \frac{4\pi CD_{AB}}{2}\ln 3.0/1.0$$

$$W_A = 2\pi R C D_{AB}\ln 3$$

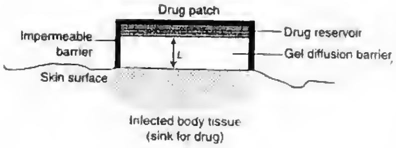
$$C = \frac{P}{RT} = \frac{1.013 \times 10^5}{8.314(1000)} = 12.18\ mol/m^3$$

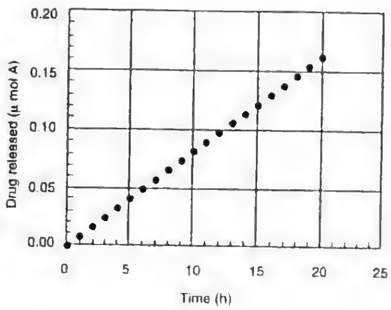
$$= 1.218 \times 10^{-5}\ mol/cm^3$$

SUBSTITUTING VALUES:

$$\underline{W_A = 4.59 \times 10^{-6}\ mol/s}$$

26.20





SLOPE OF PLOT IS W_A:

$$W_A \cong \frac{0.15\ \mu\text{mol}}{18.5\ \text{h}}\left(\frac{1}{3600}\right)$$

$$= 2.25 \times 10^{-12}\ \text{mole/s}$$

$$A_s = 9\ \text{cm}^2$$

$$N_{Az} = 2.503 \times 10^{-13}\ \text{mol/s}\cdot\text{cm}^2$$

SINCE PROFILE IS LINEAR --
ALL TRANSPORT IS DIFFUSION

$$\therefore\ N_{Az} = D_{AB}\,\frac{C_{A1} - C_{A2}}{z_2 - z_1}$$

$$D_{AB} = \frac{2.503 \times 10^{-13}(0.2)}{0.5 \times 10^{-6}}$$

$$= \underline{1.00 \times 10^{-7}\ \text{cm}^2/\text{s}}$$

MODIFIED WILKE-CHANG - EQN (24-54)

$$\frac{D_{AB}\,\mu}{T} = \text{CONST}$$

$$D_{AB}|_{35} = D_{AB\,20}\left(\frac{293}{308}\right)\left(\frac{\mu_{H_2O}|_{35}}{\mu_{H_2O}|_{20}}\right)$$

$$= 1.0 \times 10^{-7}\left(\frac{293}{308}\right)\left(\frac{0.00993}{0.00742}\right)$$

$$= 1.273 \times 10^{-7}\ \text{cm}^2/\text{s}$$

26.20 CONTINUED-

ALL OTHER TERMS REMAIN THE SAME-

$$W_A|_{35} = W_A|_{20}\,\frac{D_{AB}|_{35}}{D_{AB}|_{20}}$$

$$= \left(2.25 \times 10^{-12}\right)\frac{1.273 \times 10^{-7}}{1 \times 10^{-7}}$$

$$= 2.864 \times 10^{-12}\ \text{mol/s}$$

$$= \underline{2.475 \times 10^{-7}\ \text{mol/DAY}}$$

26.21 $J_{Az} = -C D_{AB}\dfrac{dc_A}{dz} = \dfrac{D_{AB}}{\Delta z}\left(C_{A1} - C_{A2}\right)$

$$C_{A1} - C_{A2} = k\left(p_{A1}^{1/2} - p_{A2}^{1/2}\right)$$

$$\Rightarrow\ \underline{J_{Az} = D_{AB}\,k\left(p_{A1}^{1/2} - p_{A2}^{1/2}\right)}$$

AT 1 ATM $C_{A1} = k\,p_A^{1/2}$

$$= \frac{7\ \text{cm}^3}{100\ \text{g}}\left(\frac{9\ \text{g}}{\text{cm}^3}\right) = 0.63$$

$$k = 0.63\ \text{ATM}^{-1/2}$$

$D_{AB} = 6 \times 10^{-5}\ \text{cm}^2/\text{s}$ $p_{A1} = 8\ \text{atm}$ $p_{A2} = 0$

$\Delta z = 0.2\ \text{cm}$

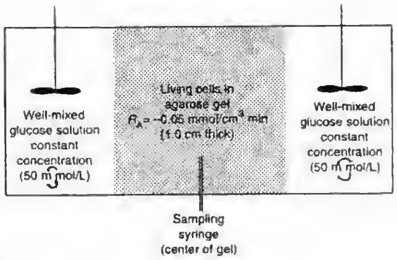
SUBSTITUTING: $J_{Az} = 5.346 \times 10^{-4}\ \text{cm/s}$

$$W_A = J_{Az} A = \left(5.346 \times 10^{-4}\right)(8)$$

$$= 4.277\ \text{cm}^3/\text{s}$$

$$= \underline{15.4\ \text{cm}^3/\text{DAY}}$$

26.22



$$\nabla \cdot \vec{N}_A - R_A = 0$$

$$\frac{dN_{Az}}{dz} - R_A = 0$$

FOR NO BULK CONTRIBUTION

$$N_{Az} = -D_{AB}\frac{dc_A}{dz}$$

$$\frac{d}{dz}\left(-D_{AB}\frac{dc_A}{dz}\right) = R_A$$

$$d\left(-D_{AB}\frac{dC_A}{dz}\right) = R_A\,dz$$

$$-D_{AB}\frac{dc_A}{dz} = R_A z + C_1$$

$$-D_{AB}C_A = R_A\frac{z^2}{2} + C_1 z + C_2$$

B.C. $C_A(0.5\text{ cm}) = C_{A0}$

$$\frac{dC_A}{dz}(0) = 0 \quad \Rightarrow \quad C_1 = 0$$

$$C_2 = -D_{AB}(C_{A0}) - \frac{R_A}{2}(0.5)^2$$

$$C_A = -\frac{1}{D_{AB}}\left[R_A\frac{z^2}{2} + C_1 z + C_2\right]$$

WITH VALUES SUBSTITUTED

$$\underline{\underline{C_A = C_{A0} - \frac{R_A}{D_{AB}}\left(z^2/2 - 0.125\right)}}$$

26.23 CYLINDRICAL GEOMETRY --

$T = 1100$ K $\quad P = 2$ ATM

δ (B.L. FILM) $= 5$ mm $\quad L = 25$ cm

$d = 2$ cm

$$\nabla \cdot \vec{N}_A = \frac{1}{r}\frac{d}{dr}(rN_{Ar}) = 0 \qquad rN_{Ar} \sim \text{CONST.}$$

$$C(s) + O_2(g) = CO_2(g)$$

O_2 IS A $\quad N_{Br} = -N_{Ar}$

CO_2 IS B

$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr}$$

$$\underbrace{2\pi L r N_{Ar}}_{W_A}\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi L c D_{AB}\int_0^{0.21} dy_A$$

$$W_A \ln\frac{r_2}{r_1} = -2\pi L c D_{AB}(0.21)$$

$$W_A = -\frac{2\pi L c D_{AB}(0.21)}{\ln(r_2/r_1)}$$

$$c = \frac{P}{RT} = \frac{2.026\times10^5}{(8.314)(1100)} = 22.1 \text{ mol}/\text{m}^3$$

$$= 2.21\times10^{-5} \text{ mol}/\text{cm}^3$$

$$D_{AB} = 0.175\left(\frac{1}{2}\right)\left(\frac{1100}{273}\right)^{3/2} = 0.708 \text{ cm}^2/\text{s}$$

AT $t=0$ $\quad r_2/r_1 = 1.5$

SOLVING FOR W_A AT $t=0$

$$\underline{\underline{W_A = -1.28\times10^{-3} \text{ mol/s}}}$$

FOR $t>0$ ~ r_1 DECREASES

$$W_A = -\frac{2\pi L c D_{AB}(0.21)}{\ln\left(\frac{r+0.5}{r}\right)}$$

26.23 CONTINUED –

FOR SOLID C: RATE OF DEPLETION = $\frac{\rho}{M}\frac{dV}{dt}$

PER MOLE: $W_A = -\frac{\rho}{M}\frac{dV}{dt}$

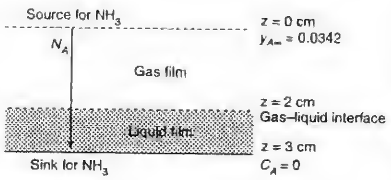
$$\frac{2\pi L c D_{AB}(0.21)}{\ln\left[(r+0.5)/r\right]} = \frac{\rho}{M}(2\pi r L)\frac{dr}{dt}$$

INTEGRATING BETWEEN $r = r_i$ @ $t=0$
$r = 0$ @ t

THE SOLUTION ~ A BIT MESSY ~

$$t = 18000\text{ s} \quad (5\text{ h})$$

26.24



EQUILIBRIUM DATA →

P_A(mmHg)	5.0	10.0	15.0	20.0	25.0	30.0
c_A(mol/m^3)	6.1	11.9	20.0	32.1	53.6	84.8

NH_3 (A) DIFFUSES, IN SERIES, THROUGH GAS & LIQUID LAYERS

THROUGH GAS: $N_{Az} = \frac{c D_{AB}}{\delta_G}\ln\left(\frac{1-y_{Ai}}{1-y_{A1}}\right)$

LIQUID: $N_{Az} = \frac{D_L}{\delta_L}(C_{Ai} - C_{AS})$

$$D_{AB} = 0.198\left(\frac{288}{273}\right)^{3/2} = 0.215\text{ cm}^2/\text{s}$$

$$D_L = 1.77\times10^{-5}\text{ cm}^2/\text{s}$$

$$N_{AG} = N_{AL}$$

$$\frac{c D_{AB}}{\delta_L}\ln\frac{1-y_{Ai}}{1-y_{A1}} = \frac{D_L}{\delta_L}(C_{Ai} - C_{AS})$$

26.24 CONTINUED –

$$c = \frac{P}{RT} = \frac{1.013\times10^5}{(8.314)(288)} = 42.3\text{ mol/m}^3$$
$$= 4.23\times10^{-5}\text{ mol/cm}^3$$

INSERTING VALUES:

$$0.257\ln\frac{1-y_{Ai}}{0.9658} = C_{Ai}$$

VALUES OF y_{Ai}, C_{Ai} MUST AGREE WITH PLOT OF DATA --

TRIAL & ERROR IS NECESSARY --

~ RESULT IS $p_{Ai} = 25.88$ mm

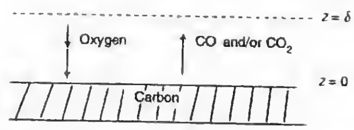
~ $y_{Ai} = \frac{25.88}{760} = 0.0339$

$$C_{Ai} = 5.58\times10^{-5}\text{ mol/cm}^3$$

WITH THESE VALUES --

$$N_{Az} = \frac{(1.77\times10^{-5})(5.58\times10^{-5})}{1}$$
$$= 9.88\times10^{-10}\text{ mol/cm}^2\cdot\text{s}$$

26.25



CONSTITUENT A IS O_2

$\nabla\cdot\vec{N}_A = \frac{d}{dz}N_{Az} = 0$ $\quad N_{Az}$ -- CONSTANT

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + N_{Bz})$$

REACTION AT SURFACE IS $C + O_2 = 2CO$

~ $2N_{Az} = -N_{Bz}$

$$y_A(N_{Az} + N_{Bz}) = y_A N_{Az}(-1)$$

~

26.25 CONTINUED –

$$N_{Az}(1+y_A) = -cD_{AB}\frac{dy_A}{dz}$$

SEPARATING VARIABLES & INTEGRATING:

FROM $z = 0$ TO δ y_A FROM 0 TO 0.21

$$N_{Az} = -\frac{cD_{AB}}{\delta}\ln 1.21 \quad (a)$$

IF REACTION AT SURFACE IS

$$C + O_2 = CO_2$$

THEN $y_A(N_{Az}+N_{Bz}) = 0$

& SOLUTION IS

$$N_{Az} = -\frac{cD_{AB}}{\delta}(0.21) \quad (b)$$

IF REACTION AT SURFACE IS

$$4C + \underset{(A)}{3O_2} = \underset{(B)}{2CO} + \underset{(C)}{2CO_2}$$

THEN $N_{Bz} = N_{Cz} = -\frac{2}{3}N_{Az}$

$$y_A(N_{Az}+N_{Bz}+N_{Cz}) = y_A N_{Az}\left(-\frac{1}{3}\right)$$

FICK'S LAW EXPRESSION BECOMES

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} - y_A\frac{N_{Az}}{3}$$

$$N_{Az}(1+y_A/3) = -cD_{AB}\frac{dy_A}{dz}$$

& SOLUTION IS

$$N_{Az} = -\frac{cD_{AB}}{\delta}\left[3\ln 1.07\right]$$

$$= \frac{cD_{AB}}{\delta}(0.203) \quad (c)$$

26.26

Time (h)	Measured SiO^2 (100) Si	film thickness (μm) (111) Si
1	0.049	0.070
2	0.078	0.105
4	0.124	0.154
7	0.180	0.212
16	0.298	0.339

SYSTEM CONSIDERED WAS EVALUATED IN TEXT – EXAMPLE 2.

$$\delta^2 = \frac{2M_B D_{AB} c_{As}}{\rho_B}t$$

FROM DATA IN TABLE –

$$\frac{d}{dt}\delta^2 = \frac{2M_B D_{AB} c_{As}}{\rho_B} = A$$

$$D_{AB} = \frac{A\rho_B}{2M_B c_{As}}$$

A EVALUATED FROM DATA VARIES FROM 0.0049 TO 0.00718 ~ TAKE MIDDLE VALUE (CONDITION @ $t = 4$ h)

$$A = 0.00593\ \mu m^2/h$$

$$= 1.646\times10^{-14}\ cm^2/s$$

$$D_{AB} = \frac{(1.646\times10^{-14})(2.27)}{2(60)(9.68\times10^{-8})}$$

$$= 3.24\times10^{-9}\ cm^2/s$$

26.27 CYLINDRICAL GEOMETRY –

$$\nabla\cdot\vec{N}_A = \frac{1}{r}\frac{d}{dr}(rN_{Ar}) = 0 \quad rN_{Ar} \sim \text{CONST.}$$

$A \sim O_2$, $B \sim CO$ $\quad N_A = -cD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar}+N_{Br})$

REACTION IS $2C + O_2 = 2CO$

$$N_{Br} = -2N_{Ar}$$

26.27 CONTINUED -

$$N_{Ar} = -\frac{cD_{AB}}{1+y_A}\frac{dy_A}{dr}$$

$$\underbrace{2\pi L\, r N_{Ar}}_{W_A}\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi L c D_{AB}\int_0^{0.4}\frac{dy_A}{1+y_A}$$

$$W_A = -\frac{2\pi L c D_{AB}\ln(1.4)}{\ln(r_2/r_1)}$$

$$c = \frac{P}{RT} = \frac{1\,\text{ATM}}{(82.04)(1145)} = 1.065\times10^{-5}\ \text{mol}/\text{cm}^3$$

$$D_{AB} = 1.0\times10^{-5}\ \text{m}^2/\text{s} = 0.10\ \text{cm}^2/\text{s}$$

SUBSTITUTING NUMERICAL VALUES -

$$W_A = -4.099\times10^{-5}\ \text{mol}/\text{s} \quad (a)$$

$$W_B = W_{CO} = -2W_A = \underline{\underline{8.20\times10^{-5}\ \text{mol}/\text{s}}}$$

FOR CONCENTRATION PROFILE:

$$\frac{d}{dr} rN_{Ar} = \frac{d}{dr}\left[-r\frac{cD_{AB}}{1+y_A}\frac{dy_A}{dr}\right] = 0$$

INTEGRATE ONCE:

$$\frac{r}{1+y_A}\frac{dy_A}{dr} = C_1$$

& AGAIN: $\ln(1+y_A) = C_1\ln r + C_2$

B.C. $y_A(r_1=0.5) = 0$

$y_A(r_2=1.5) = 0.4$

SOLVING: $C_1 = 0.306 \quad C_2 = 0.212$

NOW - FOR $r=1$

26.27 CONTINUED -

$$\ln(1+y_A) = C_1\cancelto{0}{\ln(1)} + C_2$$

$$y_A = e^{0.212} - 1 = \underline{\underline{0.236}}$$

26.28 PROBLEM STATEMENT REFERS TO EXAMPLE 4 IN CHAPTER

FOR SPHERICAL GEOMETRY -

$$\frac{1}{r^2}\frac{d}{dr}(r^2 N_{Ar}) = -k_1 C_A$$

WITH PURE DIFFUSION:

$$N_{Ar} = -D_{AB}\frac{dC_A}{dr}$$

$$\& \quad -\frac{D_{AB}}{r^2}\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) = -k_1 C_A$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) = \frac{k_1}{D_{AB}}C_A \qquad (1)$$

LETTING $y = C_A r \sim \frac{dy}{dr} = C_A + r\frac{dC_A}{dr}$

WE HAVE: $r^2\frac{dC_A}{dr} = r\frac{dy}{dr} - y \qquad (2)$

SUBSTITUTING (2) INTO (1) WE HAVE

$$\frac{d^2y}{dr^2} - \frac{k_1}{D_{AB}}y = 0$$

SOLN. IS

$$y = C_1\cosh\left(\sqrt{k_1/D_{AB}}\,r\right) + C_2\sinh\left(\sqrt{k_1/D_{AB}}\,r\right)$$

B.C. $y(0) = 0 \quad y(R) = C_{A0}$

$$\therefore C_1 = 0 \qquad C_2 = \frac{C_{A0}R}{\sinh\left(\sqrt{k_1/D_{AB}}\,R\right)}$$

26.28 CONTINUED -

$$\frac{c_A}{c_{A0}} = \frac{R}{r}\,\frac{\sinh\left(\sqrt{k_1/D_{AB}}\,r\right)}{\sinh\left(\sqrt{k_1/D_{AB}}\,R\right)} \qquad (a)$$

TO EVALUATE N_{Ar} — MUST KNOW $\frac{dc_A}{dr}$

FROM (a)

$$\frac{dc_A}{dr} = \frac{c_{A0}R}{\sinh\left(\sqrt{k_1/D_{AB}}\,R\right)} \times \left[-\frac{1}{r^2}\sinh\left(\sqrt{k_1/D_{AB}}\,r\right) + \frac{\sqrt{k_1/D_{AB}}}{r}\cosh\left(\sqrt{k_1/D_{AB}}\,r\right)\right]$$

EVALUATING AT $r=R$:

$$\left.\frac{dc_A}{dr}\right|_r = \frac{c_{A0}}{R} + c_{A0}\sqrt{k_1/D_{AB}}\coth\left(\sqrt{k_1/D_{AB}}\,R\right)$$

& FINALLY —

$$N_{Ar} = \frac{D_{AB}c_{A0}}{R}\left[1 - R\sqrt{k_1/D_{AB}}\coth\left(\sqrt{\frac{k_1}{D_{AB}}}\,R\right)\right]$$

FROM EXAMPLE 4: $D_{AB} = 2\times10^{-10}\ m^2/s$

$R = 0.002$

$c_{A0} = 0.02\ mol/m^2$

$k_1 = 0.019\ s$

SUBSTITUTING:

$$N_{Ar} = 1.02\times10^{-12}\ mol/m^2\cdot s$$

26.29 FLAT CATALYTIC SURFACE:

$$\frac{dN_{Az}}{dz} - k_1 y_B = 0 \qquad (1)$$

$$\frac{dN_{Bz}}{dz} + k_1 y_B = 0 \qquad (2)$$

ADDING: $\frac{d}{dz}(N_{Az}+N_{Bz}) = 0 \quad \{N_{Bz} = -N_{Az}\}$

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz}$$

EQN (1) BECOMES:

$$cD_{AB}\frac{d^2y_B}{dz^2} - k_1 y_B = 0$$

OR $$\frac{d^2y_B}{dz^2} - \frac{k_1}{cD_{AB}}y_B = 0$$

LETTING $b^2 = k_1/cD_{AB}$

$$\frac{d^2y_B}{dz^2} - b^2 y_B = 0$$

& SOLUTION IS

$$y_B = C_1\cosh bz + C_2\sinh bz$$

B.C. $y(0) = y_{B0}$ $\quad y(\delta) = 1$

GIVING $C_1 = y_{B0}$

$$C_2 = \frac{1 - y_{B0}\cosh b\delta}{\sinh b\delta}$$

DOING THE MATH!

$$N_{Az} = bcD_{AB}\left[\frac{1 - y_{B0}\cosh b\delta}{\sinh b\delta}\right]$$

26.30 SAME CONFIGURATION AS IN PROB 26.29 EXCEPT IN FILM

$A \underset{k_1}{\overset{k_1'}{\rightleftharpoons}} B \qquad R_A = k_1 y_B - k_1' y_A$

FICK'S LAW: $N_{Az} = -CD_{AB}\dfrac{dy_A}{dz}$

CONSERVATION OF MASS:

$$\frac{dN_{Az}}{dz} - k_1 y_B + k_1' y_A = 0$$

$$\therefore \quad -CD_{AB}\frac{d^2y_A}{dz^2} - k_1(1-y_A) + k_1' y_A = 0$$

WITH A LITTLE ALGEBRA WE GET

$$\frac{d^2y_A}{dz^2} - \frac{k_1+k_1'}{CD_{AB}}y_A = -\frac{k_1}{CD_{AB}}$$

DEFINING $M^2 = \dfrac{k_1+k_1'}{CD_{AB}}$

$$N^2 = k_1/CD_{AB}$$

OUR EQN FOR $y_A(z)$ IS

$$\frac{d^2y_A}{dz^2} - M^2 y_A = -N^2$$

SOLN IS:

$$\underline{\underline{y_A = C_1 \cosh Mz + C_2 \sinh Mz + \frac{N^2}{M^2}}}$$

B.C. $y_A(0) = y_{A0}$

$y_A(\delta) = 0$

~ DOING THE MATH ~

$$\underline{\underline{C_1 = y_{A0} - \frac{N^2}{M^2}}}$$

26.30 CONTINUED -

$$\underline{\underline{C_2 = \frac{\left(N^2/M^2 - y_{A0}\right)\cosh M\delta - N^2/M^2}{\sinh M\delta}}}$$

CHAPTER 27

27.1 – SEMI INFINITE BODY OF LIQUID

– THIS CASE IS DISCUSSED IN TEXT EQN (27-9) APPLIES:

$$\frac{\rho_A - \rho_{A0}}{\rho_{AS} - \rho_{A0}} = 1 - \text{erf}\left(\frac{z}{2\sqrt{D_{AB}t}}\right)$$

FOR O_2 IN H_2O ·· WILKE-CHANG, EQN. (24-52) APPLIES

$$D_{AB} = \frac{7.4\times10^{-8}(\phi_B M_B)^{1/2} T}{(V_b)^{0.6}\,\mu_B}$$

VALUES: $\phi_B = 2.26$ $M_B = 18$

$T = 283$ $V_B = 25.6$

$\mu_B = 0.01394$ cp

$D_{AB} = 1.469\times10^{-3}\ \text{cm}^2/\text{s}$

@ $t = 3600$ s – $2\sqrt{D_{AB}t} = 4.60$

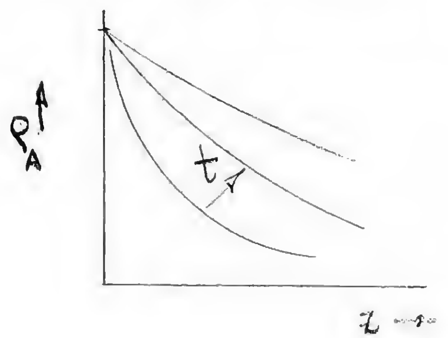
@ 36000 – $= 14.54$

@ 360000 $= 46.0$

EQN – @ 3600 s –

$$\frac{\rho_A - 2}{9 - 2} = 1 - \text{erf}\left(z/46.0\right)$$

CONC. PROFILES APPEAR AS



27.2 O_2 DISSOLVING INTO POLYMER FILM

$C_{AS} = 3.16(1.5) = 4.74\ \text{gmol/m}^3$

FOR $t = 10$ s – VERY SHORT PENETRATION – EQN (27-11) APPLIES

$$N_{Az} = \sqrt{\frac{D_{AB}}{\pi t}}\,(C_{AS} - C_{A0})$$

$$= \left[\frac{1\times10^{-11}}{\pi(10)}\right]^{1/2}(4.74 - 0.39)$$

$$= \underline{\underline{2.45\ \text{gmol/m}^2\cdot\text{s}}} \quad a)$$

FOR $C_A = 3\ \text{gmol/m}^3$ @ $z = 4$ mm

$$\frac{C_A - C_{A0}}{C_{AS} - C_{A0}} = 1 - \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

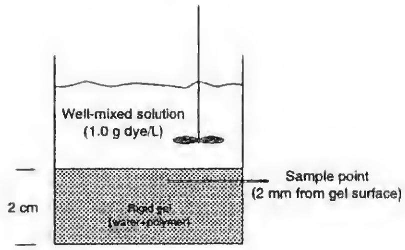
$$\frac{3 - 0.39}{4.74 - 0.39} = 0.6 = 1 - \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

$$\text{erf}\,\frac{z}{2\sqrt{D_{AB}t}} = 0.4$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 0.372$$

$$\boxed{\begin{aligned} t &= 2.89\times10^{6}\ \text{s} \\ &= 802.8\ \text{h} \\ &= 33.4\ \text{DAYS}\end{aligned}}$$

27.3



$C_{A0} = 0$

$C_{A\infty} = 1.0$

FOR $t = 24$ h

$C_A = 0.203$ @ $z = 2$ mm

27.3 CONTINUED -

$$\frac{C_{AS}-C_A}{C_{AS}-C_{A0}} = \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{1.0-0.203}{1.0} = 0.797 = \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

FOR $t = 24\,h = 86400\,s$

$z = 0.2\,cm$

$\underline{\underline{D_{AB} = 1.43\times10^{-7}\ cm^2/s}}$ a)

ASSUMPTIONS:

1. SURFACE CONCENTRATION CONSTANT — $\neq C_{AS}(t)$
2. ONE DIRECTIONAL DIFFUSION
3. CONSTANT D_{AB} b)

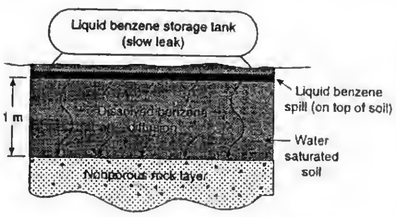
USING WILKE-CHANG EQN: (24-54)

$$D_{AB}|_{T_2} = D_{AB}|_{t_1}\,\frac{\mu_{B1}\,T_2}{\mu_{B2}\,T_1}$$

$$= 1.43\times10^{-7}\left(\frac{9.93\times10^{-4}}{6.58\times10^{-4}}\right)\left(\frac{313}{293}\right)$$

$$= \underline{\underline{2.30\times10^{-7}\ cm^2/s}} \quad (c)$$

27.4



$T = 293\,K$

$C_A^* = 24\ mol/m^3$

$C_{A0} = 0$

$D_{A\,eff} = 1\times10^{-9}\ m^2/s$

$t = 72\,h$

$= 2.59\times10^5\,s$

27.4 CONTINUED -

$$\frac{z}{2\sqrt{D_{AB}t}} = \frac{0.05}{2\left[(1\times10^{-9})(2.59\times10^5)\right]}$$

$$= 1.553$$

$$\text{erf}(1.553) = 0.972 = \frac{C_{AS}-C_A}{C_{AS}-C_{A0}}$$

$$\underline{\underline{C_A = 0.672\ mol/m^3}}$$

27.5 H_2 INTO Fe

$D_{AB} = 1.24\times10^{-11}\ cm^2/s$

$C_{A0} = 0$

C_A @ 0.1 cm $= 1.76\times10^{-7}$ mol/g Fe

$C_{AS} = 2.2\times10^{-7}$ mol/g Fe

$T = 373$ $P = 1\,ATM$

$$\frac{C_{AS}-C_A}{C_{AS}-C_{A0}} = \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{2.2-1.76}{2.2-0} = 0.2 = \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 0.179$$

$$\underline{\underline{t = 6.27\times10^5\,S = 174\,h}}$$

27.6 HERBICIDE INTO SOIL -

$D_{AB} = 1\times10^{-8}\ m^2/s$ $t = 1800\,s$

$C_{AS} = 1$ $C_{A0} = 0$ $C_A = 0.001$

$$\frac{C_A-C_{A0}}{C_{AS}-C_{A0}} = \frac{0.001}{1} = 1-\text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

27.6 CONTINUED –

$$\operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}} = 0.999$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 2.25$$

$$z = 2(2.25)\left[(1\times10^{-3})(1800)\right]^{1/2}$$

$$= \underline{6.037 \text{ m}}$$

27.7 BORON DIFFUSING INTO Si

$C_{AS} = 5\times10^{20}$ Atoms/cm^3

$C_{Az} = 0.17\times10^{20}$ " @ $z = 2\times10^{-7}$ m

$C_{Ao} = 0$ $t = 1800$ s

$$\frac{C_{AS}-C_{Az}}{C_{AS}-C_{Ao}} = \frac{(5-0.17)\times10^{20}}{5\times10^{20}} = 0.966$$

$$= \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

From Appendix L -- $\frac{z}{2\sqrt{D_{AB}t}} = 1.5$

$$D_{AB} = \left(\frac{2\times10^{-7}}{3}\right)^2 (1800)^{-1}$$

$$= \underline{2.469\times10^{-18} \text{ m}^2/\text{s}}$$

As Stated — $D_{AB} = D_o e^{-Q_o/RT}$

$$\ln \frac{D_{AB}}{D_o} = -\frac{Q_o}{RT}$$

$$T = \frac{Q_o}{R \ln D_o/D_{AB}}$$

$$= \frac{2.74\times10^5}{(8.314)\ln \dfrac{1.9\times10^{-6}}{2.469\times10^{-18}}}$$

$$= \underline{\underline{1204 \text{ K}}}$$

27.8 CARBON DIFFUSING INTO STEEL

$$\left.\begin{array}{l}\omega_{AS} = 0.007\\ \omega_{Ao} = 0.002\end{array}\right\} \frac{0.007-\omega_A}{0.007-0.002} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{0.007-\omega_A}{0.005} = \operatorname{erf}\left[\frac{z}{2\left((1\times10^{-11})(3600)\right)}\right]$$

$$\omega_A = 0.007 - 0.005 \operatorname{erf}\left[\frac{z}{3.7\times10^{-4} \text{ m}}\right]$$

For $z = 0.01$ cm

$$\operatorname{erf}[\ ] = \operatorname{erf}(0.264) = 0.291$$

$$\omega_A = 0.007 - 0.005(0.291)$$

$$= \underline{\underline{0.55 \text{ wt \% C}}}$$

For $z = 0.02$ cm

$$\operatorname{erf}[\ ] = \operatorname{erf}(0.528) = 0.545$$

$$\omega_A = 0.007 - 0.005(0.528) = \underline{\underline{0.427 \text{ wt \% C}}}$$

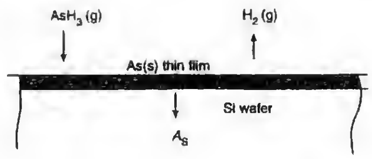
For $z = 0.04$ cm

$$\operatorname{erf}[\ ] = \operatorname{erf}(1.056) = 0.866$$

$$\omega_A = 0.007 - 0.005(0.866) = \underline{0.267 \text{ wt \% C}}$$

27.9

As into Si:



$$\frac{C_{AS}-C_A}{C_{AS}-C_{Ao}} = \operatorname{erf} \frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = \frac{2\times10^{-4}}{2\left[(5\times10^{-13})(3600)\right]^{1/2}} = 2.357$$

27.9 CONTINUED

$$\text{Erf}(2.357) = 0.9990 = \frac{2\times10^{21} - C_A}{2\times10^{21} - 1\times10^{12}}$$

$$\underline{C_A = 2.0\times10^{18}\ \text{ATOMS/cm}^3}$$

$$N_{Az} = \left[\frac{D_{AB}}{\pi t}\right]^{1/2}(C_{As} - C_{Ao})$$

$$= \left[\frac{5\times10^{-13}}{\pi(3600)}\right]^{1/2}\left(2\times10^{21} - 1\times10^{12}\right)$$

$$= 1.33\times10^{13}\ \text{ATOMS/cm}^2\cdot\text{s}$$

27.10 SAME SITUATION AS PROB 27.9

$C_{As} = 2\times10^{21}$ ATOMS/cm^3

$C_{Ao} = 1\times10^{12}$ "

$C_A = 2\times10^{17}$ "

$$\text{Erf}\frac{z}{2\sqrt{D_{AB}t}} = \frac{2\times10^{21} - 2\times10^{17}}{2\times10^{21} - 1\times10^{12}}$$

$$= 0.9999$$

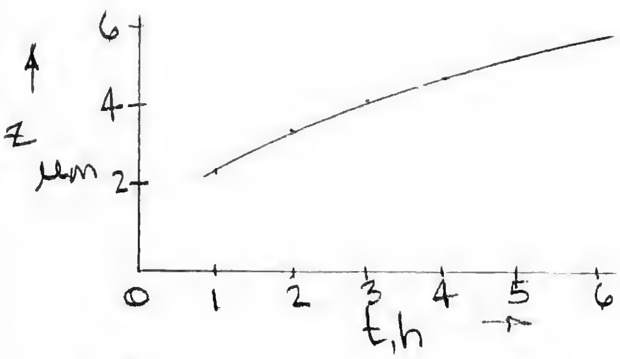
$$\frac{z}{2\sqrt{D_{AB}t}} = 2.8$$

$$z = 5.6\left(5\times10^{-13}\right)^{1/2} t^{1/2}$$

$$= 3.960\times10^{-6}\, t^{1/2}$$

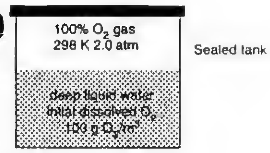
t(s)	$t^{1/2}$	z (cm)$\times10^4$
3600	60	2.346
7200	84.8	3.36
10800	103.9	4.115
14400	120	4.75
18000	134.2	5.31
21600	147.0	5.82

27.10 - CONTINUED



$z(t^{1/2})$ IS OBVIOUSLY LINEAR WITH SLOPE $= 0.0396\ \mu\text{m/s}^{1/2}$

27.11 O_2 (A) INTO H_2O (B)



$T = 298\text{K}$, $P = 2$ ATM

$C_{Ao} = 10\ \text{g/m}^3$

$C_{As} = 2.0/0.8 = 2.5\ \text{mol/m}^3$

$= 2.5(32) = 80\ \text{g/m}^3$

$$\frac{C_{As} - C_A}{C_{As} - C_{Ao}} = \frac{80-20}{80-10} = 0.857 = \text{Erf}\left[\frac{z}{2\sqrt{D_{AB}t}}\right]$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 1.038$$

$$t = \left[\frac{z}{2(1.038)}\right]^2\left[\frac{1}{D_{AB}}\right]$$

$$= \left[\frac{0.3}{2(1.038)}\right]^2\left[\frac{1}{2.1\times10^{-5}}\right]$$

$$\underline{= 994\ \text{s} = 16.6\ \text{m}}$$

27.12 A DIFFUSING INTO SEMI-INFINITE MEDIUM

$C_{AS} = 2\ mol/m^3$

$C_{AO} = 0$ "

$C_A = 0.2$ " @ $z = 5\ mm$

$$\frac{C_{AS}-C_A}{C_{AS}-C_{AO}} = \frac{2-0.2}{2} = 9.0 = \text{erf}\,\frac{z}{2\sqrt{D_{AB}t}}$$

$$\frac{z}{2\sqrt{D_{AB}t}} = 1.165$$

$$t = \left[\frac{z}{2(1.165)}\right]^2 \left(D_{AB}^{-1}\right)$$

$$= \left[\frac{0.5}{2(1.165)}\right]^2 \left(1\times10^{-6}\right)^{-1}$$

$$= 46053\ s = 12.79\ h$$

27.13 REFER TO PROB 27.4 —

C_6H_6 DIFFUSING INTO H_2O SATURATED SOIL.

ANALYTICAL SOLN GIVEN BY EQ (27-16):

$$\frac{C_A-C_{AS}}{C_{AO}-C_{AS}} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi z}{L}\right)e^{-(n\pi/2)^2 X_D}$$

FOR n ODD ; $X_D = D_{AB}t/x_1^2$

IN THE PRESENT CASE: $x_1 = \frac{L}{2} = 1\ m$

CALCULATIONS MADE USING SPREAD SHEET — R.H. COLUMN

PROCEDURE IS TO GUESS A VALUE OF t & SOLVE CONTINUOUSLY UNTIL $C_A = 1\ g/m^3$ @ $z = x_1$

27.13 CONTINUED —

EXCEL SPREADSHEET

$T = 293\ K$

$M_A = 78$ $\quad D_{AB} = 1\times10^{-9}\ m^2/s$

$C_{AS} = 24.0\ mol/m^3$

$C_{AO} = 0$

n	Term	Summation	% Change
1	1.16E+00	1.160E+00	
3	-1.84E-01	9.763E-01	18.8
5	2.50E-02	1.001E+00	2.5
7	-1.92E-03	9.994E-01	0.2
9	7.67E-05	9.995E-01	0.0
11	-1.53E-06	9.995E-01	0.0
13	1.50E-08	9.995E-01	0.0
15	-7.19E-11	9.995E-01	0.0
17	1.67E-13	9.995E-01	0.0
19	-1.86E-16	9.995E-01	0.0
21	1.00E-19	9.995E-01	0.0
23	-2.59E-23	9.995E-01	0.0
25	3.21E-27	9.995E-01	0.0

RESULT: $t = 3.763\times10^7\ s$

$= 10452\ h$

$= 435.5$ DAYS

27.14 REFER TO PROBLEM 27.4

FLUX EXPRESSION GIVEN AS EQN (27-17).

$$N_{Az} = \frac{4D_{AB}}{L}(C_{AS}-C_{AO})\sum_{n=1}^{\infty}\cos\left(\frac{n\pi z}{L}\right)e^{-\frac{n^2\pi^2 D_{AB}t}{4x_1^2}}$$

WITH $x_1 = \frac{L}{2} = 1\ m$ $\quad N_{Az} = 0$ @ $x_1 = 1$

$$m_A(t) = W^2\int_0^t N_A(t)\Big|_{z=0}\,dt$$

27.14 CONTINUED --

SPREADSHEET FOR SUMMATION:

n	Term	Summation	% Change
1	1.17E-01	1.168E-01	
3	6.79E-02	1.847E-01	36.7
5	3.18E-02	2.164E-01	14.7
7	1.65E-02	2.330E-01	7.1
9	1.00E-02	2.430E-01	4.1
11	6.70E-03	2.497E-01	2.7
13	4.80E-03	2.545E-01	1.9
15	3.60E-03	2.581E-01	1.4
17	2.80E-03	2.609E-01	1.1
19	2.25E-03	2.631E-01	0.9
21	1.84E-03	2.650E-01	0.7
23	1.53E-03	2.665E-01	0.6
25	1.30E-03	2.678E-01	0.5
27	1.11E-03	2.689E-01	0.4
29	9.64E-04	2.699E-01	0.4
31	8.43E-04	2.707E-01	0.3
33	7.44E-04	2.715E-01	0.3
35	6.62E-04	2.721E-01	0.2
37	5.92E-04	2.727E-01	0.2
39	5.33E-04	2.733E-01	0.2
41	4.82E-04	2.737E-01	0.2
43	4.38E-04	2.742E-01	0.2
45	4.00E-04	2.746E-01	0.1
47	3.67E-04	2.749E-01	0.1
49	3.38E-04	2.753E-01	0.1
51	3.12E-04	2.756E-01	0.1

RESULT: FOR $t = 2\,Y = 6.3\times10^{7}$ s

$M_A = 516$ grams

27.15. CONCENTRATION PROFILE IN A SLAB W/ NO SURFACE RESISTANCE IS EXPRESSED BY EQN (27-16).

$$\frac{C_A - C_{AS}}{C_{A0}-C_{AS}} = \frac{4}{\pi}\sum_1^\infty \frac{1}{n}\sin\frac{n\pi z}{L}\, e^{-\left(\frac{n\pi}{2}\right)^2 X_D}$$

n ODD -

27.15 CONTINUED -

MEAN CONCENTRATION: $\bar{C} = \dfrac{\int_0^L C_A\,dz}{\int_0^L dz}$

SUBSTITUTING:

$$\bar{C}_A = \frac{4}{\pi}(C_{A0}-C_{AS})\left[\int_0^L \frac{1}{n} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \times \sin\left(\frac{n\pi z}{L}\right)dz + C_{AS}\,dz\right]$$

$$= -\frac{4}{\pi}(C_{AS}-C_{AS})\left[\sum_1^\infty \frac{L}{n^2\pi} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \times \cos\left(\frac{n\pi z}{L} + C_{AS}L\right)\right]$$

$$\frac{\bar{C}_A - C_{AS}}{C_{A0}-C_{AS}} = -\frac{4}{\pi^2}\sum^\infty \frac{1}{n^2} e^{-\left(\frac{n\pi}{2}\right)^2 X_D}\left[-1-(-1)\right]$$

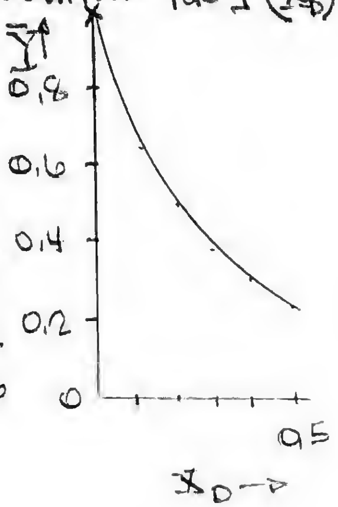
$$= \frac{8}{\pi^2}\sum_1^\infty \frac{1}{n^2} e^{-\left(\frac{n\pi}{2}\right)^2 X_D} \quad \text{- n ODD}$$

FOR $\bar{Y} = \dfrac{\bar{C}_A - C_{AS}}{C_{A0}-C_{AS}}$

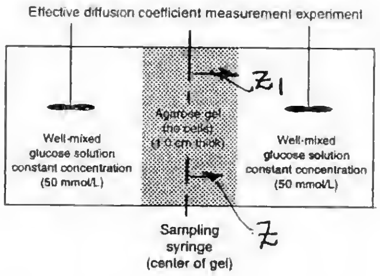
$$\bar{Y} = 0.8106\left[e^{-\left(\frac{\pi}{2}\right)^2 X_D} + \frac{1}{9}e^{-\left(\frac{3\pi}{2}\right)^2 X_D} + \frac{1}{25}e^{-\left(\frac{5\pi}{2}\right)^2 X_D} + \cdots\right]$$

DOING THE CALCULATION FOR $\bar{Y}(X_D)$:

X_D	$\bar{Y}$
0	1.0
0.1	0.643
0.2	0.496
0.3	0.387
0.4	0.302
0.5	0.236



27.16



$C_{A0} = 0$

$C_A|_{t=42h} = 48.5$ mmol/L

GOVERNING DIFFERENTIAL EQN:

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2} \qquad (a)$$

CHARTS WILL BE USED –

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{50 - 48.5}{50} = 0.03$$

FIGURE F.4

$$\text{AT } \frac{z}{z_1} = 0 \quad X = \frac{D_{AB}t}{x_1^2} \cong 1.6$$

$$D_{AB} = \frac{1.6(0.5)^2}{42(3600)} = \underline{\underline{2.64 \times 10^{-6}\ cm^2/s}}$$

27.17 CYLINDRICAL GEOMETRY

$D_{AB} = 4 \times 10^{-7}\ cm^2/s$

$L = 5$ cm $\qquad C_A' = 2.0$ g mol/m^3

$r = 0.5''$ $\qquad K = 1.5 \dfrac{cm^3 \text{ FLUID}}{cm^3 \text{ ABSORBENT}}$

$C_A = KC_A'$

$C_{A0} = 0$

$C_{AS} = 1.5\, C_A' = 3.0$ gmol/m^3

$C_A = 2.94$ gmol/m^3 @ $x = 0.1$ cm

27.17 CONTINUED –

USING CHARTS – SINCE $L \gg r$ WE ASSUME DIFFUSION IS ONLY SIGNIFICANT IN THE r-DIRECTION

$$Y = \frac{C_{AS} - C_A}{C_{AS} - C_{A0}} = \frac{3.0 - 2.94}{3.0} = 0.02$$

FIGURE F.2 @ $\dfrac{x}{x_1} = 0.2 \quad m = 0$

$$X \cong 0.75 = \frac{D_{AB}t}{r^2}$$

$$t = \frac{0.75(0.5)^2}{4 \times 10^{-7}} = \underline{\underline{4.69 \times 10^5\ s}}$$

$= 130.3$ h

$= 5.428$ DAYS

27.18 SPHERICAL GEOMETRY

$D_{AB} = 1.5 \times 10^{-7}\ cm^2/s$

AT $r = 0$ $\quad C_A = 0.02$

$C_{A0} = 0.2$

$r_1 = 0.05$ cm

$$\frac{C_A - C_{AS}}{C_{A0} - C_{AS}} = 0.10$$

USING CHARTS – FIG F.9

$$n = \frac{r}{r_1} = 0 \qquad m = 0$$

$$X = 0.3 = \frac{D_{AB}t}{r_1^2}$$

$$t = \frac{0.3(0.05)^2}{1.5 \times 10^{-7}} = \underline{\underline{5 \times 10^3\ s}}$$

$= \underline{\underline{1.389\ h}}$

27.19 - TRANSIENT DRYING OF A SLAB

$D_{AB} = 1.3 \times 10^{-4}\ cm^2/s$

@ $t=0$ $\omega = 15\%$ BY WT

@ $x = x_1$ $\omega = 4\%$ BY WT

DESIRED ω @ $x = \frac{x_1}{2} = 10\%$ BY WT.

MOISTURE CONCENTRATIONS MUST BE EXPRESSED IN CONSISTENT TERMS — WT H_2O PER WT DRY SOLID

$$\therefore \omega_0' = \frac{0.15}{1-0.15} = 0.1765\ \frac{g\,H_2O}{g\,D.S.}$$

$$\omega_s' = \frac{0.04}{1-0.04} = 0.0417 \quad "$$

$$\omega_\Delta' = \frac{0.10}{1-0.10} = 0.111 \quad "$$

FOR CHART SOLN:

$$Y = \frac{0.111 - 0.0417}{0.1765 - 0.0417} = 0.515$$

$$n = x/x_1 = 0.5$$

$$m = 0$$

FIG F.7 — @ $n = 0.4$ $\frac{D_{AB}t}{x_1^2} \cong 0.24$

@ $n = 0.6$ " $\cong 0.16$

$\therefore$ @ $n = 0.5$ $\frac{D_{AB}t}{x_1^2} \cong 0.20$

$$t = \frac{0.20(5)^2}{1.3 \times 10^{-4}} = \underline{\underline{38460\ s}}$$

$$= \underline{\underline{10.68\ h}}$$

27.20

AL DIFFUSES INTO Si

$T = 1250\ K$ $t = 10h = 3.6 \times 10^4\ s$

FIG 24.6 ~ $D_{AB} \cong 1.1 \times 10^{-13}\ cm^2/s$

$x_1 = 0.5\ \mu m$

CONDITION SOUGHT IS ω_A @ $\frac{x}{x_1} = 0.5$

CHART SOLUTION ISN'T POSSIBLE SINCE B.C. ON TOP Si SURFACE IS UNKNOWN

PRESUMING Si THICKNESS IS LARGE COMPARED TO PENETRATION DEPTH — CONSIDER THIS A SEMI-INFINITE SITUATION:

$$\frac{\omega_{AS} - \omega_A}{\omega_{AS} - \omega_{A0}} = \text{erf}\ \frac{z}{2\sqrt{D_{AB}t}} \qquad \{\text{EQ } (27\text{-}10)\}$$

$$\frac{z}{2[D_{AB}t]^{1/2}} = \frac{0.25 \times 10^{-4}}{2\left[(1.1 \times 10^{-13})(3.6 \times 10^4)\right]^{1/2}}$$

$$\cong 0.1986$$

$$\text{Erf}(0.1986) \cong 0.2212$$

$$= \frac{0.01 - \omega_A}{0.01} \qquad \omega_A = 0.00779$$

$$\underline{\underline{\cong 0.779\ \%}}$$

27.21 SPHERICAL GEOMETRY

$$D_{AB} = 2\times10^{-6}\ cm^2/s$$

$$r_1 = 0.25\ cm$$

$$C_{AS} = 0.1(150) = 15\ mol/m^3$$

$$C_A(r=0) = 12\ mol/m^3$$

$$\frac{C_{AS}-C_A}{C_{AS}-C_{A0}} = \frac{15-12}{15} = 0.2$$

$$n = \frac{x}{x_1} = 0 \qquad m = 0$$

FIG F.9 $\quad X_D = \frac{D_{AB}t}{r_1^2} \hat{=} 0.25$

$$t = \frac{0.25(0.25)^2}{2\times10^{-6}} = \underline{\underline{7812\ s}}$$

$$= \underline{\underline{2.17\ h}}$$

27.22 RECTANGULAR BLOCK
- EDGES SEALED -

$$C_{A0} = 64\ mg/cm^3$$

$$C_{AS} = 0$$

$$D_{AB} = 3\times10^{-7}\ cm^2/s$$

$$V = (1\ cm)(0.652\ cm)L$$

L = OTHER DIMENSION

$$\frac{41.7}{64} = (1)(0.652)L$$

$$L = 1\ cm$$

$$\frac{D_{AB}t}{x_1^2} = \frac{(3\times10^{-7})(96)(3600)}{(0.652/2)^2}$$

$$= 0.976$$

27.22 CONTINUED -

$$n = 0 \qquad m = 0$$

FIG F.7 $\quad Y \cong 0.11$

$$\frac{C_{AS}-C_A}{C_{AS}-C_{A0}} = \frac{0-C_A}{0-64} = 0.11 \qquad \underline{\underline{C_A = 7.04\ mg/cm^3}}$$

27.23 CYLINDER: $r = 1.25$ cm
$L = 80$ cm

$$C_{A0} = 30\ wt\% = \frac{0.3}{1-0.3} = 0.429\ g_A/g_{D.S.}$$

$$C_{AS} = 1\ wt\% = \frac{0.01}{1-0.01} = 0.0101 \quad ''$$

SINCE $L \gg r$ – VIRTUALLY ALL DIFFUSION IS IN r-DIRECTION

$$C_A(r=0) = 18\ wt\% = \frac{0.18}{1-0.18} = 0.2195$$

AT $t = 36000$ s (10h)

$$Y = \frac{w_A' - w_{AS}'}{w_{A0}' - w_{AS}'} = \frac{0.219-0.0101}{0.429-0.0101} \cong 0.50$$

FOR $n = m = 0$ $\quad X = \frac{D_{AB}t}{r^2} = 0.2$

$$\frac{D_{AB}}{r^2} = \frac{0.2}{10h}$$

AFTER 15 h. $\quad X = \frac{D_{AB}}{r^2}(15) = 0.3$

FIG F.8 $\quad Y \cong 0.29 = \frac{w_A' - 0.0101}{0.429-0.0101}$

$$w_A' = 0.1316$$

$$wt\% = \frac{w_A}{1-w_A} = 0.1316 \qquad \underline{\underline{w_A = 11.6\%}}$$

27.24 SPHERICAL GEOMETRY -

$r = 0.1$ cm

FOR H_2O IN AIR — $D_{AB} = 0.260\ cm^2/s$ @ 298 K, 1 ATM

$\omega_{A0} = 0$

$\omega_A(r=0) = 0.9\ \omega_{AS}$

$$Y = \frac{\omega_{AS} - \omega_A}{\omega_{AS} - \omega_{A0}} = \frac{1-0.9}{1} = 0.1$$

FIG F.9 — $n = m = 0$

$$X_D = \frac{D_{AB}t}{r^2} \cong 0.3$$

$$t = \frac{0.3(0.1)^2}{0.260} = \underline{\underline{0.0115\ s}}$$

27.25 RECTANGULAR SOLID -

10 cm × 10 cm × 45 cm

H_2O DIFFUSES: $D_{AB} = 1.04 \times 10^{-5}\ cm^2/s$

$C_{A0} = 45$ wt %, $\omega_{A0} = \frac{0.45}{1-0.45} = 0.818$

$C_{AS} = 15$ " $\quad \omega_{AS} = \frac{0.15}{1-0.15} = 0.176$

$C_A = 25$ " $\quad \omega_A = \frac{0.25}{1-0.25} = 0.333$

$$Y = \frac{\omega_{AS} - \omega_A}{\omega_{AS} - \omega_{A0}} = \frac{0.176 - 0.333}{0.176 - 0.818} = 0.244$$

$$= Y_1 Y_2 = Y_S^2$$ SINCE SIDES HAVE SAME DIMENSIONS

$$\therefore\ Y_S = (0.244)^{1/2} = 0.494$$

27.25 CONTINUED -

USING FIG F.7

$n = m = 0 \quad X_D \cong 0.39 = \frac{D_{AB}t}{x_1^2}$

$$t = \frac{0.39(5)^2}{1.04 \times 10^{-5}} = \underline{\underline{9.375 \times 10^5\ s}}$$

$$= \underline{260.4\ h}$$

$$= \underline{10.85\ \text{DAYS}}$$

IF ALL DIFFUSION IS FROM ENDS:

$Y = 0.244$

$X_D \cong 0.72 = \frac{D_{AB}t}{x_1^2}$

$$t = \frac{0.72(22.5)^2}{1.04 \times 10^{-5}} = \underline{3.50 \times 10^7\ s}$$

$$= \underline{9736\ h}$$

$$= \underline{\underline{405.6\ \text{DAYS}}}$$

CHAPTER 28

28.1 FOR O_2 DIFFUSING IN AIR @ 300 K, 1 ATM

$D_{AB}(273K) = 0.175\ cm^2/s$

$D_{AB}(300K) = 0.175\left(\frac{300}{273}\right)^{3/2} = 0.202\ cm^2/s$

@ 300 K — $\nu_{AIR} = 0.1569\ cm^2/s$

$Sc = \frac{0.1569}{0.202} = \underline{\underline{0.777}}$ (a)

FOR O_2 IN H_2O @ 300 K

$D_{AB} = 1.5 \times 10^{-9}\ m^2/s$

$\nu_{H_2O} = 0.880 \times 10^{-6}\ m^2/s$

$Sc = \frac{0.880 \times 10^{-6}}{1.5 \times 10^{-9}} = \underline{\underline{587}}$ (b)

28.2 SiH_4 IN He 900 K
(A) (B) 100 Pa

$y_{SiH_4} = 0.01$

D_{AB} @ 298 K, 101.3 kPa $= 0.518\ cm^2/s$

$D_{AB\,T,P} = (0.518)\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)^{3/2}\left(\frac{\Omega_{D1}}{\Omega_{D2}}\right)$

VALUES: $\epsilon_{AB}/k = 46.06$

@ 298 K $\epsilon_{AB}/kT = 6.470$ $\Omega_D = 0.802$

@ 900 K $\epsilon_{AB}/kT = 19.54$ $\Omega_D = 0.1668$

$D_{AB\,T,P} = (0.518)\left(\frac{1.013 \times 10^5}{1100}\right)\left(\frac{900}{298}\right)^{3/2} \times \frac{0.802}{0.1668}$

28.2 CONTINUED -

$D_{AB\,T,P} = 3.31 \times 10^3\ cm^2/s$

ν_{He} @ 900 K $= 6 \times 10^{-3}\ ft^2/s\left(\frac{0.3048\ m}{ft}\right)^2$

$= 5.574 \times 10^{-4}\ m^2/s$

$= 5.574\ cm^2/s$

$Sc = \frac{5.574}{3310} = \underline{\underline{0.001684}}$

28.3 Cl_2 IN $SiCl_4$ (l)
(A) (B)

FOR D_{AB} — USE WILKE-CHANG EQN.
— EQ (24-52)

$D_{AB} = \frac{7.4 \times 10^{-8}(M_B \phi_B)^{1/2}}{V_A^{0.6}} \frac{T}{\mu_B}$

VALUES: $\phi_B = 1.0$ $M_B = 170$

$\mu_B = 5.2 \times 10^{-4}\ kg/m.s$

$= 0.52\ cp$

$V_A = 48.4$

SUBSTITUTING: $D_{AB} = 5.395 \times 10^{-5}\ cm^2/s$

$\nu = \frac{\mu}{\rho} = \frac{5.2 \times 10^{-5}\ g/m.s}{1.47\ g/cm^3} = 3.54 \times 10^{-3}\ cm^2/s$

$Sc = \frac{3.54 \times 10^{-3}}{5.395 \times 10^{-5}} = \underline{\underline{65.6}}$

28.4 $St = k_c / v_\infty$

$$= \frac{k_c L}{D_{AB}} \frac{\nu}{L v_\infty} \frac{D_{AB}}{\nu}$$

$$= Sh / Re\,Sc$$

$$Pe = \frac{v_\infty L}{D_{AB}}$$

$$= \frac{v_\infty L}{\nu} \frac{\nu}{D_{AB}} = Re\,Sc$$

28.5

VARIABLE	SYMBOL	DIM.
MASS TX COEF	k_c	Lt^{-1}
LENGTH	L	L
VELOCITY	v	Lt^{-1}
VISCOSITY	μ	$ML^{-1}t^{-1}$
DIFFUSIVITY	D_{AB}	L^2t^{-1}
DENSITY	ρ	ML^{-3}

$i = n - r = 6 - 3 = 3$ π GROUPS

CORE – D_{AB}, ρ, L

$$\pi_1 = D_{AB}^a \rho^b L^c k_c$$

$$= \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{L}{t}$$

M: $0 = b$
L: $0 = 2a - 3b + c + 1$
t: $0 = -a - 1$

$a = -1 \quad b = 0 \quad c = 1$

$$\pi_1 = k_c L / D_{AB} = Sh$$

28.5 CONTINUED –

$$\pi_2 = D_{AB}^d \rho^e L^f v = \left(\frac{L^2}{t}\right)^d \left(\frac{M}{L^3}\right)^e L^f \frac{L}{t}$$

M: $0 = e$
L: $0 = 2d - 3e + f + 1$
t: $0 = -d - 1$

$d = -1 \quad e = 0 \quad f = 1$

$$\pi_2 = vL / D_{AB}$$

$$\pi_3 = D_{AB}^g \rho^h L^i \mu = \left(\frac{L^2}{t}\right)^g \left(\frac{M}{L^3}\right)^h L^i \frac{M}{Lt}$$

M: $0 = h + 1$
L: $0 = 2g - 3h + i - 1$
t: $0 = -g - 1$

$g = -1 \quad h = -1 \quad i = 0$

$$\pi_3 = \frac{\mu}{\rho D_{AB}} = Sc$$

– NOTE THAT $\pi_2 / \pi_3 = \frac{v \rho L}{\mu} = Re$

28.6

VARIABLE	SYMBOL	DIMENSIONS
MASS	M	M
DIAMETER	D	L
g	g	L/t^2
SURFACE TENSION	σ	M/t^2
DENSITY (L)	ρ_L	M/L^3
VISCOSITY (L)	μ_L	M/Lt
VELOCITY	v	L/t
DENSITY (g)	ρ_g	M/L^3
VISCOSITY (g)	μ_g	M/Lt

$i = n - r = 9 - 3 = 6$ π GROUPS

CORE – ρ_L, μ_L, D

28.6 CONTINUED -

$$\pi_1 = \rho_L^a \mu_L^b D^c v = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c \frac{L}{t}$$

M: $0 = a + b$ $\qquad a = 1$

L: $0 = -3a - b + c + 1$ $\qquad c = 1$

t: $0 = -b - 1$ $\qquad b = -1$

$$\underline{\pi_1 = \rho_L D v / \mu_L = Re}$$

$$\pi_2 = \rho_L^a \mu^b D^c g = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c \frac{L}{t^2}$$

M: $0 = a + b$ $\qquad a = 2$

L: $0 = -3a - b + c + 1$ $\qquad c = 3$

t: $0 = -b - 2$ $\qquad b = -2$

$$\underline{\pi_2 = \frac{\rho^2 D^3 g}{\mu_L^2} = \frac{D^3 g}{\nu^2}}$$

$$\pi_3 = \rho_L^a \mu_L^b D^c \sigma = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c \frac{M}{t^2}$$

M: $0 = a + b + 1$ $\qquad a = 1$

L: $0 = -3a - b + c$ $\qquad c = 1$

t: $0 = -b - 2$ $\qquad b = -2$

$$\underline{\pi_3 = \frac{\rho_L D \sigma}{\mu_L^2}}$$

$$\pi_4 = \rho_L^a \mu_L^b D^c \mu_g$$

~ BY INSPECTION - $\underline{\pi_4 = \mu_L / \mu_g}$

$$\pi_5 = \rho_L^a \mu_L^b D^c \rho_g$$

~ BY INSPECTION $\underline{\pi_5 = \rho_g / \rho_L}$

$$\pi_6 = \rho_L^a \mu_L^b D^c M = \left(\frac{M}{L^3}\right)^a \left(\frac{M}{Lt}\right)^b L^c M$$

M: $0 = a + b + 1$ $\qquad a = -1$

L: $0 = -3a - b + c$ $\qquad c = -3$

t: $0 = -b$ $\qquad b = 0$

$$\underline{\pi_6 = M / \rho_L D^3}$$

28.7

VARIABLE	SYMBOL	DIMENSIONS
MASS TX COEF.	k_c	L/t
VELOCITY	v	L/t
ROD DIAMETER	D_i	L
PIPE "	D_o	L
DENSITY	ρ	M/L^3
VISCOSITY	μ	M/Lt
DIFFUSIVITY	D_{AB}	L^2/t

$i = n - r = 7 - 3 = 4$ π GROUPS

CORE - D_{AB}, ρ, D_o

$$\pi_1 = D_{AB}^a \rho^b D_o^c k_c = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{L}{t}$$

M: $0 = b$ $\qquad b = 0$

L: $0 = 2a - 3b + c + 1$ $\qquad c = 1$

t: $0 = -a - 1$ $\qquad a = -1$

$$\underline{\pi_1 = k_c D_o / D_{AB} = Sh}$$

$$\pi_2 = D_{AB}^a \rho^b D_o^c v = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{L}{t}$$

SAME FORM AS π_1 --

$$\underline{\pi_2 = D_o v / D_{AB}}$$

28.7 CONTINUED --

$$\Pi_3 = D_{AB}^a \rho^b D_o^c \mu = \left(\frac{L^2}{t}\right)^a \left(\frac{M}{L^3}\right)^b L^c \frac{M}{Lt}$$

M: $0 = b+1$ $\quad b=-1$

L: $0 = 2a-3b+c-1$ $\quad c=0$

t: $0 = -a-1$ $\quad a=-1$

$$\Pi_3 = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}} = Sc$$

$$\Pi_4 = D_{AB}^a \rho^b D_o^c D_i$$

~ BY INSPECTION - $\Pi_4 = \frac{D_i}{D_o}$

NOTE THAT $\frac{\Pi_2}{\Pi_3} = \frac{D_o v \rho}{\mu} = Re$

28.8

VARIABLE	SYMBOL	DIMENSION
CONCENTRATION DIFFERENCE	$C_A - C_{A\infty}$	M/L^3
OVERALL "	$C_{A0} - C_{A\infty}$	M/L^3
RADIUS	r	L
REFERENCE RADIUS	R	L
DIFFUSIVITY	D_{AB}	L^2/t
MASS TX. COEF	k_c	L/t
TIME	t	t

$$i = n - r = 7 - 3 = 4$$

CORE --- $C_{A0}-C_{A\infty}, R, D_{AB}$

$$\Pi_1 = (C_{A0}-C_{A\infty})^a R^b D_{AB}^c (C_A - C_{A\infty})$$

BY INSPECTION - $\Pi_1 = \frac{C_A - C_{A\infty}}{C_{A0}-C_{A\infty}}$

28.8 CONTINUED

$$\Pi_2 = (C_{A0}-C_{A\infty})^a R^b D_{AB}^c r$$

-- BY INSPECTION - $\Pi_2 = r/R$

$$\Pi_3 = (C_{A0}-C_{A\infty})^a R^b D_{AB}^c k_c = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L^2}{t}\right)^c \frac{L}{t}$$

M: $0 = a$ $\quad a=0$

L: $0 = -3a+b+2c+1$ $\quad b=1$

t: $0 = -c-1$ $\quad c=-1$

$$\Pi_3 = k_c R / D_{AB}$$

$$\Pi_4 = (C_{A0}-C_{A\infty})^a R^b D_{AB}^c t = \left(\frac{M}{L^3}\right)^a L^b \left(\frac{L^2}{t}\right)^c t$$

M: $0 = a$ $\quad a=0$

L: $0 = -3a+b+2c$ $\quad b=-2$

t: $0 = -c+1$ $\quad c=1$

$$\Pi_4 = D_{AB} t / R^2$$

28.9 B.L. EQUATIONS:

LAMINAR: $\frac{k_c x}{D_{AB}} = 0.332 Re_x^{1/2} Sc^{1/3}$

TURBULENT: $\frac{k_c x}{D_{AB}} = 0.0292 Re_x^{4/5} Sc^{1/3}$

$$Re_x|_{tr} = 2\times10^5$$

$$\left.\begin{array}{l}\text{FRACTION OF}\\ \text{MASS TX}\\ \text{WHICH IS}\\ \text{LAMINAR}\end{array}\right\} = \frac{N_{AL}}{N_{AL}+N_{At}} = \frac{\bar{k}_{cL}}{\bar{k}_{cL}+\bar{k}_{ct}}$$

28.9 CONTINUED

$$\bar{k}_{cL} = \frac{1}{L}\int_0^{L_{tr}} k_{cx}\,dx$$

$$= \frac{1}{L}(0.332)\left(\frac{v_\infty}{\nu}\right)^{1/2} Sc^{1/3}\int_0^{L_{tr}} x^{-1/2}\,dx$$

$$= \frac{0.664}{L}\, Re_{tr}^{1/2}\, Sc^{1/3}$$

$$\bar{k}_{ct} = \frac{1}{L}\int_{L_{tr}}^{L} k_{ctx}\,dx$$

$$= \frac{1}{L}(0.0292)\left(\frac{v_\infty}{\nu}\right)^{4/5} Sc^{1/3}\int_{L_{tr}}^{L} x^{-1/5}\,dx$$

$$= \frac{0.0365}{L}\left(Re_L^{4/5} - Re_{tr}^{4/5}\right) Sc^{1/3}$$

$$\text{LAMINAR FRACTION} = \frac{0.664\, Re_{tr}^{1/2}}{0.664\, Re_{tr}^{1/2} + 0.0365\left(Re_L^{4/5} - Re_{tr}^{4/5}\right)}$$

$$Re_{tr}^{1/2} = (2\times10^5)^{1/2} = 447.2$$

$$Re_{tr}^{4/5} = (2\times10^5)^{4/5} = 17411$$

$$Re^{4/5} = (3\times10^6)^{4/5} = 151950$$

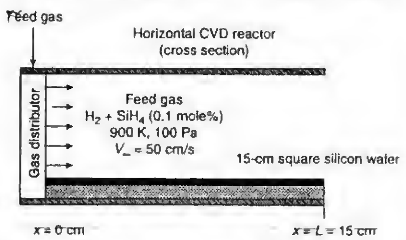
SUBSTITUTING & SOLVING

$$\text{LAMINAR FRACTION} = 0.057 = 5.7\,\%$$

28.10

AT SURFACE

$SiH_4 = Si + 2H_2$



$T = 900\text{ K}$ $\quad P = 100\text{ Pa}$ $\quad v_\infty = 50\text{ cm/s}$

$L = 15\text{ cm}$ $\quad D_{AB} = 0.4036\times10^{4}\text{ cm}^2/\text{s}$

$y_{A\infty} = 0.001$

28.10 CONTINUED

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.8\times10^{-4}}{2.67\times10^{-8}}\left(\frac{1}{0.4036\times10^{4}}\right) = \underline{1.67}$$

$$Re = \frac{(50)(15)(2.67\times10^{-8})}{1.8\times10^{-4}} = \underline{0.1125} \quad \text{(LAMINAR)}$$

$$Sh = \frac{\bar{k}_c L}{D_{AB}} = 0.664\, Re^{1/2} Sc^{1/3} = \underline{0.264}$$

$$k_c = \frac{0.4036\times10^{4}}{15}(0.264) = 71.1\text{ cm}^2/\text{s}$$

$$c = \frac{P}{RT} = \frac{100/1.0135\times10^5}{(82.06)(900)} = 1.336\times10^{-8}\text{ mol/cm}^3$$

$$c_{A\infty} = 0.001(1.336\times10^{-8}) = 1.336\times10^{-11} \text{ ''}$$

$$W_A = N_A A = k_c(c_{A\infty} - c_{AS})(15)(15)$$

$$= (71.1)(1.336\times10^{-11})(225)$$

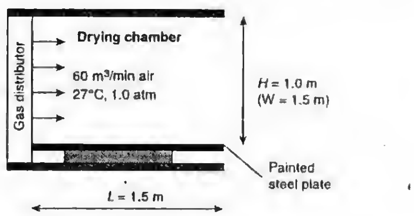
$$= 2.136\times10^{-7}\text{ mol/s}$$

$$= \underline{1.28\times10^{-5}\text{ mol/m}}$$

THICKEST Si LAYER WILL OCCUR WHERE k_{cx} IS LARGEST

$\sim$ AT $x = 0$

28.11



$T = 300\text{ K}$

$P = 1\text{ ATM}$

$P_{AS} = 0.137\text{ ATM}$

$$v = \dot{V}/(W)(H) = \frac{60}{(1)(1.5)(60)} = 0.67\text{ m/s}$$

$$= 67\text{ cm/s}$$

28.11 CONTINUED...

$$D_{AB} = 0.0962\left(\frac{300}{298}\right)^{3/2} = 0.0972 \text{ cm}^2/\text{s}$$

$$\nu = 1.569 \times 10^{-5} \text{ m}^2/\text{s}$$

$$Sc = \frac{(1.569\times10^{-5})(10^4)}{0.0972} = 1.614$$

$$Re = \frac{vL}{\nu} = \frac{(67)(150)}{0.1569} = 6.41\times10^4$$

(LAMINAR)

$$k_c = 0.664\,\frac{(0.0972)}{150}\left(6.41\times10^4\right)^{1/2}(1.614)^{1/3}$$

$$= 0.128 \text{ cm/s}$$

$$C_{AS} = \frac{p_{AS}}{RT} = \frac{0.137}{(82.06)(300)} = 5.565\times10^{-6} \text{ mol/cm}^3$$

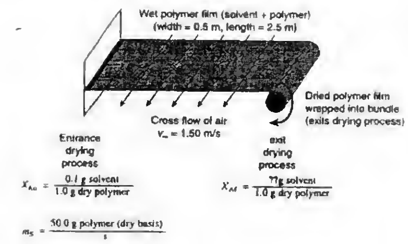
$$W_A = N_A \times A = k_c\left(C_{AS} - C_{A\infty}\right)(A)$$

$$= (0.128)(5.565\times10^{-6} - 0)(150)(150)$$

$$= 0.0160 \text{ mol/s}$$

$$= \underline{\underline{4500 \text{ g/h}}}$$

28.12



$T = 293$ K

$P = 1$ ATM

$v = 150$ cm/s

$D_{AB} = 0.080 \text{ cm}^2/\text{s}$ $\quad p_W^\circ = 0.16$ ATM

$\nu_{AIR} = 0.15 \text{ cm}^2/\text{s}$

$$Sc = \frac{0.15}{0.080} = 1.875$$

28.12 CONTINUED -

$$Re_L = \frac{vW}{\nu} = \frac{150(50)}{0.15} = 50{,}000$$

(LAMINAR)

$$k_c = 0.664\,\frac{D_{AB}}{W}\,Re_L^{1/2}\,Sc^{1/3}$$

$$= 0.664\,\frac{0.080}{50}\left(5\times10^4\right)^{1/2}(1.875)^{1/3}$$

$$= 0.293 \text{ cm/s}$$

$$W_A = N_A(A) = k_c\,\Delta C_A\,(2WL)$$

$$C_{AS} = \frac{p_A^\circ}{RT} = \frac{0.16}{(82.06)(293)} = 6.65\times10^{-6} \text{ mol/cm}^3$$

$$W_A = (0.293)(6.65\times10^{-6} - 0)(2\times50\times250)$$

$$= 0.0487 \text{ mol/s}$$

$$= (0.0487)(86) = \underline{\underline{4.19 \text{ g/s}}} \quad \text{(b)}$$

$$Sh = \frac{k_c W}{D_{AB}} = \frac{(0.293)(50)}{0.080} = \underline{\underline{183.1}} \quad \text{(a)}$$

$$\text{INPUT} = \frac{0.1 \text{ g SOLVENT}}{\text{g DRY SOL.}}\left(\frac{50 \text{ g DRY SOLV}}{\text{s}}\right)$$

$$= 5 \text{ g/s SOLVENT}$$

$$\text{OUTPUT} = 4.19 \text{ g/s} + \dot{m} \text{ g/s (IN POLYMER)}$$

$$\dot{m} = 5 - 4.19 = 0.81 \text{ g/s}$$

$$X = \frac{0.81}{50} = 0.0162 \;\frac{\text{g SOLVENT}}{\text{g DRY SOLID}} \quad \text{(c)}$$

28.13 ACETONE (A) IN AIR (B)

$T = 298\ K$

$P = 1.013 \times 10^5\ Pa \quad p_A^\circ = 3.066 \times 10^4\ Pa$

$v = 600\ cm/s \qquad L = 100\ cm$

$D_{AB} = 0.93 \times 10^{-5}\ m^2/s$

AT 298 K ... $\nu_{AIR} = 1.55 \times 10^{-5}\ m^2/s$

$$Sc = \frac{1.55 \times 10^{-5}}{0.93 \times 10^{-5}} = 1.67$$

$$Re_x = \frac{(6)(0.4)}{1.55 \times 10^{-4}} = 1.548 \times 10^5$$

(LAMINAR)

$$\frac{k_c x}{D_{AB}} = 0.332\, Re_x^{1/2}\, Sc^{1/3}$$

$$k_c = \frac{0.93 \times 10^{-5}}{0.4}(0.332)(1.548 \times 10^5)^{1/2} \times (1.67)^{1/3}$$

$$= \underline{\underline{3.6 \times 10^{-3}\ m/s}} \quad (a)$$

FOR $L = 1\ m$

$$Re_L = \frac{(6)(1)}{1.55 \times 10^{-5}} = 3.87 \times 10^5$$

TURBULENT FOR $Re_x > 2 \times 10^5$

ASSUMING B.L. IS

LAMINAR FOR $0 < Re_x < 2 \times 10^5$

TURBULENT FOR $2 \times 10^5 < Re_x$

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Re_{tr}^{1/2} Sc^{1/3} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[Re_L^{4/5} - Re_{tr}^{4/5} \right]$$

28.13 CONTINUED –

$Re_{tr} = 2 \times 10^5$

$Re_L = 3.87 \times 10^5$

SUBSTITUTING & SOLVING:

$$\bar{k}_c = 8.15 \times 10^{-3}\ m^2/s$$

$$W_A = \bar{k}_c A (C_{AS} - C_{A0})$$

$$C_{AS} = \frac{p_A^\circ}{RT} = \frac{3.066 \times 10^4}{(8.314)(298)} = 12.37\ mol/m^3$$

$$W_A = (8.15 \times 10^{-3})(12.37 - 0)(1)$$

$$= 0.101\ mol/s$$

$$= (0.101)(58) = \underline{\underline{5.86\ g/s}}$$

28.14 GAS STREAM CONTAINING CO (A), O_2 (B), CO_2 (C)

$y_A = 0.009 \qquad T = 300\ K$

$y_B = 0.001 \qquad P = 1\ ATM$

$y_C = 0.99$

$$y'_A = \frac{0.009}{0.999} = 0.00901$$

$$y'_C = \frac{0.99}{0.999} = 0.991$$

$D_{AB} = 0.213\ cm^2/s \qquad \nu_A = 0.158\ cm^2/s$

$D_{AC} = 0.155\ " \qquad \nu_B = 0.159\ "$

$D_{BC} = 0.166\ " \qquad \nu_C = 0.0832\ "$

28.14 CONTINUED –

$$D_{B-MIX} = \frac{1}{y'_A/D_{BA} + y'_C/D_{BC}}$$

SUBSTITUTING & SOLVING:

$$D_{B-MIX} = 0.166 \text{ cm}^2/\text{s}$$

USE VISCOSITY ~ ν_C – THE DOMINANT COMPONENT

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.0832}{0.166} = \underline{\underline{0.501}} \quad (a)$$

$$Re = \frac{vL}{\nu} = \frac{(1200)(300)}{0.0832} = 4.327 \times 10^6$$

{VERY MUCH INTO TURBULENT REGIME}

PRESUMING B.L. FLOW TO BE
LAMINAR FOR $0 < Re < 2 \times 10^5$
TURBULENT $2 \times 10^5 < Re$

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Sc^{1/3} Re_{tr}^{1/2} + 0.0365 \frac{D_{AB}}{L} Sc^{1/3} \left[Re_L^{4/5} - Re_{tr}^{4/5} \right]$$

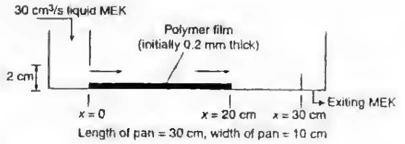
SUBSTITUTING VALUES & SOLVING:

$$\underline{\underline{\bar{k}_c = 3.277 \text{ cm/s}}} \quad (c)$$

$\underline{\underline{\text{TURBULENT EFFECTS DOMINATE}}}$ (b)

28.15

SOLUTE (A) INTO MEK (B)



28.15 CONTINUED –

$C_{A\infty} = 0 \qquad D_{AB} = 3 \times 10^{-6} \text{ cm}^2/\text{s}$

$\nu = 6 \times 10^{-3}$ "

$\rho_{SOLID} = 1.05 \text{ g/cm}^3 \qquad \rho_L = 0.80 \text{ g/cm}^3$

$\rho_A^* = 0.04 \text{ g/cm}^3 \qquad \dot{V} = 30 \text{ cm}^3/\text{s}$

$$v = \frac{30}{(1.98)(10)} = 1.515 \text{ cm/s}$$

{UNIT DEPTH}

$$Sc = \frac{6 \times 10^{-3}}{3 \times 10^{-6}} = 2000$$

$$Re = \frac{(1.515)(20)}{6 \times 10^{-3}} = 5050 \quad \text{\{LAMINAR\}}$$

$$\bar{k}_c = 0.664 \frac{D_{AB}}{L} Re_L^{1/2} Sc^{1/3}$$

$$= 88.94 \times 10^{-6} \text{ cm/s}$$

$$n_A = k_c(\rho_A^* - \rho_{A\infty})$$

$$= (88.94 \times 10^{-6})(0.04)$$

$$= \underline{\underline{3.558 \times 10^{-6} \text{ g/cm}^2 \cdot \text{s}}} \quad (a)$$

$$W_A = n_A A$$

$$= (3.558 \times 10^{-6})(10 \times 20)$$

$$= \underline{\underline{7.116 \times 10^{-4} \text{ g/s}}}$$

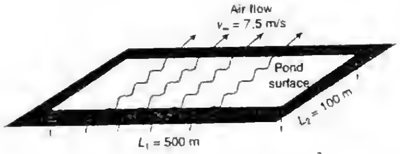
$$m_{SOLID} = (0.02)(10)(20)(1.05)$$

$$= 4.20 \text{ g}$$

$$t = \frac{4.20}{7.116 \times 10^{-4}} = \underline{\underline{5902 \text{ s}}} \quad (b)$$

$$= \underline{\underline{1.64 \text{ h}}}$$

28.16



METHYLENE CHLORIDE (A) IN AIR (B)

$T = 293\ K \quad P = 1\ ATM$

$D_{AB} = 0.085\ cm^2/s \quad v = 7.5\ m/s$

$\nu = 0.15\ cm^2/s$

$$Sc = \frac{0.15}{0.085} = 1.765$$

$$Re = \frac{vL}{\nu} = \frac{(7.5)(100)}{0.15\times10^{-4}} = 5\times10^7$$

ASSUMING B.L. FLOW TO BE LAMINAR FOR $0 < Re < 2\times10^5$

TURBULENT FOR $2\times10^5 < Re$

$$\overline{k}_c = 0.664\frac{D_{AB}}{L} Sc^{1/3} Re_{tr}^{1/2} + \frac{0.0365}{L} Sc^{1/3}\left[Re_L^{4/5} - Re_{tr}^{4/5}\right]$$

$Re_{tr} = 2\times10^5 \quad Re_L = 5\times10^7$

SUBSTITUTING & SOLVING:

$$\overline{k}_c = 0.5372\ cm/s \quad (b)$$

FOR $Re = 2\times10^5 = \frac{vL_{tr}}{\nu}$

$$L_{tr} = \frac{(2\times10^5)(0.15\times10^{-4})}{7.5} = 0.4\ m \quad (a)$$

THIS IS THE EXTENT OF THE LAMINAR B.L.

$$Sc(L) = \frac{0.010}{1.07\times10^{-5}} = 934 \quad (c)$$

28.17 LUBRICATING OIL (A) IN AIR (B)

$T = 386\ K \quad P = 1\ ATM \quad v = 50\ m/s$

$x_{tr} = 0.097\ m \quad D_{AB} = 0.040\ cm^2/s$

$\mu = 2.23\times10^{-5}\ kg/m.s$

$\rho = 0.917\ kg/m^3$

$P_A^0 = 0.20\ Pa$

$$\nu = \frac{2.23\times10^{-5}}{0.917} = 2.43\times10^{-5}\ m^2/s = 0.243\ cm^2/s$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.243}{0.040} = 6.075 \quad (a)$$

$$\overline{k}_c = 0.664\frac{D_{AB}}{L} Sc^{1/3} Re_{tr}^{1/2} + 0.0365\frac{D_{AB}}{L} Sc^{1/3}\left[Re_L^{4/5} - Re_{tr}^{4/5}\right]$$

$$Re_{tr} = \frac{(5000)(9.7)}{0.243} \cong 2\times10^5 \quad Re_L = \frac{(5000)(200)}{0.243} = 4.115\times10^6$$

SUBSTITUTING VALUES INTO $\overline{k}_c$ EXPRESSION:

$$\overline{k}_c = 2.48\ cm/s \quad (b)$$

AT $x = 120\ cm$

$$Re_x = \frac{(5000)(120)}{0.243} = 2.469\times10^6$$

$$k_{cx} = \frac{0.0292\,D_{AB}}{x} Sc^{1/3} Re_x^{1/2}$$

$$= \frac{0.0292(0.040)(6.075)^{1/3}(2.469\times10^6)^{4/5}}{120}$$

$$= 2.307\ cm/s \quad (c)$$

28.18 For $Re_x = 70{,}000 = \frac{xv}{\nu}$

$$x = 70{,}000 \frac{\nu}{v}$$

$$\frac{k_c x}{D_{AB}} = 0.332\, Re_x^{1/2} Sc^{1/3}$$

$$k_c = \frac{0.332\,(70{,}000)^{1/2}}{70{,}000} Sc^{1/3} \frac{D_{AB}\, v}{\nu}$$

$$= 0.00125\, v\, Sc^{-2/3} \quad (a)$$

For $Re_L = 70{,}000 = Lv/\nu$

$$L = 70{,}000 \frac{\nu}{v}$$

$$\frac{\bar{k}_c L}{D_{AB}} = 0.664\, Re_L^{1/2} Sc^{1/3}$$

$$= \frac{0.664\,(70{,}000)^{1/2}}{70{,}000} Sc^{1/3} \frac{D_{AB}\, v}{\nu}$$

$$= 0.0025\, v\, Sc^{-2/3} \quad (b)$$

For $Re_x = 7\times10^5$

$$x = 7\times10^5 \frac{\nu}{v}$$

$$\frac{k_c x}{D_{AB}} = 0.0292\, Re_x^{4/5} Sc^{1/3}$$

$$k_c = \frac{0.0292\,(7\times10^5)^{4/5}}{(7\times10^5)} Sc^{1/3} \frac{D_{AB}}{\nu} v$$

$$= 0.00198\, v\, Sc^{-2/3} \quad (c)$$

28.19 – von Karman B.L. Analysis

Given $v = \alpha + \beta y^{1/7}$

B.C. $v(y=0) = 0 \qquad \alpha = 0$

$v(y=\delta) = v_\infty \qquad \beta = v/\delta^{1/7}$

$$\therefore \quad v_x = v_\infty \left(\frac{y}{\delta}\right)^{1/7}$$

& Given $c_A - c_{A\infty} = \eta + \xi y^{1/7}$

@ $y=0$ $\quad c_A - c_{A\infty} = c_{As} - c_{A\infty}$

@ $y=\delta_c$ $\quad c_A - c_{A\infty} = 0$

$$\eta = c_{As} - c_{A\infty}$$

$$\xi = -\frac{\eta}{\delta_c^{1/7}} = -\frac{c_{As}-c_{A\infty}}{\delta_c^{1/7}}$$

Now – $$\frac{c_A - c_{A\infty}}{c_{As}-c_{A\infty}} = 1 - \left(\frac{y}{\delta_c}\right)^{1/7}$$

Eqn (28-29)

$$\frac{d}{dx}\int_0^{\delta_c} (c_A - c_{A\infty})\, v\, dy = k_c (c_{As} - c_{A\infty})$$

Divide by $(c_{As}-c_{A\infty})v_\infty$

$$\frac{d}{dx}\int_0^{\delta_c} \left(\frac{c_A - c_{A}}{c_{As}-c_{A\infty}}\right) \frac{v}{v_\infty}\, dy = \frac{k_c}{v_\infty}$$

Evaluating the integral...

$$\int_0^{\delta_c} \left[1 - \left(\frac{y}{\delta_c}\right)^{1/7}\right]\left(\frac{y}{\delta}\right)^{1/7} dy$$

$$\int_0^{\delta_c} \left[\left(\frac{y}{\delta}\right)^{1/7} - \frac{y^{2/7}}{\delta_c^{1/7}\delta^{1/7}}\right] dy$$

$$\left[\frac{7}{8}\frac{y^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{y^{9/7}}{\delta_c^{1/7}\delta^{1/7}}\right]_0^{\delta_c}$$

28.19 CONTINUED

INTEGRAL EVALUATION – BETWEEN 0 & δ_c

$$\frac{7}{8}\frac{\delta_c^{8/7}}{\delta^{1/7}} - \frac{7}{9}\frac{\delta_c^{8/7}}{\delta^{1/7}} = \frac{7}{92}\frac{\delta_c^{8/7}}{\delta^{1/7}}$$

BACK INTO GOVERNING EQN.:

$$\frac{d}{dx}\left[\frac{7}{92}\frac{\delta_c^{8/7}}{\delta^{1/7}}\right] = \frac{k_c}{v_\infty}$$

LETTING $Sc = 1$ $\delta_c = \delta$

$$\frac{7}{92}\frac{d}{dx}\delta_c = \frac{k_c}{v_\infty}$$

& SINCE WE KNOW $\delta = \frac{0.371\,x}{Re_x^{1/5}}$

$$\frac{d\delta_c}{dx} = \frac{d\delta}{dx} = \frac{0.371}{(v/\nu)^{1/5}}\left(\frac{4}{5}x^{-1/5}\right)$$

$$= 0.297\,Re_x^{-1/5}$$

WE NOW HAVE

$$\frac{7}{92}\left(0.297\,Re_x^{-1/5}\right) = k_c/v_\infty$$

& FINALLY $k_c = 0.0289\,v_\infty Re_x^{-1/5}$

28.20 $v_x = a + by$

B.C. $v_x(0) = 0$

$v_x(\delta) = v_\infty$

$v_x = v_\infty(y/\delta)$

$C_A = \alpha + \beta y$

B.C. $C_A = C_{As}$ @ $y = 0$

$C_A = C_{A\infty}$ @ $y = \delta_c$

28.20 CONTINUED

$$\frac{C_A - C_{As}}{C_{A\infty} - C_{As}} = \frac{y}{\delta_c}$$

ANOTHER PHYSICAL SITUATION THAT A PROFILE SHOULD PROVIDE IS

$$\frac{dC_A}{dy} = 0 \text{ @ } y = 0$$

THE LINEAR MODEL DOES YIELD THIS RESULT & WILL NOT SATISFY ALL OF THE PHYSICAL REQUIREMENTS

28.21 FOR A SPHERICAL PELLET ($d = 1$ cm)

$$Nu = 0.37\,Re_d^{0.6}\,Pr^{1/3}$$

FROM DATA PROVIDED IN PROBLEM STATEMENT:

$$Re_{d_p} = \frac{v_\infty d}{\nu} = \frac{(1)(0.01)}{1.569\times10^{-5}} = 637$$

$$Pr = \frac{\nu}{\alpha} = \frac{1.569\times10^{-5}}{2.216\times10^{-5}} = 0.708$$

$$h = Nu\frac{k}{d} = \frac{0.317(637)^{0.6}(0.708)^{1/3}(0.02642)}{0.01}$$

$$= 41.64\ W/m^2 \quad (a)$$

HT & MASS TX ANALOGS:

$$\frac{h}{\rho c_p v}Pr^{2/3} = \frac{k_c}{v}Sc^{2/3}$$

$$k_c = \frac{h}{\rho c_p}\left(\frac{Pr}{Sc}\right)^{2/3}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.569\times10^{-5}}{9.62\times10^{-6}} = 1.63$$

28.21 CONTINUED -

$$k_c = \frac{41.64}{(1177)(1.006)}\left(\frac{0.708}{1.63}\right)^{2/3}$$

$$= \underline{\underline{0.020 \text{ m/s}}} \qquad (b)$$

$$N_A = k_c(C_{AS} - C_{A\infty})$$

$$C_{AS} = \frac{P_A^0}{RT} = \frac{1.27 \times 10^4}{(8.314)(300)} = 5.09 \text{ mol/m}^3$$

$$C_{A\infty} = 0 \qquad (c)$$

$$N_A = (0.020)(5.09) = \underline{\underline{0.102 \text{ mol/m}^2\text{s}}}$$

28.22 - GIVEN IN PROBLEM STATEMENT

$$\frac{hd_1}{k} = 0.031\, Re_{d_1}^{0.8}\, Pr^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}\left(\frac{d_2}{d_1}\right)^{0.15}$$

HEAT & MASS TR ARE RELATED BY

$$\frac{h}{\rho c_p v} Pr^{2/3} = \frac{k_c}{v} Sc^{2/3}$$

SUBSTITUTING & SOLVING:

$$Sh = \frac{k_c d_1}{D_{AB}}$$

$$= \underline{\underline{0.031\, Re_{d_1}^{0.8}\, Sc^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}\left(\frac{d_2}{d_1}\right)^{0.15}}}$$

28.23 AS GIVEN:

$$Nu = \frac{hd_p}{k} = 0.37\, Re_{d_p}^{0.6}\, Pr^{1/3}$$

& FROM CHILTON - COLBURN ANALOGY

$$\frac{h}{\rho c_p v} Pr^{2/3} = \frac{k_c}{v} Sc^{2/3}$$

28.23 CONTINUED

COMBINING RELATIONS -

$$\underline{Sh = \frac{k_c d_p}{D_{AB}} = 0.37\, Re_{d_p}^{0.6}\, Sc^{1/3}}$$

FOR SLOW FLOW - NO BULK CONTRIBUTION:

STEADY STATE: $\frac{d}{dr}(r^2 N_{Ar}) = 0$

$$N_{Ar} = -D_{AB}\frac{dC_A}{dr}$$

$$r^2 N_{Ar}\int_R^\infty \frac{dr}{r^2} = -D_{AB}\int_{C_{AS}}^{C_{A\infty}} dC_A$$

$$r^2 N_{Ar}\left(-\frac{1}{r}\right)\Big|_R^\infty = D_{AB}(C_{AS} - C_{A\infty})$$

AT $r = R$: $N_{Ar} = \frac{D_{AB}}{R}(C_{AS} - C_{A\infty}) = k_c \Delta C_A$

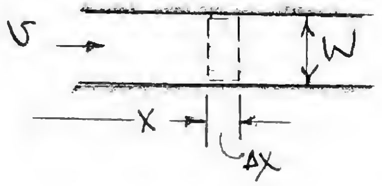
$$k_c = \frac{D_{AB}}{R} = \frac{2D_{AB}}{d_p}$$

GIVING: $Sh = \frac{k_c d_p}{D_{AB}} = 2$

MODIFIED EQUATION IS, THUS

$$\underline{\underline{Sh = 2 + 0.37\, Re_{d_p}^{0.6}\, Sc^{1/3}}}$$

28.24 FOR FLOW IN A CHANNEL BETWEEN 2 PLANES (PER UNIT DEPTH)



28.24 CONTINUED -

MASS BALANCE FOR CONTROL VOLUME

$$C_A v W|_x + 2k_c(C_{AS}-C_A)\Delta x = C_A v W|_{x+\Delta x}$$

$$\frac{C_A|_{x+\Delta x} - C_A|_x}{\Delta x} = \frac{2}{W}\frac{k_c}{v}(C_{AS}-C_A)$$

IN LIMIT AS $\Delta x \to 0$

$$\frac{dC_A}{dx} = \frac{2}{W}\frac{k_c}{v}(C_{AS}-C_A)$$

LET $\theta = C_A - C_{AS}$ ~ $\frac{dC_A}{dx} = \frac{d\theta}{dx}$

$$\frac{d\theta}{dx} = -\frac{2}{W}\frac{k_c}{v}\theta$$

$$\int_{\theta_0}^{\theta_L}\frac{d\theta}{\theta} = -\frac{2}{W}\frac{k_c}{v}\int_0^L dx$$

$$\ln\frac{\theta_L}{\theta_0} = -\frac{2k_cL}{Wv}$$

$$\frac{\theta_L}{\theta_0} = \exp\left(-2\frac{k_c}{v}\frac{L}{W}\right)$$

$$\frac{C_{AL}-C_{AS}}{C_{A0}-C_{AS}} = \exp\left(-2\frac{k_c}{v}\frac{L}{W}\right)$$

FOR $C_{A0}=0$

$$C_{AL} = C_{AS}\left[1-e^{-2\frac{k_c}{v}\frac{L}{W}}\right]$$

NAPHTHALENE IN AIR:

$T = 273\text{ K}$ $\quad P = 1.013\times10^5\text{ Pa}$

$P_A^0 = 1\text{ Pa}$ $\quad D_{AB} = 5.14\times10^{-6}\text{ m}^2/\text{s}$

$Sc = 2.5$ $\quad \nu = Sc\,D_{AB} = 1.32\times10^{-5}$ "

$v = 15\text{ m/s}$

28.24 CONTINUED

$$C_{AS} = \frac{P_A}{RT} = \frac{1}{(8.314)(273)} = 4.406\times10^{-4}\text{ MOL/m}^3$$

USING REYNOLDS ANALOGY

$$Re = \frac{D_{EQUIV}\,v}{\nu}$$

$$D_{EQUIV} = \frac{4(l)(W)}{2} = 2W$$

$$Re = \frac{2(0.0075)(15)}{1.32\times10^{-5}} = 1.7\times10^4$$

FIG (13.1) $f_F = C_f \cong 0.0064$

$$\frac{k_c}{v} = \frac{0.0064}{2} = 0.0032$$

$$C_{AL} = (4.406\times10^{-4})\left[1-e^{-2(0.0032)(0.1/0.0075)}\right]$$

$$= \underline{\underline{3.60\times10^{-5}\text{ MOL/m}^3}} \qquad (a)$$

USING THE VON KARMAN ANALOGY:

$$\frac{k_c}{v} = \frac{C_f/2}{1+5(C_f/2)^{1/2}\left[Sc-1+\ln\left(1+\frac{5}{6}Sc\right)\right]}$$

$C_f/2 = 0.0032 \qquad Sc = 2.5$

$$\frac{k_c}{v} = 0.00184$$

$$C_{AL} = (4.406\times10^{-4})\left[1-e^{-2(0.00184)(0.1/0.0075)}\right]$$

$$= \underline{\underline{2.11\times10^{-5}}}\text{ MOL/m}^3 \qquad (b)$$

CHILTON COLBURN: $\frac{k_c}{v} = \frac{C_f}{2}Sc^{-2/3}$

$k_c/v = 0.00174$ - $\qquad$ (c)

GIVING $\underline{\underline{C_{AL} = 2.0\times10^{-5}\text{ MOL/m}^3}}$

28.24 CONTINUED —

PARTS (a), (b) & (c) COMPARE CONCENTRATIONS AT/NEAR STARTING CONDITIONS — BEFORE NAPHTHALENE SHEETS HAVE CHANGED DIMENSIONS.

AFTER EXTENDED TIME —

ORIGINAL VOL. OF NAP

$= (10)(10)(0.25) = 25 \text{ cm}^3$

WHEN 1/2 OF VOLUME HAS BEEN SUBLIMED — 12.5 cm^3 REMAIN & 12.5 cm ARE GONE —

NEW CHANNEL WIDTH = 0.00875 m

AT AVERAGE CONDITIONS —

$D_{EQUIV} = 2W = 2(0.008125)$

$= 0.01625 \text{ m}$

$Re = \dfrac{(0.01625)(15)}{1.32\times10^{-5}} = 1.85\times10^4$

$C_f = f_F \simeq 0.0064 \quad \dfrac{C_f}{2} = 0.0032$

∴ AT AVERAGE CONDITIONS THE ANSWERS TO PARTS (a), (b) & (c) BECOME —

REYNOLDS $\quad C_{AL} = 3.34\times10^{-5} \text{ mol/m}^3$

VON KARMAN $\quad = 1.95\times10^{-5}$ "

C-COLBURN $\quad = 1.84\times10^{-5}$ "

THESE ARE PROBABLY MORE REPRESENTATIVE VALUES —

28.24 CONTINUED

TOTAL NAPHTHALENE LOST —

$= (12.5 \text{ cm}^3)(1.145 \text{ g/cm}^3)\left(\dfrac{\text{mol}}{128.1 \text{ g}}\right)$

$= 0.1117 \text{ mol}$

$W_A = C_{AL} v A$

$= C_{AL}(15 \text{ m/s})(0.1 \text{ m})(0.008125 \text{ m})$

$= 0.0122 (C_{AL}) \text{ mol/s}$

$t = \dfrac{0.1117}{0.0122\, C_{AL}}$

USING CORRECTED RESULTS FOR C_{AL}

REYNOLDS: $t = \dfrac{0.1117}{(0.0122)(3.34\times10^{-5})}$

$= 2.744\times10^5 \text{ s}$

$= \underline{\underline{76.2 \text{ h}}}$

VON-KARMAN: $t = \underline{\underline{130.6 \text{ h}}}$

C-COLBURN: $t = \underline{\underline{138.4 \text{ h}}}$

28.25 SPHERICAL DROP IN AIR —

$D_{AIR} = 1.5689\times10^{-5} \text{ m}^2/\text{s} \qquad \rho_{AIR} = 1.177 \text{ kg/m}^3$

$\alpha_{AIR} = 2.2156\times10^{-5}$ " $\qquad k = 2.64\times10^{-2} \text{ W/m·K}$

$D_{AB} = 2.63\times10^{-5}$ " $\qquad c_p = 1006 \text{ kJ/kg·K}$

$T_S = 290 \text{ K} \qquad \lambda = 2461 \text{ J/g}$

$P_w^\circ = 1940 \text{ Pa} \qquad T_\infty = 310 \text{ K}$

28,25 CONTINUED -

ENERGY BALANCE --

$$\left\{\begin{matrix}\text{HT TX TO DROP} \\ \text{BY CONVECTION}\end{matrix}\right\} = \left\{\begin{matrix}\text{HT. LOST BY} \\ \text{EVAPORATION}\end{matrix}\right\}$$

$$h(T_\infty - T_S) = \lambda k_c (C_{AS} - C_{A\infty}) M$$

USING CHILTON-COLBURN ANALOGY

$$\frac{k_c}{v_\infty} Sc^{2/3} = \frac{h}{\rho c_p v_\infty} Pr^{2/3}$$

$$\frac{h}{k_c} = \left(\frac{Sc}{Pr}\right)^{2/3} \rho c_p$$

$$C_{AS} - C_{A\infty} = \frac{h}{k_c} \frac{(T_\infty - T_S)}{\lambda M}$$

$$= \left(\frac{Sc}{Pr}\right)^{2/3} \frac{\rho c_p (T_\infty - T_S)}{\lambda M}$$

$$= \left(\frac{0.60}{0.708}\right)^{2/3} \frac{(1177)(1,006)(20)}{2461\,(18)}$$

$$= 0,478 \text{ mol/m}^3$$

$$C_{AS} = \frac{P^\circ}{RT} = \frac{1940}{(8.314)(290)} = 0,805 \text{ mol/m}^3$$

$$C_{A\infty} = 0,805 - 0,478 = \underline{\underline{0,326 \text{ mol/m}^3}}$$

28,26 - THIS IS THE SAME PHYSICAL PROCESS AS IN TEXT EXAMPLE 6

$$T_\infty = \frac{\lambda_{T_S}}{\rho c_p}\left(\frac{Pr}{Sc}\right)^{1/3}(C_{AS} - C_{A\infty}) + T_S$$

28,26 CONTINUED -

$T_S = 298$ K

$c_{p\,AIR} = 1$ J/g·K

$\mu = 1{,}84 \times 10^{-4}$ g/cm·s

$\lambda = 2{,}45$ kJ/g

$\rho = 1{,}17 \times 10^{-3}$ g/cm^3

$p^\circ_w = 1300$ Pa

$k = 2{,}62 \times 10^{-4}$ W/cm·K

$v_\infty = 0{,}22$ m/s

$D_{AB} = 3 \times 10^{-5}$ m^2/s

$$Pr = \frac{\mu c_p}{k} = \frac{(1{,}84 \times 10^{-4})(1)}{2{,}62 \times 10^{-4}} = 0{,}70$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{(1{,}84 \times 10^{-4})}{(1{,}17 \times 10^{-3})(3 \times 10^{-5})} = 0{,}524$$

$$C_{AS} = \frac{P}{RT} = \frac{1300}{(8{,}314)(298)} = 0{,}525 \text{ mol/m}^3$$

$$C_{A\infty} = 0$$

$$T_\infty = \frac{(2450)(18)}{(1{,}17 \times 10^{-3})(1)}\left(\frac{0{,}70}{0{,}524}\right)^{1/3}(0{,}525 \times 10^{-6}) + 298$$

$$= 21{,}8 + 298 = \underline{\underline{319{,}8 \text{ K}}}$$

28,27 H_2O (A) INTO AIR (B)

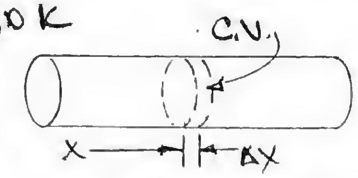
$T = 310$ K

$D = 0{,}15$ m

$P = 1{,}013 \times 10^5$ Pa

$D/\varepsilon = 10^4$

$T_S = 290$ K



MASS BALANCE FOR C.V. SHOWN

28.27 CONTINUED -

$$C_A v \frac{\pi D^2}{4}\Big|_x + k_c (C_{AS} - C_A)\pi D \Delta x = C_A v \frac{\pi D^2}{4}\Big|_{x+\Delta x}$$

$$\frac{C_A|_{x+\Delta x} - C_A|_x}{\Delta x} = \frac{4}{D}\frac{k_c}{v}(C_{AS} - C_A)$$

IN LIMIT AS $\Delta x \to 0$

$$\frac{dC_A}{dx} = \frac{4}{D}\frac{k_c}{v}(C_{AS} - C_A)$$

LET $\theta = C_A - C_{AS}$ $\quad \frac{d\theta}{dx} = \frac{dC_A}{dx}$

$$\frac{d\theta}{dx} = -\frac{4}{D}\frac{k_c}{v}\theta$$

$$\int_{\theta_0}^{\theta_L}\frac{d\theta}{\theta} = -\frac{4}{D}\frac{k_c}{v}\int_0^L dx$$

$$\ln\frac{\theta_L}{\theta_0} = -\frac{4}{D}\frac{k_c}{v}L$$

$$\frac{C_{AL} - C_{AS}}{C_{A0} - C_{AS}} = e^{-\frac{4}{D}\frac{k_c}{v}L}$$

CHILTON - COLBURN ANALOGY

$$\frac{k_c}{v} Sc^{2/3} = \frac{C_f}{2} = \frac{f}{2}$$

$$Sc = \frac{\nu}{D_{AB}}$$

$\nu = 1.569 \times 10^{-5}\ m^2/s$ @ $T_f = 300\,K$

28.27 CONTINUED -

$$D_{AB} = \frac{2.634}{1.013 \times 10^5}\left(\frac{300}{298}\right)^{3/2} = 2.626 \times 10^{-5}\ m^2/s$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.569 \times 10^{-5}}{2.626 \times 10^{-5}} = 0.597$$

$$Re = \frac{Dv}{\nu} = \frac{(0.15)(1.5)}{1.569 \times 10^{-5}} = 1.43 \times 10^4$$

FIG 13.1 $\quad f_F \cong 0.0066$

$$\frac{k_c}{v} = \frac{0.0066}{2}(0.597)^{-2/3} = 4.65 \times 10^{-3}$$

$$\frac{C_{AL} - C_{AS}}{C_{A0} - C_{AS}} = e^{-\frac{4(6)}{0.15}(4.65 \times 10^{-3})} = 0.474$$

$$C_{AS} = \frac{P'}{RT} = \frac{1895}{(8.314)(290)} = 0.786\ mol/m^3$$

$$C_{AL} = C_{AS}(1 - 0.474)$$

$$= 0.786(0.526) = \underline{\underline{0.413\ mol/m^3}}$$

28.28 SAME PHYSICAL SITUATION AS IN PROB 28.27

$$\ln\frac{C_{AL} - C_{AS}}{C_{A0} - C_{AS}} = -\frac{4}{D}\frac{k_c}{v}L$$

$$Re = \frac{Dv}{\nu} = \frac{(0.025)(15)}{1.415 \times 10^{-5}} = 2.65 \times 10^4$$

FIG 13.1 - $f_F = C_f = 0.0058$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.415 \times 10^{-5}}{540 \times 10^{-6}} = 2.62$$

28.28 CONTINUED

USE CHILTON-COLBURN ANALOGY

$$\frac{k_c}{v} = \frac{C_f/2}{Sc^{2/3}} = \frac{0.0058/2}{2.62^{2/3}}$$

$$= 0.00153$$

$$\ln\left[\frac{C_{AL}-C_{AS}}{C_{A0}-C_{AS}}\right] = -\frac{4}{0.025}(0.00153)\,L$$

$$= -0.245\ L$$

$$C_{AS} = \frac{P_A^{\circ}}{RT} = \frac{3}{(8.314)(283)} = 1.275\times10^{-3}\ \text{mol/m}^3$$

$$C_{AL} = 4.75\times10^{-4}\ \text{mol/m}^3 \qquad C_{A0} = 0$$

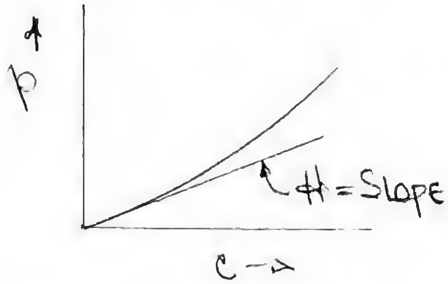
$$\ln\frac{4.75-12.75}{-12.75} = -0.466$$

$$L = \frac{0.466}{0.245} = \underline{\underline{1.90\ \text{m}}}$$

CHAPTER 29 -

29.1 EQUILIBRIUM DATA - Cl_2 IN H_2O

p, Cl_2	kg/m^3	mol/m^3
666	0.438	6.17
1330	0.575	8.10
4000	0.937	13.20
6600	1.210	
13200	1.773	



A CAREFUL PLOT WILL YIELD

$$H \triangleq 62 \ \text{Pa}/(\text{mol}/m^3)$$

29.2 EQUILIBRIUM DATA FOR TCE IN H_2O

p, TCE ATM	C mol/m^3
0.000	0
0.050	5.0
0.150	15.0
0.200	20.0

PLOT & OBSERVATION WILL SHOW LINEAR BEHAVIOR -

$$H = \frac{\Delta p}{\Delta C} = 0.010 \ \text{Atm}/(\text{mol}/m^3)$$

29.3 BENZENE (B) - 49 MOLES
TOLUENE (T) 21 MOLES

$\Sigma = 70$ "

AT 363 K, $P = 1.013 \times 10^5$ Pa

$p_B = 1.344 \times 10^5$ "

$p_T = 5.38 \times 10^5$ "

	x	$p_i \times 10^{-5}$	$p_i' \times 10^{-5}$
B	x_B	1.344	$1.344\, x_B$
T	$1-x_B$	5.38	$5.38(1-x_B)$

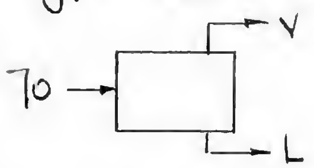
$$p_B + p_T = 1.013 \times 10^5$$

$$1.344 \times 10^5 x_B + 5.38 \times 10^5 (1-x_B) = 1.013 \times 10^5$$

$$x_B = 0.589 \quad x_T = 0.411 \quad \text{(a)}$$

$$y_B = \frac{1.344 \times 10^5}{1.013 \times 10^5}(0.589) = 0.783$$

$$y_T = 1 - y_B = 0.217$$



MASS BALANCE -

TOTAL: $70 = V + L$

B: $49 = 0.783V + 0.589L$

$= 0.783(70 - L) + 0.589L$

$$L = 29.95 \text{ mol} \quad \text{(b)}$$

29.4 Basis – 100 kg H_2O

	kg	M	mol	x_i
O_2	2×10^{-3}	32	6.25×10^{-5}	1.126×10^{-5}
H_2O	100	18	5.55	

$$p_{O_2} = y_{O_2} P = 0.21 (1.013 \times 10^5) = 0.2127 \times 10^5$$

$$p^*_{O_2} = H x_{O_2} = (4.06 \times 10^9)(1.126 \times 10^{-5})$$

$$= 4.57 \times 10^4$$

As $p^*_{O_2} > p_{O_2}$

(a) SOLUTION WILL LOSE O_2

$$p_{O_2} = H x_{O_2}$$

$$2.127 \times 10^4 = 4.06 \times 10^9 \; x_{O_2}$$

$$x_{O_2} = 5.24 \times 10^{-6}$$

FROM TABLE: TOTAL MOLES = 5.55

IN EQUILIBRIUM SOLUTION:

$$z_{O_2} = (5.24 \times 10^{-6})(5.55)$$

$$= 2.91 \times 10^{-5} \text{ kg mol}$$

BY MASS:

$$\frac{(2.91 \times 10^{-5} \text{ kg mol})(32)}{100 \text{ kg } H_2O}$$

$$= 9.3 \times 10^{-4} \text{ kg } O_2 / 100 \text{ kg } H_2O \quad \text{(b)}$$

29.5

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H k_G}$$

$$p_A = H c_A = H c x_A$$

$$c = (1 \text{ g/cm}^3)(10^6 \text{ cm}^3/\text{m}^3)\left(\frac{\text{kg}}{1000 \text{ g}}\right)\left(\frac{\text{kg mol}}{18 \text{ kg}}\right)$$

$$= 55.56 \text{ kg mol/m}^3$$

$$H' = \frac{4.06 \times 10^9}{55.56} = 7.3 \times 10^7 \; \frac{\text{Pa}}{\text{kg mol/m}^3}$$

$$\frac{1}{K_L} = \frac{1}{2.15 \times 10^{-5}} + \frac{1}{(9.28 \times 10^{-8})(7.3 \times 10^7)}$$

$$= 4.65 \times 10^5 + 0.148 \cong 4.65 \times 10^5$$

$$K_L = 2.15 \times 10^{-5} \text{ m/s}$$

ALL (100%) OF RESISTANCE IS IN GAS ϕ

29.6 INTERPHASE TRANSPORT

ClO_2 – AIR – H_2O

$P = 1.5$ ATM $\quad H = 7.7 \times 10^{-4}$ ATM/(g mol/m^3)

$y_A = 0.0401 \quad \rho_L = 992.3 \text{ kg/m}^3$

$x_A = 0.00040$

AT EQUILIBRIUM:

$$p_A = H c_A$$

$$= y_A P = 0.04(1.5) = 0.06 \text{ ATM}$$

$$c_A = x_A c = \frac{0.0004 (992.3)}{18}$$

$$= 0.022 \text{ kg mol/m}^3 = 22.0 \text{ g mol/m}^3$$

$$c_A^* = \frac{p_A}{H} = \frac{0.06}{7.7 \times 10^{-4}} = 77.9 \text{ g mol/m}^3$$

29.6 CONTINUED –

SINCE $C_A^* > C_A$ – ABSORPTION (a)

MAXIMUM $C_A = C_A^* = 77.9 \text{ gmol/m}^3$ (b)

$k_x = 1.0 \text{ gmol/m}^2 \cdot s$

$k_G = 0.010 \text{ gmol/m}^2 \cdot s \cdot atm$

$k_y = k_G P = 0.010(1.5) = 0.015 \text{ gmol/m}^2 \cdot s$

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{H'}{k_x}$$

$p_A = H c_A$

$$\frac{p_A}{P} = y_A = \frac{cH}{P} x_A = H' x_A$$

$$H' = \frac{cH}{P} = \frac{(55.13)(7.7 \times 10^{-4})}{1.5}$$

$= 0.0283$

$$\frac{1}{K_y} = \frac{1}{0.015} + \frac{0.0283}{1.0}$$

$K_y = 0.015 \text{ gmol/m}^2 \cdot s$ (c)

$N_A = K_y (y_{AG} - y_A^*)$

$y_{AG} = 0.06$

$$y_A^* = \frac{p_A^*}{P} = \frac{7.7 \times 10^{-4} (22)}{1.5}$$

$= 0.0113$

$N_A = (0.015)(0.06 - 0.0113)$

$= 7.30 \times 10^{-4} \text{ gmol/m}^2 \cdot s$ (d)

29.7 $N_A = K_L (c_A^* - c_{AL})$

$$c_A^* = \frac{p_{AG}}{H} = \frac{1.013 \times 10^4}{1.674 \times 10^3} = 6.05 \text{ kg mol/m}^3$$

$N_A = (1.26 \times 10^{-6})(6.05 - 4) = 2.58 \times 10^{-6} \text{ kg mol/m}^2 \cdot s$

(d)

$$\frac{1/k_L}{1/K_L} = \frac{K_L}{k_L} = 0.53$$

$$k_L = \frac{K_L}{0.53} = \frac{1.26 \times 10^{-6}}{0.53} = 2.38 \times 10^{-6}$$ (a)

{UNITS ARE $\text{kg mol/m}^2 \cdot s \cdot (\text{mol/m}^3)$}

$N_A = (2.58 \times 10^{-6}) = 2.38 \times 10^{-6} (c_{Ai} - 4)$

$c_{Ai} = 5.08 \text{ kg mol/m}^3$ (c)

$N_A = k_G (p_{AG} - p_{Ai})$

$p_{Ai} = H c_{Ai} = (1.674 \times 10^3)(5.08)$

$= 8.50 \times 10^3 \text{ Pa}$

$$k_G = \frac{N_A}{p_{AG} - p_{Ai}}$$

$$= \frac{2.58 \times 10^{-6}}{1.013 \times 10^4 - 0.850 \times 10^4}$$

$= 1.58 \times 10^{-9} \text{ kg mol/m}^2 \cdot s \cdot Pa$ (b)

29.8 STRIPPING TCA FROM WASTEWATER

$T = 293\ K \quad P = 1.25\ ATM$

$H' = 400\ ATM \qquad p_A = H' x_A$

$p_A = H' x_A$

$y_A = \frac{H'}{P} x_A \qquad \frac{H'}{P} = \frac{400}{1.25} = \underline{\underline{320\ \frac{\Delta y}{\Delta x}}}$ (a)

$p_A = \frac{H'}{C} c_A \qquad H = \frac{H'}{c}$

$c = \frac{P}{RT} = \frac{1.25}{(0.08206)(293)} = 0.0520\ \text{kgmol}/m^3$

$H = \frac{400}{0.0520} = \underline{\underline{7690\ \text{atm}/(\text{kgmol}/m^3)}}$

(b)

GIVEN $- k_c = 0.01$ mol/s

$k_g = \frac{k_c}{RT} = \frac{0.01}{(0.08206)(293)}$

$= \underline{\underline{4.16 \times 10^{-4}\ \text{kgmol}/m^2 \cdot s \cdot atm}}$

(c)

$N_A = k_c \Delta c_A = c k_c \frac{\Delta c_A}{c} = k_y \Delta y_A$

$k_y = c k_c = (0.0520)(0.01)$

$= \underline{\underline{5.2 \times 10^{-4}\ \text{kgmol}/m^2 \cdot s \cdot \Delta y}}$

(c)

$k_x = c\, k_L$

$\left\{ \begin{matrix} \text{IN} \\ \text{LIQUID} \end{matrix} \right\}$ $c = \rho_w = \frac{998.2}{18} = 55.46\ \text{kgmol}/m^3$

$k_x = (55.46)(0.01) = \underline{\underline{0.5546\ \text{kgmol}/m^2 \cdot s}}$

(d)

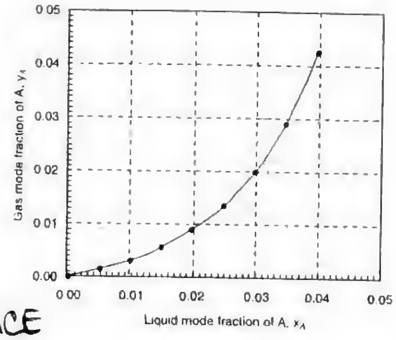
29.9

$T = 300\ K$

$P = 2\ ATM$

$y_A = 0.01$

$x_A = 0.035$



80% OF RESISTANCE IS IN LIQUID

$N_A = k_x (x_{A\infty} - x_{Ai}) = K_x (x_{A\infty} - x_A^*)$

$\frac{1/k_x}{1/K_x} = \frac{K_x}{k_x} = 0.8 = \frac{x_{A\infty} - x_{Ai}}{x_{A\infty} - x_A^*} = \frac{0.035 - x_{Ai}}{0.035 - 0.021}$

$x_{Ai} = 0.035 - 0.8(0.014) = \underline{\underline{0.0238}}$ (a)

FROM EQUILIBRIUM DIAGRAM ABOVE

$\underline{\underline{y_{Ai} = 0.0122}}$ (a)

$\frac{1/k_y}{1/K_y} = \frac{K_y}{k_y} = 0.2$

(b)

$\underline{\underline{K_y = 0.2(1.25) = 0.25\ \text{gmol}/m^2 \cdot s \cdot \Delta y}}$

$K_G = \frac{K_y}{P} = \frac{0.25}{2}$ (c)

$= \underline{\underline{0.125\ \text{gmol}/m^2 \cdot s \cdot atm}}$

$\frac{K_y}{P} = \frac{K_c}{RT} \quad \sim \quad K_c = K_y \frac{RT}{P}$

$K_c = 0.25 \frac{(82.06)(300)}{2}$

$= \underline{\underline{3077\ m^2/s}}$

29.10 SOLUTE A REMOVED FROM GAS STREAM

$T = 300\,K \qquad x_{AL} = 0.01$

$P = 2\,ATM \qquad y_{AG} = 0.035$

EQUILIBRIUM: $\quad y_A = 0.3\,x_A$

$$\frac{1/k_y}{1/K_y} = \frac{K_y}{k} = 0.6$$

$$\frac{y_{AG} - y_{Ai}}{y_{AG} - y_A^*} = 0.6$$

$$y_A^* = 0.3\,x_A = 0.3(0.01) = 0.003$$

$$\frac{0.035 - y_{Ai}}{0.035 - 0.003} = 0.6$$

$$\underline{\underline{y_{Ai} = 0.0158}}$$

$$x_{Ai} = \frac{y_{Ai}}{0.3} = \underline{\underline{0.0527}} \qquad (a)$$

$$K_y = 0.6\,k_y = 0.6(1.25)$$

$$= \underline{\underline{0.75\ \text{gmol}/\text{m}^2 \cdot \text{s} \cdot \Delta y}} \qquad (b)$$

$$\left\{\begin{aligned} \frac{1}{K_x} &= \frac{1}{H k_y} + \frac{1}{k_x} \\ \frac{1}{K_y} &= \frac{1}{k_y} + \frac{H'}{k_x} \end{aligned}\right\} \rightarrow \frac{1}{K_x} = \frac{H'}{K_y}$$

$$K_x = \frac{K_y}{H'} = \frac{0.75}{0.3}$$

$$= \underline{\underline{2.5\ \text{gmol}/\text{m}^2 \cdot \text{s} \cdot \Delta x}} \qquad (c)$$

29.11 AERATION OF H_2O

$T = 293\,K \qquad y_{AG} = 0.21$

$P = 2\,ATM$

$\rho_{H_2O} = 1000\ \text{kg}/\text{m}^3$

$H' = 40{,}100\ \text{ATM}\ \Delta y/\Delta x$

$$p_{O_2} = 0.21(2) = 0.42\ \text{ATM}$$

$$= (40{,}100)\,x_{O_2}$$

$$x_{O_2}^* = \frac{0.42}{40{,}100} = \underline{\underline{1.047 \times 10^{-5}\ \frac{\text{MOL } O_2}{\text{MOL } H_2O}}} \qquad (a)$$

$$C_A^* = (1.047 \times 10^{-5})\left(\frac{1000}{18}\right)$$

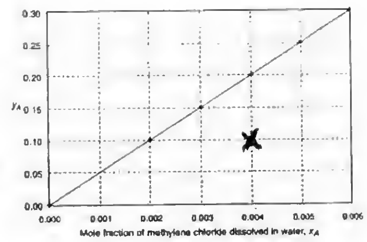
$$= \underline{\underline{5.82 \times 10^{-4}\ \text{kg mol}/\text{m}^3}} \qquad (b)$$

AS SYSTEM PRESSURE INCREASES

x^* & C_A^* WILL $\underline{\underline{\text{INCREASE}}}$ (c)

29.12

EQUILIBRIUM FOR SPECIES A IN AIR & SPECIES A DISSOLVED IN H_2O →



$T = 293\,K \qquad y_{AP} = 0.10$

$P = 2.20\ \text{ATM} \qquad x_{AP} = 0.0040$

$\rho_{H_2O} = 992.3\ \text{kg}/\text{m}^3 \qquad k_y = 0.0109\ \text{gmol}/\text{m}^2\cdot\text{s}$

$k_x = 0.125$ "

THIS IS A $\underline{\underline{\text{STRIPPING}}}$ PROCESS a)

29.12 CONTINUED –

$y_A = H' x_A'$

$p_A = \frac{H'P}{C} x_{AC}^* = H C_A^* \qquad H = \frac{H'P}{C}$

$C = \frac{992.3}{18} = 55.13 \text{ kgmol/m}^3$

$H' = 50$ – – FROM DIAGRAM

$H = \frac{50\,(2.2)}{55.13} = 1.996 \text{ Atm/(kgmol/m}^3)$ (b)

$y_A = H' x_A \quad 0.10 = 50 x_A^*$

$x_A^* = 0.002$

$C_A^* = x_A^* C = 0.002\,(55.13)$

$= 0.110 \text{ kgmol/m}^3$

$C_{AO} = x_{AO} C = (0.004)(55.13)$

$= 0.220 \text{ kgmol/m}^3$

$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y} = \frac{1}{0.125} + \frac{1}{(50)(0.01)}$

$= 8 + 2 = 10$

$K_x = 0.10 \text{ gmol/m}^2\text{·s}$ (c)

$K_L = \frac{K_x}{C} = \frac{0.10}{55.13} = 1.81 \times 10^{-6} \text{ m/s}$

$N_A = K_L (C_{AO} - C_A^*)$

$= (1.81 \times 10^{-6})(0.220 - 0.110)$

$= 8.91 \times 10^{-6} \text{ kgmol/m}^2\text{·s}$

29.12 CONTINUED –

$N_A = (8.91 \times 10^{-6}) = k_x (x_{AO} - x_{Ai})$

$= (0.125)(0.004 - x_{Ai})$

$x_{Ai} = 3.93 \times 10^{-3}$ (d)

$y_{Ai} = H' x_{Ai} = 50\,(3.93 \times 10^{-3}) = 0.196$

29.13 HEXANE (A) ABSORBED FROM AIR

$T = 293\text{ K} \qquad y_{AO} = 0.015 \text{ ATM}$

$P = 1.5 \text{ ATM} \qquad x_{AO} = 0.05$

$\rho_L = 0.80 \text{ g/cm}^3 \qquad k_y = 0.02 \text{ kgmol/m}^2\text{·s}$

$M_L = 180 \qquad k_x = 0.01$ "

$C = \frac{\rho_L}{M} = \frac{0.80}{180} = 4.44 \times 10^{-3} \text{ gmol/cm}^3$

$p_A = 0.15\, x_A^*$

$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y} = \frac{1}{0.01} + \frac{1}{(0.15)(0.02)}$

$K_x = 2.31 \times 10^{-3} \text{ kgmol/m}^2\text{·s}$

$K_L = K_x / C = \frac{2.31 \times 10^{-3}\,(10)^{-3}}{4.44 \times 10^{-3}\,(10)^4}$

$= 0.52 \times 10^{-7} \text{ cm/s}$ (a)

$k_y (y_{AO} - y_{Ai}) = k_x (x_{Ai} - x_{AL})$

$0.02\,(0.015 - y_{Ai}) = 0.01\,(x_{Ai} - 0.05)$

ALSO; $y_{Ai} = \frac{p_{Ai}}{P} = \frac{0.15}{P} x_{Ai}$

$y_{Ai} = \frac{0.15}{1.5} = 0.10\, x_{Ai}$

29.13 CONTINUED –

COMBINING THESE EXPRESSIONS –

$x_{Ai} = \underline{\underline{0.0667}}$ (b)

$y_{Ai} = 0.1 x_{Ai} = \underline{\underline{0.00667}}$

29.14 SO_2 ABSORBED INTO H_2O

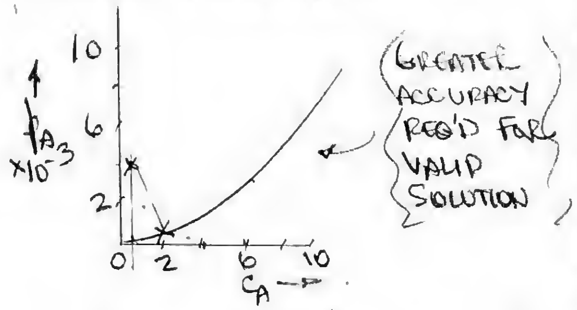
$p_{AG} = 4 \times 10^3$ Pa

$c_{AL} = 0.55$ kg mol/m^3

$k_G = 3.95 \times 10^{-9}$ kg mol/m^2·s·Pa

$k_L = 1.1 \times 10^{-4}$ kg mol/m^2·s·(kg mol/m^3)

EQUILIBRIUM DATA – GIVEN IN PROBLEM STATEMENT –



$$-\frac{k_L}{k_G} = -\frac{1.1 \times 10^{-4}}{3.95 \times 10^{-9}} = -2.78 \times 10^{4}$$

FROM PLOT – $\underline{\underline{p_{Ai} \cong 213 \text{ Pa}}}$ (a)

$\underline{\underline{c_{Ai} \cong 0.69 \text{ kg mol/m}^3}}$

$N_A = k_G(p_{AG} - p_{Ai})$

$= 3.95 \times 10^{-9}(4000 - 213)$

$= 1.496 \times 10^{-5}$ kg mol/m^2·s

29.14 CONTINUED

FROM EQUILIBRIUM PLOT –

FOR $c_{AL} = 0.55$ $p_A^* \cong 164$

$$K_G = \frac{N_A}{p_{AG} - p_A^*} = \frac{1.496 \times 10^{-5}}{4000 - 164}$$

$= 3.9 \times 10^{-9}$ kg mol/m^2·s·Pa

& ALSO FROM EQUILIBRIUM PLOT

FOR $p_{AG} = 4000$ $c_A^* \cong 6.9$

$$K_L = \frac{N_A}{c_A^* - c_{AL}} = \frac{1.496 \times 10^{-5}}{6.9 - 0.055}$$

$= 2.36 \times 10^{-6}$ kg mol/m^2·s·(kg mol/m^3)

SUMMARY:

$k_G = 3.95 \times 10^{-9}$ kg mol/m^2·s·Pa

$k_L = 1.1 \times 10^{-4}$ kg mol/m^2·s·(kg mol/m^3)

$K_G = 3.9 \times 10^{-9}$ kg mol/m^2·s·Pa

$K_L = 2.36 \times 10^{-6}$ kg mol/m^2·s·(kg mol/m^3)

$p_{AG} - p_{Ai} = 3787$ Pa (b)

$c_{Ai} - c_{AL} = 0.14$ kg mol/m^3

$p_{AG} - p_A^* = 3836$ Pa

$c_A^* - c_{AL} = 6.35$ kg mol/m^3

$$\frac{1/k_G}{1/K_G} = \frac{K_G}{k_G} = \frac{3.9 \times 10^{-9}}{3.95 \times 10^{-9}} = 0.987$$

~ $\underline{\underline{98.7\%}}$ OF RESISTANCE IS IN GAS φ (c)

29.15 Cl_2 FROM GAS STREAM INTO LIQUID

$P = 1.013 \times 10^5$ Pa $\quad y_{A\infty} = 0.002$

$k_y = 1.0$ kg mol/m^2·h·Δy

$k_x = 10$ kg mol/m^2·h·Δx

$H = 6.13 \times 10^4$ Pa/(kg mol/m^3)

$C_{AL} = 2.6 \times 10^{-3}$ kg mol/m^3

$p_{Ai} = HC_{Ai} \quad y_{Ai} = \frac{HC}{P} x_A = H' x_A$

$C_L = \frac{1000}{18} = 55.55$ kg mol/m^3

$H' = \frac{HC}{P} = \frac{(6.13 \times 10^4)(55.55)}{1.013 \times 10^5} = 33.6$

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y}$$

$$= \frac{1}{10} + \frac{1}{33.6(1)}$$

$K_x = 7.71$ kg mol/m^2·h·Δx (a)

$x_{AL} = \frac{C_{AL}}{C} = \frac{2.6 \times 10^{-3}}{55.55} = 4.68 \times 10^{-5}$

$x_A^* = \frac{y_{AG}}{H'} = \frac{0.002}{33.6} = 5.95 \times 10^{-5}$

$N_A = K_x (x_A^* - x_{AL}) = 7.71(5.95 - 4.68) \times 10^{-5}$

$= 9.79 \times 10^{-5}$ kg mol/m^2·h (b)

$N_A = k_x (x_{Ai} - x_{AL})$

$9.79 \times 10^{-5} = 10(x_{Ai} - 4.68 \times 10^{-5})$

$x_{Ai} = 5.66 \times 10^{-5}$ (c)

29.15 CONTINUED -

$y_{Ai} = H' x_{Ai} = 33.6(5.66 \times 10^{-5})$

$= 1.90 \times 10^{-3}$ (c)

FRACTION OF RESISTANCE IN LIQUID φ:

$$\frac{1/k_x}{1/K_x} = \frac{K_x}{k_x} = \frac{7.71}{10} = 0.771$$

$= 77.1\%$ (d)

29.16 COMPONENT A - FROM LIQ TO GAS

$T = 290$ K $\quad p_{AG} = 4000$ Pa

$P = 1.013 \times 10^5$ Pa $\quad C_{AL} = 4$ kg mol/m^3

60% of RESISTANCE IS IN GAS PHASE

$K_G = 2.46 \times 10^{-8}$ kg mol/m^2·s·Pa

$H = 1400$ Pa/(kg mol/m^3)

$\frac{1/k_G}{1/K_G} = \frac{K_G}{k_G} = 0.6 \quad k_G = \frac{2.46 \times 10^{-8}}{0.6}$

$= 4.1 \times 10^{-8}$ kg mol/m^2·s·Pa (a)

$p_A^* = HC_{AL} = 1400(4) = 5600$ Pa

$N_A = K_G(p_A^* - p_{AG}) = (2.46 \times 10^{-8})(5600 - 4000)$

$= 3.94 \times 10^{-5}$ kg mol/m^2·s

$= k_G(p_{Ai} - p_{AG}) = (4.1 \times 10^{-8})(p_{Ai} - 4000)$

$p_{Ai} = 4961$ Pa (c)

29.16 CONTINUED -

$$c_{Ai} = \frac{p_{Ai}}{H} = \frac{4961}{1400} = 3.54 \ \text{kg mol}/\text{m}^3$$

$$N_A = k_L(c_{AL} - c_{Ai})$$

$$3.94 \times 10^{-5} = k_L(4 - 3.54)$$

$$k_L = 8.56 \times 10^{-5} \ \text{kg mol}/\text{m}^2\cdot\text{s} \qquad (b)$$

$$N_A = K_L(c_{AL} - c_A^*)$$

$$c_A^* = \frac{p_{AG}}{H} = \frac{4000}{1400} = 2.86$$

$$K_L = \frac{3.94 \times 10^{-5}}{4 - 2.86} \qquad (d)$$

$$= 3.46 \times 10^{-5} \ \text{kg mol}/\text{m}^2\cdot\text{s}\cdot(\text{kg mol}/\text{m}^3)$$

29.17 Cl_2 FROM GAS PHASE INTO WATER

(EQUILIBRIUM DATA FOR THIS SYSTEM GIVEN IN PROB 29.1)

$T = 293 \ \text{K}$ $\qquad p_{AG} = 4 \times 10^4 \ \text{Pa}$

$P = 1.013 \times 10^5 \ \text{Pa}$ $\qquad c_{AL} = 1 \ \text{kg}/\text{m}^3$

75% OF RESISTANCE IS IN LIQUID.

$$\frac{p_{AG} - p_{Ai}}{p_{AG} - p_A^*} = 0.25$$

FROM PLOT OF PROB 29.1 DATA

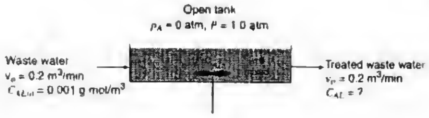
$$p_A^* = 4480 \ \text{Pa}$$

$$\frac{40000 - p_{Ai}}{40000 - 4480} = 0.25 \qquad p_{Ai} = 3.12 \times 10^4 \ \text{Pa}$$

& FROM PLOT — $c_{Ai} = 3.0 \ \text{kg}/\text{m}^3$

29.18

SYSTEM →



$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H k_g}$$

$k_g = 0.01 \ \text{kg mol}/\text{m}^2\cdot\text{s}\cdot\text{atm}$

$H = 10 \ \text{atm}/(\text{kg mol}/\text{m}^3)$

$k_L = 5 \times 10^{-4} (\text{kg mol}/\ldots)$

$$\frac{1}{K_L} = \frac{1}{5 \times 10^{-4}} + \frac{1}{(10)(0.01)} = 2010$$

$$K_L = 4.975 \times 10^{-4} \ \text{kg mol}/\text{m}^2\cdot\text{s}\cdot(\text{kg mol}/\text{m}^3)$$

FRACTION OF RESISTANCE IN LIQUID

$$= \frac{1/k_L}{1/K_L} = \frac{K_L}{k_L} = \frac{4.975 \times 10^{-4}}{5 \times 10^{-4}} = 0.995$$

$$= 99.5\%$$

29.19 Tx FROM BENZENE PHASE TO AQUEOUS ϕ

$$\underset{(Bz)}{c_A'} = 170 \underset{(Aq)}{c_A''}$$

$$k_L' = 3.5 \times 10^{-6} \ \text{kg mol}/\text{m}^2\cdot\text{s}\cdot(\text{kg mol}/\text{m}^3)$$

$$k_L'' = 2.5 \times 10^{-5} \quad "$$

$$\frac{1}{K_L'} = \frac{1}{k_L'} + \frac{A}{k_L''} = \frac{1}{3.5 \times 10^{-6}} + \frac{170}{2.5 \times 10^{-5}} \qquad (a)$$

$$K_L' = 1.41 \times 10^{-7} \ \text{kg mol}/\text{m}^2\cdot\text{s}\cdot(\text{kg mol}/\text{m}^3)$$

29.19 CONTINUED -

$$\frac{1}{K_L''} = \frac{1}{k_L''} + \frac{1}{H k_L'}$$

$$= \frac{1}{2.5\times10^{-5}} + \frac{1}{(170)(3.5\times10^{-6})} \qquad \text{(b)}$$

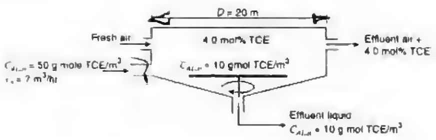
$$\underline{\underline{K_L'' = 2.40\times10^{-5}\ \text{kg mol}/\text{m}^2\cdot\text{s}\cdot(\text{kg mol}/\text{m}^3)}}$$

FRACTION OF RESISTANCE IN AQ FILM

$$\frac{1/k_L''}{1/K_L''} = \frac{K_L''}{k_L''} = \frac{2.40\times10^{-5}}{2.50\times10^{-5}} = \underline{\underline{0.96}}$$

$$= \underline{\underline{96\%}} \qquad \text{(c)}$$

29.20



TCE TRANSFERRED FROM LIQUID TO GAS PHASE -

$T = 293$ K

$P = 1$ ATM

$D = 20$ m

$A = \frac{\pi}{4}(20)^2$

$= 314\ \text{m}^2$

$C_{A\infty} = 10\ \text{gmol}/\text{m}^3$

$y_{A\infty} = 0.04$

$k_x = 200\ \text{gmol}/\text{m}^2\cdot\text{s}$

$k_y = 0.1$ "

$H = 550$ ATM$/\Delta x$

$C_L = 66\ \text{gmol}/\text{m}^3$

$$p_A = H x_A$$

$$y_A = \frac{p_A}{P} = \frac{H}{P} x_A = H' x_A$$

$$H' = \frac{550}{1} = 550$$

29.20 CONTINUED -

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H' k_y} = \frac{1}{200} + \frac{1}{(550)(0.1)}$$

$$K_x = 43.10\ \text{gmol}/\text{m}^2\cdot\text{s}$$

$$K_L = \frac{K_x}{C} = \frac{43.10}{66} = \underline{\underline{0.653\ \text{m/s}}} \qquad \text{(a)}$$

$$N_A = K_x(x_{A\infty} - x_A^*)$$

$$x_{A\infty} = \frac{C_{A\infty}}{C} = \frac{10}{66} = 0.1515$$

$$y_{A\infty} = 550\, x_A^*$$

$$x_A^* = \frac{0.04}{550} = 7.27\times10^{-5}$$

$$N_A = (43.10)(0.1515 - 7.27\times10^{-5})$$

$$= \underline{\underline{6.53\ \text{gmol}/\text{m}^2\cdot\text{s}}} \qquad \text{(b)}$$

$$W_A = N_A \cdot A = 6.53(314)$$

$$= 2050\ \text{gmol/s}$$

MASS BALANCE FOR LIQUID -

$$\dot{V}_0 C_A\big|_{IN} = 2050(3600) + \dot{V}_0 C_A\big|_{OUT}$$

$$\dot{V}_0 = \frac{2050(3600)}{50-10}$$

$$= 1.845\times10^5\ \text{gmol/h}$$

$$= \frac{1.845\times10^5}{66}\ \text{m}^3/\text{h}$$

$$= \underline{\underline{2795\ \text{m}^3/\text{h}}} \qquad \text{(c)}$$

29.21 NH_3 & H_2S STRIPPED FROM H_2O

FOR BOTH -

$k_G = 3.20 \times 10^{-9}$ kgmol/m^2·s·Pa

$k_L = 1.73 \times 10^{-9}$ kgmol/m^2·s·(kgmol/m^3)

$H_{NH_3} = 1.36 \times 10^{3}$ Pa/(kgmol/m^3)

$H_{H_2S} = 8.81 \times 10^{5}$ "

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L}$$

NH_3: $$\frac{1}{K_G} = \frac{1}{3.20 \times 10^{-9}} + \frac{1.36 \times 10^{3}}{1.73 \times 10^{-9}}$$

$$K_G = 2.556 \times 10^{-9} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{Pa}$$

H_2S: $$\frac{1}{K_G} = \frac{1}{3.20 \times 10^{-9}} - \frac{8.81 \times 10^{5}}{1.73 \times 10^{-9}}$$

$$K_G = 1.95 \times 10^{-11} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{Pa}$$

$$\frac{K_{G\,NH_3}}{K_{G\,H_2S}} = \frac{255.6}{1.95} = \underline{131 \text{ TO } 1}$$

29.22 NH_3 ABSORBED

$K_G = 3.12 \times 10^{-9}$ kgmol/m^2·s·Pa

$C_{AL} = 4$ kgmol/m^3

$p_{AG} = 3040$ Pa

$p_{Ai} = (1360 \text{ Pa/(kgmol/m}^3)) C_{Ai}$

75% OF RESISTANCE IS IN GAS ϕ ...

29.22 CONTINUED -

$$\frac{1/k_G}{1/K_G} = \frac{K_G}{k_G} = 0.75 = \frac{3.12 \times 10^{-9}}{k_G}$$

$$k_G = 4.16 \times 10^{-9} \text{ kgmol/m}^2\cdot\text{s}\cdot\text{Pa} \quad (a)$$

$$K_L = HK_G = (1360)(3.12 \times 10^{-9}) \quad (c)$$

$$= 4.24 \times 10^{-6} \text{ kgmol/m}^2\cdot\text{s}\cdot(\text{kgmol/m}^3)$$

25% OF RESISTANCE IN LIQUID PHASE

$$0.25 = \frac{K_L}{k_L} \qquad k_L = \frac{4.24 \times 10^{-6}}{0.25}$$

(b)

$$k_L = 16.96 \times 10^{-6} \text{ kgmol/m}^2\cdot\text{s}\cdot(\text{kgmol/m}^3)$$

$$N_A = K_G(p_A^* - p_{AG})$$

$$p_A^* = HC_{AL} = (1360)(4) = 5440 \text{ Pa}$$

$$N_A = (3.12 \times 10^{-9})(5440 - 3040)$$

$$= 7.488 \times 10^{-6} \text{ kgmol/m}^2\cdot\text{s}$$

$$= k_G(p_{Ai} - p_{AG})$$

$$= (4.16 \times 10^{-9})(p_{Ai} - 3040)$$

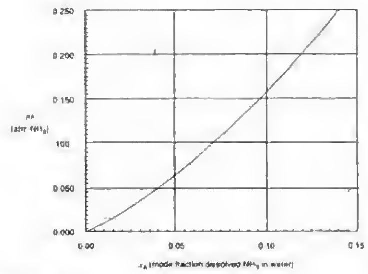
$$p_{Ai} = 4840 \text{ Pa}$$

(d)

$$C_{Ai} = \frac{p_{Ai}}{H} = \frac{4840}{1360} = 3.56 \text{ kgmol/m}^3$$

29.23

NH_3 REMOVAL



$T = 303\ K$

$P = 2\ ATM$

$C_L = 55.6\ kg\ mol/m^3$

$x_{AL} = 0.04 \qquad p_{AG} = 0.2\ ATM$

$k_G = 1.0\ kg\ mol/m^2 \cdot s \cdot ATM$

$k_L = 0.045\ m/s$

$$k_x = C\,k_L = (55.6)(0.045)$$

(a)

$$= \underline{\underline{2.50\ kg\ mol/m^2 \cdot s \cdot ATM}}$$

VALUES FROM EQUILIBRIUM CURVE:

$$p_A = 0.02\ ATM \quad x_A = 0.018$$

$$C_A = C x_A = 55.6(0.018)$$

$$= 1.0\ kg\ mol/m^3$$

$$H = p_A/C_A = 0.02/1$$

$$= 0.02\ ATM/(kg\ mol/m^3)$$

$$\frac{1}{K_g} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{1.0} + \frac{0.02}{0.045}$$

(c)

$$\underline{\underline{K_G = 0.692\ kg\ mol/m^2 \cdot s \cdot ATM}}$$

FRACTION OF RESISTANCE IN GAS ϕ:

$$\frac{1/k_G}{1/K_g} = \frac{K_g}{k_G} = \frac{0.692}{1} = 0.692$$

29.23 CONTINUED –

FOR OPERATING POINT AT $x_A = 0.04$,

$p_A = 0.2 \quad - \quad p_A^* = 0.050\ ATM$

$$\frac{p_{AG} - p_{Ai}}{p_{AG} - p_A^*} = 0.692 = \frac{0.20 - p_{Ai}}{0.20 - 0.05}$$

$$p_{Ai} = 0.096\ ATM \quad \cdot \quad \underline{\underline{x_{Ai} = 0.067}}$$

(FROM CURVE)

(b)

$$y_{Ai} = \frac{p_{Ai}}{P} = \frac{0.096}{2} = \underline{\underline{0.048}}$$

$$N_A = K_G(p_{AG} - p_A^*)$$

$$= (0.692)(0.20 - 0.05)$$

(d)

$$= \underline{\underline{0.104\ kg\ mol/m^2 \cdot s}}$$

29.24 ABSORPTION TOWER – SOLUTE (A)
SOLVENT (B)

$p_{AG} = 1.519 \times 10^4\ Pa$

$C_{AL} = 1.0 \times 10^{-3}\ kg\ mol/m^3$

$N_A = 4 \times 10^{-5}\ kg\ mol/m^2 \cdot s$

$k_G = 3.95 \times 10^{-9}\ kg\ mol/m^2 \cdot s \cdot Pa$

$$H = \frac{3.04 \times 10^3}{1 \times 10^{-3}} = 3.04 \times 10^6\ Pa/(kg\ mol/m^3)$$

$$N_A = k_G(p_{AG} - p_{Ai})$$

$$4 \times 10^{-5} = (3.95 \times 10^{-9})(1.59 \times 10^4 - p_{Ai})$$

$$p_{Ai} = 5070\ Pa$$

$$p_{AG} - p_{Ai} = 15190 - 5070 = \underline{\underline{10120\ Pa}}$$

29.24 CONTINUED-

$$N_A = k_L(C_{Ai} - C_{AL})$$

$$C_{Ai} = \frac{p_{Ai}}{H} = \frac{5.07\times10^{3}}{3.04\times10^{6}}$$

$$= 1.67\times10^{-3}\ \text{kgmol}/m^3$$

$$4\times10^{-5} = k_L\left(1.67\times10^{-3} - 1.0\times10^{-3}\right)$$

$$\underline{\underline{k_L = 0.0597\ m/s}}$$

$$C_{Ai} - C_{AL} = 1.67\times10^{-3} - 1.0\times10^{-3}$$

$$\underline{\underline{= 0.67\times10^{-3}\ \text{kgmol}/m^3}}$$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{3.95\times10^{-9}} + \frac{3.04\times10^{6}}{0.0597}$$

$$\underline{\underline{K_G = 3.29\times10^{-9}\ \text{kgmol}/m^2\cdot s\cdot Pa}}$$

$$N_A = K_G(p_{AG} - p_A^*)$$

$$p_{AG} - p_A^* = \frac{4\times10^{-5}}{3.29\times10^{-9}}$$

$$\underline{\underline{= 1.216\times10^{4}\ Pa}}$$

$$\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{H k_G}$$

$$= \frac{1}{0.0597} + \frac{1}{(3.95\times10^{-9})(3.04\times10^{6})}$$

$$\underline{\underline{K_L = 0.010\ \text{kgmol}/m^2\cdot s\cdot(\text{kgmol}/m^3)}}$$

29.24 CONTINUED-

$$N_A = K_L(C_A^* - C_{AL})$$

$$C_A^* - C_{AL} = \frac{4\times10^{-5}}{0.01} = \underline{\underline{4\times10^{-3}\ \text{kgmol}/m^3}}$$

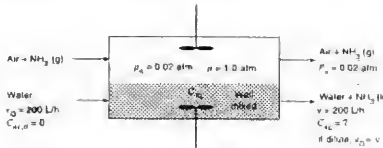
FRACTION OF RESISTANCE IN LIQUID ϕ

$$= \frac{1/k_L}{1/K_L} = \frac{K_L}{k_L} = \frac{0.010}{0.0597} = \underline{\underline{0.167}}$$

(16.7%)

29.25

NH_3 INTO H_2O



$T = 293\ K$ $\qquad$ $p_{AG} = 0.02$ ATM

$P = 1$ Atm $\qquad$ $\mathring{V} = 200\ L/h = 0.2\ m^3/h$

$D = 4\ m$ $\qquad$ $H = 0.02$ Atm$/(\text{kg mol}/m^3)$

$k_G = 1.25\ \text{kgmol}/m^2\cdot h\cdot\text{Atm}$

$k_L = 0.05\ m/h$

$$\frac{1}{K_G} = \frac{1}{k_G} + \frac{H}{k_L} = \frac{1}{1.25} + \frac{0.02}{0.05}$$

$$\underline{\underline{K_G = 0.833\ \text{kgmol}/m^2\cdot h\cdot\text{Atm}}} \quad (a)$$

$$\frac{1/k_G}{1/K_G} = \frac{K_G}{k_G} = \frac{0.0833}{1.25} = \frac{p_{AG} - p_{Ai}}{p_{AG} - p_A^*}$$

$$\frac{0.02 - p_{Ai}}{0.02 - 0} = 0.666 \qquad \underline{\underline{p_{Ai} = 0.0067}} \quad (b)$$

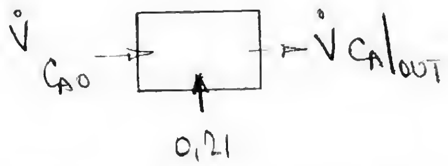
29.25 CONTINUED -

$$W_A = N_A \cdot A_x = K_G(p_{AG} - p_A^*)A$$

$$= (0.833)(0.02-0)\left(\frac{\pi}{4}\right)(4)^2$$

$$\underline{\underline{= 0.21 \text{ kg mol}/h}} \qquad (c)$$

MASS BALANCE:



FOR NH_3: $\dot{V}(0) + 0.21 = \dot{V}\, c_{A\,OUT}$

$$c_{A\,OUT} = \frac{0.21}{0.2} = \underline{\underline{1.05 \text{ kg mol}/m^3}}$$

(d)

CHAPTER 30

30.1 SOLVENT EVAPORATING INTO AIR

$P_0 = 1$ ATM

$T_s = 313\ K$, $T_\infty = 293\ K$ } $T_f = 303\ K$, $P_s = 0.05$ ATM

$C_{AL} = 0.001\ mol/cm^3$

AIR @ 303 K: $\nu = 0.158\ cm^2/s$

$\rho = 1.17 \times 10^{-3}\ g/cm^3$

$Re = \frac{L v_\infty}{\nu} = \frac{(20)(5.0)}{0.158} = 633$ } LAMINAR (a)

$D_{AB} = 0.1\left(\frac{303}{298}\right)^{3/2} = 1.025\ cm^2/s$

$Sc = \frac{0.158}{1.025} = \underline{0.154}$ (a)

$Sh = \frac{\bar{k}_c L}{D_{AB}} = 0.664\, Re_L^{1/2} Sc^{1/3}$

$= \underline{9.16}$ (a)

$\bar{k}_c = \frac{9.16\,(1.025)}{20} = 0.469\ cm/s$

$C = \frac{P}{RT} = \frac{1}{(82.06)(303)} = 4.02 \times 10^{-5}\ gmol/cm^3$

$\bar{k}_y = c\bar{k}_c = (4.02 \times 10^{-5})(0.469)$ (b)

$= \underline{1.89 \times 10^{-5}\ gmol/cm^2 \cdot s \cdot \Delta y}$

$W_A = k_y(y_A^* - y_{A\infty})A_x$

$y_A^* = \frac{P_{A0}}{P} = 0.05$

$W_A = (1.89 \times 10^{-5})(0.05 - 0)(20 \times 10)$

$= 1.89 \times 10^{-4}\ gmol/s$

30.1 CONTINUED -

MOLES OF SOLVENT $= \rho V$

$= (0.001)(20)(10)(0.01) = 0.002$

$t = \frac{0.002}{1.89 \times 10^{-4}} = 10.58\ s$ (c)

30.2 NAPHTHALENE SUBLIMING INTO AIR -

$T_s = 290\ K$, $T_\infty = 310\ K$ } $T_f = 300\ K$; $P_A^0 = 26\ Pa$ @ 290 K

$\nu = 1.569 \times 10^{-5}\ m^2/s$ $v = 20\ m/s$

$D_{AB} = 5.61 \times 10^{-6}\left(\frac{300}{290}\right)^{3/2} = 5.90 \times 10^{-6}\ m^2/s$

AT $x = 3\ m$ · $Re_x = \frac{0.3(20)}{1.569 \times 10^{-5}} = 3.82 \times 10^5$

$Sc = \frac{1.569 \times 10^{-5}}{5.90 \times 10^{-6}} = 2.66$

$k_{c\,x} = \frac{D_{AB}}{x} Re^{4/5} Sc^{1/3}$

$= \frac{5.90 \times 10^{-6}}{0.3}(3.82 \times 10^5)^{4/5}(2.66)^{1/3}$

$= \underline{0.0232\ m/s}$ (a)

FROM $0.5\ m < x < 0.75\ m$

$\bar{k}_c = \frac{0.0365\, D_{AB} Sc^{1/3}\left[Re_{0.75}^{4/5} - Re_{0.5}^{4/5}\right]}{0.75 - 0.5}$

$Re_{0.75} = \frac{0.75(20)}{1.569 \times 10^{-5}} = 9.56 \times 10^5$

$Re_{0.5} = \frac{0.5(20)}{1.569 \times 10^{-5}} = 6.37 \times 10^5$

30.2 CONTINUED-

SUBSTITUTING VALUES:

$$\bar{k}_c = 0.020 \text{ m/s}$$

$$W_A = N_A A = \bar{k}_c (c_{As} - c_{A\infty}) A$$

$$c_{As} = \frac{P}{RT} = \frac{26}{(8.314)(290)} = 0.0108 \text{ mol/m}^3$$

$$c_{A\infty} = 0$$

$$W_A = (0.020)(0.0108)(0.25)(1)$$

$$= 5.4 \times 10^{-5} \text{ mol/s}$$

$$= 0.1944 \text{ mol/h}$$

30.3 C_2H_5OH INTO AIR

$$T_f = \frac{289 + 303}{2} = 296 \text{ K} \quad P_{A0} = 6.45 \times 10^{-2} \text{ Atm}$$

$$D_{AB} = 1.32 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\nu = 1.53 \times 10^{-5} \text{ "}$$

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(3)(2)}{1.53 \times 10^{-5}} = 3.92 \times 10^5$$

TAKING FLOW AS LAMINAR FOR $Re \le 2 \times 10^5$
& TURBULENT FOR $Re > 2 \times 10^5$

$$\bar{k}_c = \frac{D_{AB}}{L}\left[0.664\, Re_t^{1/2} Sc^{1/3} + 0.0365\, Sc^{1/3}\left(Re_L^{4/5} - Re_t^{4/5}\right)\right]$$

$$Re_t = 2 \times 10^5 \quad Re_L = 3.92 \times 10^5$$

SUBSTITUTING VALUES:

$$k_c = 5.16 \times 10^{-3} \text{ m/s}$$

30.3 CONTINUED-

$$W_A = N_A A = \bar{k}_c (c_{As} - c_{A\infty}) A$$

$$c_{As} = \frac{P_A^\circ}{RT} = \frac{(6.45 \times 10^{-2})(1.013 \times 10^5)}{(8.314)(289)} = 2.72 \text{ mol/m}^3$$

$$W_A = (5.16 \times 10^{-3})(2.72 - 0)(2 \times 4)$$

$$= 0.112 \text{ mol/s}$$

$$= 0.112(46) = 5.15 \text{ g/s}$$

30.4 MOLECULAR DIFFUSION THROUGH GRAVEL — THEN CONVECTIVE TRANSFER TO AIR —

$T = 288$ K $\qquad v = 2$ cm/s

$P_A^\circ = 1039$ Pa $\qquad L = 10$ m

GRAVEL DEPTH = 1 m

THROUGH GRAVEL- $N_A = \frac{D_{AB}}{\delta}(c_{A1} - c_{A2})$

AT SURFACE $\quad N_A = \bar{k}_c (c_{A2} - c_{A\infty})$

$$Re_L = \frac{L v_\infty}{\nu} = \frac{(10)(0.02)}{1.46 \times 10^{-5}} = 1.37 \times 10^4$$

(LAMINAR)

$$\bar{k}_c = \frac{D_{AB}}{L}(0.664) Re_L^{1/2} Sc^{1/3}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.46 \times 10^{-5}}{5.72 \times 10^{-6}} = 2.55$$

$$\bar{k}_c = \frac{5.72 \times 10^{-6}}{10}(0.664)(1.37 \times 10^4)^{1/2} 2.55^{1/3}$$

$$= 6.07 \times 10^{-5} \text{ m/s}$$

$$c_{A1} = \frac{P_A^\circ}{RT} = \frac{1039}{(8.314)(288)} = 0.434 \text{ mol/m}^3$$

30.4 CONTINUED –

AT STEADY STATE –

$$N_A = \frac{D_{AB}}{\delta}(C_{A1} - C_{A2}) = \bar{k}_c (C_{A2} - 0)$$

$$\frac{5.72\times10^{-6}}{1}(0.434 - C_{A2}) = 6.07\times10^{-5}\, C_{A2}$$

$$C_{A2} = 0.0374 \text{ mol/m}^3$$

$$c = \frac{P}{RT} = \frac{1.013\times10^{5}}{(8.314)(288)} = 42.31 \text{ mol/m}^3$$

$$y_{A2} = \frac{0.034}{42.31} = \underline{\underline{8.84\times10^{-4}}} \qquad (a)$$

$$N_A = (6.07\times10^{-5})(0.0374) = \underline{\underline{2.27\times10^{-6} \text{ mol/m}^2\text{s}}}$$

FOR SAME CONFIGURATION & PROCESS BUT $V_\infty = 50$ cm/s

$$Re_L = \frac{(10)(50)}{1.46\times10^{-5}} = 3.42\times10^{5}$$

{ INTO TURBULENT FLOW REGIME }

FOR $Re \le 2\times10^5$ LAMINAR B.L.

$Re >$ " TURBULENT "

$$\bar{k}_c = \frac{D_{AB}Sc^{1/3}}{L}\left[0.664 Re_{tr}^{1/2} + 0.0365\left(Re_L^{4/5} - Re_{tr}^{4/5}\right)\right]$$

$$Re_{tr} = 2\times10^5$$

$$Re_L = 3.42\times10^5$$

SUBSTITUTING & SOLVING:

$$\bar{k}_c = 4.98\times10^{-4} \text{ m/s}$$

30.4 CONTINUED –

$$\frac{5.72\times10^{-6}}{1}(C_{A1} - C_{A2}) = 4.98\times10^{-4}(C_{A2} - 0)$$

$$C_{A2} = 4.93\times10^{-3} \text{ mol/m}^3$$

$$y_{A2} = \frac{4.93\times10^{-3}}{42.31} = \underline{\underline{1.16\times10^{-4}}} \qquad (b)$$

$$N_A = (4.98\times10^{-4})(4.93\times10^{-3}) = \underline{\underline{2.45\times10^{-6} \text{ mol/m}^2\text{s}}}$$

MASS TX BIOT NO. $= \frac{k_c\delta}{D_{AB}}$

$$= \frac{(6.07\times10^{-5})(1)}{5.72\times10^{-6}} = 10.61 \quad \{\text{CASE (a)}\}$$

$$\frac{1/k_c}{1/\frac{D_{AB}}{\delta}} = \frac{1}{10.61} = 0.094$$

– 9.4 % RESISTANCE IN FLOWING STREAM

FOR CASE (b) $Bi = \frac{k_c\delta}{D_{AB}} = 87.06$

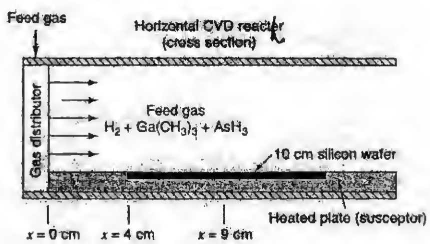
$$\frac{1}{87.06} = 0.0115$$

– 1.15 % RESISTANCE IN AIR STREAM

30.5 REFER TO CHAPTER – EXAMPLE 1 – FOR PROBLEM SPECIFICATIONS –

ARSINE (A)

TMG (B)



$$c = \frac{P}{RT} = \frac{1.013\times10^{5}}{(8.314)(800)} = 15.23 \text{ mol/m}^3$$

30.5 CONTINUED -

$$C_{A\infty} = C_{B\infty} = 0.001(15.23)$$
$$= 0.0152 \text{ mol/m}^3$$

FOR TMG - 1-800 K

$$\left.\begin{aligned} D_{H_2} &= 5.686 \text{ cm}^2/\text{s} \\ D_{AB} &= 1.55 \text{ cm}^2/\text{s} \end{aligned}\right\} Sc = 3.67$$

AT $x = 4$ cm $\quad \underline{Sh_x = k_c = N_A = 0}$

$x = 9$ cm — SEE EXAMPLE 1 —

$Sh_x = 8.375 \quad k_c = 0.0144$ m/s

$$N_B = 0.0144\,(0.0152)$$
$$\underline{= 2.19 \times 10^{-4} \text{ mol B/m}^2\cdot\text{s}}$$

$x = 14$ cm

$$Re_x = \frac{V_\infty x}{\nu} = \frac{(100)(14)}{5.686} = 246$$

$$k_{cB} = \frac{D_{AB}}{x}\left[0.332\, Re_x^{1/2}\left(\frac{Sc}{1 - [X/x]^{3/4}}\right)^{1/3}\right]$$

$X/x = 4/14$ — OTHER QUANTITIES KNOWN -

SUBSTITUTING & SOLVING:

$k_c = 0.01048$ m/s

$$\underline{N_B = (0.01048)(0.0152) = 1.59 \times 10^{-4} \text{ mol/m}^2\cdot\text{s}}$$

FOR A:

AT $x = 4$ cm: $\quad \underline{Sh_x = k_c = N_A = 0}$

30.5 CONTINUED -

$$x = 9 \text{ cm} \quad Re_x = \frac{(100)(9)}{5.686} = 158.3$$

$$k_{cA} = \frac{3.17}{9}\left[0.332(158.3)^{1/2}\left(\frac{1.784}{1 - (4/9)^{3/4}}\right)^{1/3}\right]$$
$$= 0.0232 \text{ m/s}$$

$$\underline{N_A = (0.0232)(0.0152) = 3.53 \times 10^{-4} \text{ mol/m}^2\cdot\text{s}}$$

AT 14 cm $\quad Re_x = \dfrac{(100)(14)}{5.686} = 246$

SAME FORMULA BUT $x = 14 \quad Re = 246$

$k_c = 0.0169$ m/s

$$N_A = (0.0169)(0.0152) = \underline{2.57 \times 10^{-4} \text{ mol/m}^2\cdot\text{s}}$$

TO PRODUCE $\dfrac{N_A}{N_B} = 1 \Rightarrow \dfrac{k_{cA}(C_{As} - C_{A\infty})}{k_{cB}(C_{Bs} - C_{B\infty})} = 1$

@ 9 cm: $\quad \dfrac{k_{cA}}{k_{cB}} = \dfrac{0.0232}{0.0144}\,\dfrac{\Delta C_A}{\Delta C_B}$

$$= 1 \quad \text{IF} \quad \frac{\Delta C_A}{\Delta C_B} = 0.62$$

@ 14 cm - $\quad \dfrac{\Delta C_A}{\Delta C_B} = \dfrac{k_{cB}}{k_{cA}} = \dfrac{0.01048}{0.0169} = 0.62$

FOR BOTH CASES -

$$\Delta C_B = C_{Bs} - C_{B\infty} = 0.0152 - C_{B\infty}$$
$$\Delta C_A = C_{As} - C_{A\infty} = 0.0152$$

SO $C_{B\infty}$ SHOULD BE $\dfrac{0.0152 - C_{B\infty}}{0.0152} = 0.62$

OR $\quad \underline{C_B = 0.00578 \text{ mol/m}^3}$

30.5 CONTINUED -

THICKNESS OF GaAs FILM AFTER 120S

- @ $x = 4$ cm $\delta = 0$

@ $x = 9$ cm

GaAs DEPOSITED -

$$= (2.19\times10^{-4})(144)(120) = 3.78\ g/m^2$$

$$\rho_s = 5.8(100)^3 = 5.8\times10^6\ g/m^2$$

$$\delta = \frac{3.78}{5.68\times10^6} = 0.652\times10^{-6}\ m$$

$$\underline{\underline{\sim 0.652\ \mu m}}$$

AT $x = 14$ cm

$$\delta = \frac{(1.59\times10^{-4})(144)(120)}{5.68\times10^6}$$

$$= 0.474\times10^{-6}\ m = \underline{\underline{0.474\ \mu m}}$$

30.6 MASS TX FROM SPHERICAL SURFACE -

$D = 1$ cm $\quad p_A^\circ = 1.17\times10^4$ Pa

$T = 298$ K $\quad M_A = 78$

$P = 1$ ATM $\quad D_{AB} = 0.0962\ cm^2/s$

MASS OF SOLVENT $= (0.12\ g/cm^2)A$

$$= 0.12(\pi)(1)^2 = 0.377\ g$$

$$= 3.77\times10^{-4}\ kg$$

$$W_A = N_A A = k_c(C_{AS} - C_{A\infty})\pi D^2$$

$$\nu = \frac{\mu}{\rho} = \frac{1.85\times10^{-4}}{1.18\times10^{-3}} = 0.1568\ cm^2/s$$

$$D_{AB} = 0.0962\ cm^2/s$$

30.6 CONTINUED -

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.1568}{0.0962} = 1.63$$

IN STAGNANT AIR - $\dfrac{k_c D}{D_{AB}} = 2$

$$k_c = \frac{2(0.0962)}{1} = 0.1924\ cm/s$$

$$C_{AS} = \frac{p_A^\circ}{RT} = \frac{1.17\times10^4}{(8.314)(298)} = 4.72\ mol/m^3$$

$$W = (0.1924)(4.72 - 0)\pi(1)(78)(10^{-6})$$

$$= 2.225\times10^{-4}\ g/s = 2.225\times10^{-7}\ kg/s$$

$$t = \frac{3.77\times10^{-4}}{2.225\times10^{-7}} = \underline{\underline{1694\ s}} \qquad (a)$$

$$= \underline{\underline{0.471\ h}}$$

FOR $v_\infty = 1$ m/s

$$Re = \frac{1(100)}{0.1568} = 638$$

$$Sh = \frac{k_c D}{D_{AB}} = 2 + 0.552 Re^{1/2} Sc^{1/3}$$

$$k_c = \frac{D_{AB}}{D}(\quad) = 1.77\ cm/s$$

$$W_A = 2.225\times10^{-4}\left(\frac{1.77}{0.1924}\right)$$

$$= 20.43\times10^{-7}\ kg/s$$

$$t = \frac{3.77\times10^{-4}}{20.43\times10^{-7}} = \underline{\underline{184\ s}} \qquad (b)$$

30.7 A DIFFUSING THROUGH STAGNANT B — $N_B = 0$

$$N_{Ar} = -\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dr}$$

$$\nabla^2 N_A = \frac{1}{r^2}\frac{d}{dr}\left(r^2 N_{Ar}\right) = 0$$

$\sim r^2 N_{Ar} = \text{CONSTANT}$

$$N_{Ar} R^2 \int_R^\infty \frac{dr}{r^2} = cD_{AB}\int_{y_{AS}}^{y_{A\infty}} -\frac{dy_A}{1-y_A}$$

$$N_{Ar}R^2\left[\frac{1}{R} - 0\right] = cD_{AB}\ln\left[\frac{1-y_{A\infty}}{1-y_{AS}}\right]$$

$$N_{Ar} R = \frac{cD_{AB}}{y_{B\,LM}}\left(y_{AS} - y_{A\infty}\right)$$

$$N_{Ar}\frac{D}{2} = \frac{cD_{AB}}{y_{B\,LM}}\left(y_{AS} - y_{A\infty}\right)$$

$$N_{Ar} = \frac{2D_{AB}}{y_{B\,LM}}\left(c_{AS} - c_{A\infty}\right)$$

$$\Rightarrow k_c = \frac{2D_{AB}}{(y_{B\,LM})D}$$

& FOR $y_{B\,LM} \cong 1$ (DILUTE SOLN OF A)

$$Sh = \frac{k_c D}{D_{AB}} = 2$$

30.8 SPHERICAL PELLET IN CROSSFLOW —

$T = 293\text{ K}$ $\quad \nu = 9.95\times10^{-3}\text{ cm}^2/\text{s}$

$D = 1\text{ cm}$ $\quad D_{AB} = 1.2\times10^{-5}$ "

$v_\infty = 5\text{ cm/s}$ $\quad c_{A\infty} = 0$

30.8 CONTINUED —

$$\frac{k_c D}{D_{AB}} = 2.0 + 0.552\, Re^{1/2} Sc^{1/3}$$

$$Re = \frac{Dv_\infty}{\nu} = \frac{(1)(5)}{9.95\times10^{-3}} = 502$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{9.95\times10^{-3}}{1.2\times10^{-5}} = 9.37$$

SOLVING FOR k_c: $\quad k_c = 0.00141\text{ cm/s}$ (a)

$$V = \frac{\pi D^3}{6} \qquad \frac{dV}{dt} = 2\pi D^2\frac{dD}{dt}$$

$$W_A = \frac{\rho_A}{M_A}\frac{dV}{dt} = k_c\left(c_{AS} - \cancelto{0}{c_{A\infty}}\right)\pi D^2$$

$$\frac{\rho_A}{M_A} 2\cancel{\pi}\cancel{D^2}\frac{dD}{dt} = \cancel{\pi}\cancel{D^2}\, k_c c_{AS}$$

$$\frac{dD}{dt} = \frac{(0.00141)(7\times10^{-4})(110)}{2(2)}$$

$$= 2.71\times10^{-5}\text{ cm/s} = 0.0977\text{ cm/h}$$

$$\frac{dD}{dt} = 0.195\text{ cm/h} \qquad \text{(b)}$$

FOR $D = 0.5\text{ cm}$

$Re_D = 251 \quad k_c = 2.0 + 0.552\, Re^{1/2} Sc^{1/3}$

$= 0.00201\text{ cm/s}$

$$\frac{W_A|_{1.0}}{W_A|_{0.5}} = \frac{0.00141\, \cancel{c_{AS}}\, \cancel{\pi}(1)^2}{0.00201\, \cancel{c_{AS}}\, \cancel{\pi}(0.5)^2}$$

$= 2.806$ (c) {INCREASE BY THIS FACTOR}

30.9 GLUCOSE (A) INTO AQUEOUS STREAM

$T = 298\,K$ $\quad D = 0.3\,cm$

$v = 0.15\,m/s$ $\quad$ {SPHERE}

$D_{AB} = 6.9 \times 10^{-10}\,m^2/s$

FOR Tx INTO A LIQUID STREAM:

$$Re_D = \frac{Dv}{\nu}$$

$$\nu = \frac{0.00091\,kg/m.s}{997\,kg/m^3} = 9.127 \times 10^{-7}\,m^2/s$$

$$Re = \frac{(0.003)(0.15)}{9.127 \times 10^{-7}} = 493$$

$$Sc = \frac{9.127 \times 10^{-7}}{6.9 \times 10^{-10}} = 1322$$

$$Pe = Re\,Sc = 6.521 \times 10^5$$

EQN. (30-8) APPLIES

$$\frac{k_L D}{D_{AB}} = 1.01\,Pe^{1/3}$$

$$k_L = \frac{1.01\,(6.521 \times 10^5)^{1/3}\,6.9 \times 10^{-10}}{0.003}$$

$$= 2.0 \times 10^{-5}\,m/s$$

$$k_L \sim \frac{1}{D}\left(D^{1/3} v^{1/3}\right) \sim \frac{v^{1/3}}{D^{2/3}}$$

FOR D INCREASING k_c DECREASES

FOR v " k_c INCREASES

LARGER EFFECT IS ΔD (b)

30.10 Cl_2 (A) INTO LIQUID (B)

BUBBLE $D = 0.002\,m$

$\rho_B = 1.47\,g/cm^3$ $\quad H = 6.76\,Atm/\Delta x_A$

$\mu_B = 5.2 \times 10^{-4}\,kg/m.s$

$D_{AB} = 5.6 \times 10^{-5}\,cm^2/s$

FOR D < 2.5 mm - EQ (30-14a) APPLIES -

$$k_c = \frac{D_{AB}}{D}\left[0.131\,Gr_{AB}^{1/3}\,Sc^{1/3}\right]$$

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{5.2 \times 10^{-4}}{(1.47 \times 10^3)(5.6 \times 10^{-9})}$$

$$= 63.2$$

$$Gr = \frac{D^3 \rho_L (\rho_L - \rho_G) g}{\mu_L^2}$$

$$\rho_g = \frac{PM}{RT} = \frac{(1.013 \times 10^5)(71)}{(8.314)(298)} = 2.9\,kg/m^3$$

$$\rho_L = 1470\,kg/m^3$$

SUBSTITUTING VALUES

$$k = 2.95 \times 10^{-4}\,m/s$$

$$N_A = k_c(c_{As} - \cancel{c_{A\infty}}^{\,0})$$

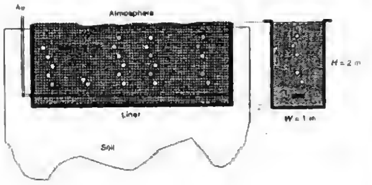
$$c_{As} = x_A c_L = \frac{p_A}{H} c_L = \frac{1}{6.76} c_L$$

$$c_L = \frac{(1470)(1000)}{169.8} = 8660\,mol/m^3$$

$$N_A = (2.95 \times 10^{-4})\left(\frac{8660}{6.76}\right)$$

$$= 0.378\,mol/m^2.s$$

30.11



TCE (A) BEING STRIPPED –

MASS BALANCE FOR A:

$$\left\{\begin{matrix}\text{RATE OF Tx}\\ \text{FROM } H_2O\end{matrix}\right\} = \left\{\begin{matrix}\text{RATE OF DEPLETION}\\ \text{IN } H_2O \text{ PHASE}\end{matrix}\right\}$$

$$N_A A_i = -\frac{dC_A}{dt} \quad \{\text{PER } m^3\}$$

$$K_L A_i C_A = -\frac{dC_A}{dt}$$

FOR LIQUID PHASE Tx CONTROLLING

$$k_L \simeq K_L$$

$$-\frac{dC_A}{dt} = k_L A_i C_A$$

$$-\int_{C_{Ai}}^{C_A} \frac{dC_A}{C_A} = k_L A_i \int_0^t dt$$

$$\ln\left(\frac{C_{Ai}}{C_A}\right) = k_L A_i t \qquad (a)$$

$T = 293\,K \qquad M_A = 131.4$

$$\left.\begin{matrix}\mu_L = 9.93\times10^{-4}\ kg/m.s\\ \rho_L = 998.2\ kg/m^3\end{matrix}\right\} \nu = 9.95\times10^{-7}\ m^2/s$$

$\rho_G = 1.19\ kg/m^3$

$H = 9.97\ \text{Atm}/(\text{kg mol}/m^3)$

$D_{AB} = 8.9\times10^{-10}\ m^2/s$

$$Sc = \frac{9.95\times10^{-7}}{8.9\times10^{-10}} = 1117$$

BUBBLE DIAM $\cong$ 0.005 m

30.11 CONTINUED –

$$A_i\ (\text{Per } m^3) = 0.015\ \frac{m^3\ AIR}{m^3 H_2O}\ \frac{6}{0.005}\ \frac{m^2}{m^3}$$

$$= 18\ m^2/m^3$$

EQN (30-14b) APPLIES:

$$k_L = \frac{D_{AB}}{d_p}(0.42)\,Gr^{1/3}\,Sc^{1/2}$$

$$Gr = \frac{d_p^3 \rho_L g(\rho_L - \rho_G)}{\mu_L^2}$$

SUBSTITUTING VALUES . .

$$k_L = 2.682\times10^{-4}\ m/s$$

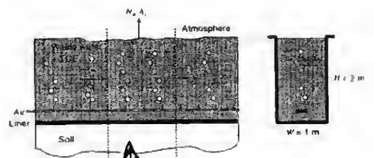
$$k_L A_i = (2.682\times10^{-4})(18) = 0.00482\ s^{-1}$$

$$\ln\left(\frac{50}{0.005}\right) = 0.00482\,t$$

$$t = 1911\ s$$

30.12

SAME SYSTEM AS IN PROB 13.11



MASS BALANCE FOR C.V. {CONSTITUENT A}

$$C_A A v|_z - C_A A v|_{z+\Delta z} = N_A A_i A \Delta z$$

DIVIDE BY $A\Delta z$ & EVALUATE IN LIMIT AS $\Delta z \to 0$

$$-v\frac{dC_A}{dz} = K_L A_i (C_A - C_A^*) \qquad (a)$$

$$C_A^* = \frac{p_A}{H} = 0 \qquad k_L = K_L - \left\{\begin{matrix}\text{LIQUID}\\ \text{PHASE}\\ \text{CONTROLS}\end{matrix}\right\}$$

30.12 CONTINUED -

$$-\int_{C_{A0}}^{C_A} \frac{dC_A}{C_A} = \frac{k_L A_i}{v}\int_0^L dz$$

$A_i = 18\ m^2/m^3$ {SEE SOLN TO PROB 30.11}

$$\ln \frac{C_{A0}}{C_A} = k_L v A_i L$$

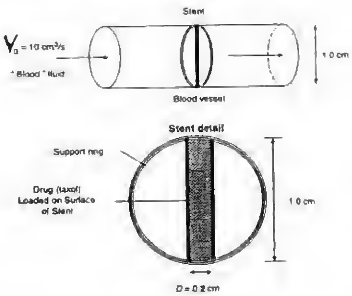
$k_L = 2.682\times10^{-4}\ m/s$

{SEE PROB 30.11 FOR DETAILS}

SUBSTITUTE VALUES & SOLVE -

$\underline{\underline{L = 191.5\ m}}$

30.13



THICKNESS OF COATING = 0.01 CM

MASS OF COATING = 5 mg

$\mu_L = 0.040\ g/cm\cdot s$ $\quad \dot{V} = 10\ cm^3/s$

$\rho_L = 1.05\ g/cm^3$

$D_{AB} = 1\times10^{-6}\ cm^2/s$ $\quad M = 18$

$$v = \frac{\dot{V}}{A} = \frac{10}{\frac{\pi}{4}(1)^2} = 12.73\ cm/s$$

FOR A SINGLE CYLINDER - EQN (30-16) -

$$\frac{k_L Sc^{0.56}}{v} = 0.281\ Re^{-0.4}$$

30.13 CONTINUED

$$Sc = \frac{0.040}{(1.05)(1\times10^{-6})} = 3.8\times10^4$$

$$Re = \frac{(0.2)(12.73)(1.05)}{0.04} = 66.8$$

SUBSTITUTING VALUES: $\underline{\underline{k_L = 0.00181\ cm/s}}$ (a)

$$W = N_A A = k_L(C_A^* - \cancel{C_{A\infty}})(\pi D L)$$

$\cancel{C_{A\infty}} = 0$

$$C_A^* = 2.5\times10^{-4}\ mg/cm^3$$

$$W = (0.00181)(2.5\times10^{-4})(\pi)(0.2)(1)$$

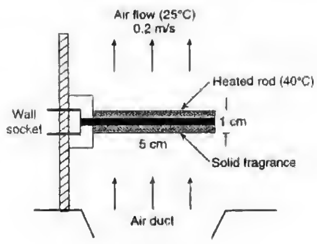
$$= 2.84\times10^{-7}\ mg/s$$

$$t = \frac{5}{2.84\times10^{-7}} = \underline{1.76\times10^7\ s}$$

$= 4890\ h$ (b)

$\cong 204$ DAYS

30.14



$P_A^o = 428\ Pa$

$M_A = 106$

$$T_f = \frac{298+313}{2} = 305.5\ K$$

$\rho_{AIR} = 1.156\ kg/m^3$

$$D_{AB} = (0.08)\left(\frac{305.5}{313}\right)^{3/2} = 0.077\ cm^2/s$$

INITIALLY: $Re = \frac{(1)(20)}{0.01621} = 1234$

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.01621}{0.077} = 2.104$$

EQN (30-16) APPLIES

30.14 CONTINUED -

$$\frac{k_G P Sc^{0.56}}{G_m} = 0.281\, Re^{-0.4}$$

$$P = 1.013 \times 10^5 \text{ Pa}$$

$$G_m = \frac{\rho v}{M} = \frac{(1.156)(0.2)}{29}$$

$$= 0.00797 \text{ kg mol/m}^2\cdot\text{s}$$

SUBSTITUTING VALUES

$$k_G = 2.12 \times 10^{-6} \text{ kg mol/m}^2\cdot\text{s}\cdot\text{Pa}$$

$$N_A = k_G (p_{As} - \cancel{p_{A\infty}}^{\,0})$$

$$= (2.12 \times 10^{-9})(428) = 9.07 \times 10^{-7} \text{ kg mol/m}^2\text{s}$$

$$W_A = N_A A$$

$$= (9.07 \times 10^{-7})(\pi)(0.01)(0.05)$$

$$= 1.425 \times 10^{-9} \text{ kg mol/s}$$

$$= (1.425 \times 10^{-9})(106) = 1.51 \times 10^{-7} \text{ kg/s}$$

$$= \underline{\underline{0.544 \text{ g/h}}} \quad \text{(a)}$$

WHEN A IS DEPLETED

$D = 0.5$ cm ~ $Re_D = 61.7$

SAME PROCEDURE AS IN PART (a)

NEW VALUES:

$$k_G = 2.79 \times 10^{-6} \text{ kg mol/m}^2\cdot\text{s}\cdot\text{Pa}$$

$$N_A = 1.19 \times 10^{-6} \text{ kg mol/m}^2\cdot\text{s}$$

30.14 CONTINUED -

$$W_A = (1.19 \times 10^{-6})(\pi)(0.005)(0.05)$$

$$= 9.35 \times 10^{-10} \text{ kg mol/s}$$

$$W_{A,AVG} = \frac{(1.425 + 0.935) \times 10^{-9}}{2}$$

$$= 1.18 \times 10^{-9} \text{ kg mol/s}$$

TOTAL MASS OF A DEPLETED —

$$m = \frac{\frac{\pi}{4}(D_i^2 - D_f^2)(1.1)(0.4)}{106}$$

{ 0.4 IS FRACTION OF A IN SOLID }

$$= 0.0122 \text{ g mol}$$

$$t = \frac{0.0122}{(1.18 \times 10^{-9})(1000)} = \underline{\underline{10340 \text{ s}}}$$

$$= \underline{\underline{2.87 \text{ h}}}$$

30.15 CONSTITUENT A INTO WATER (B)

CYLINDRICAL FILM — $D_i = 1.8$ cm
$D_o = 2.0$ "

$T = 293$ K

$$\dot{V}_{H_2O} = 314 \text{ cm}^3/\text{s} \quad v = \frac{314}{\frac{\pi}{4}(1.80)^2} = 123 \text{ cm/s}$$

$$\nu_{H_2O} = 0.00995 \text{ cm}^2/\text{s}$$

$$D_{AB} = 1.2 \times 10^{-9} \text{ m}^2/\text{s}$$

$$Sc = \frac{0.00995}{(1.2 \times 10^{-9})(100)^2} = 829$$

$$Re = \frac{(1.8)(123)}{0.00995} = 22{,}250$$

30.15 CONTINUED –

EQN (30-18) APPLIES

$$k_L = \frac{D_{AB}}{D}(0.023)Re^{0.83}Sc^{1/3}$$

SUBSTITUTING VALUES: $k_L = 0.00583$ cm/s

WHEN SCALE HAS BEEN REMOVED

$D = 2$ cm

$$v = \frac{3.14}{\pi/4(2)^2} = 100 \text{ cm/s}$$

SAME PROCEDURE AS ABOVE –

NEW VALUES:

$Re = 20{,}100$

$k_L = 0.00482$ cm/s

$$k_{L_{AVG}} = \frac{0.00583 + 0.00482}{2}$$

$$= 0.005325 \text{ cm/s}$$

$$W_A = k_L(C_{AS} - 0)(\pi D L)$$

$$= (0.005325)(0.14\times10^{-6})(\pi)(2)(100)$$

$$= 4.68\times10^{-7} \text{ gmol/s}$$

MASS OF $CaCO_3$ REMOVED –

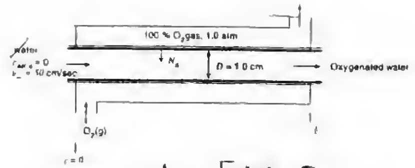
$$m = \rho V = \frac{(2.7)\left(\frac{\pi}{4}\right)(2^2 - 1.8^2)(100)}{100}$$

$$= 1.611 \text{ g mol}$$

$$t = \frac{1.611}{4.68\times10^{-7}} = 3.44\times10^{6} \text{ s}$$

$$= \underline{956 \text{ h}}$$

30.16



O_2 INTO H_2O:

$T = 298$ K

$P = 1$ ATM

$\nu = 9.12\times10^{-3}$ cm²/s

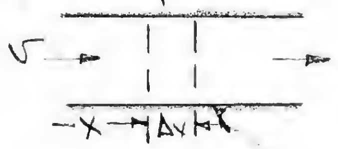
$D_{AB} = 2.10\times10^{-5}$ "

$L = 500$ cm

$H = 0.78$ ATM/(MOL/m³)

$v = 50$ cm/s

FOR MASS TX FROM CYLINDRICAL INTERFACE IN A PIPE:



MASS BALANCE FOR C.V. YIELDS

$$\ln\frac{C_{AS} - C_{A0}}{C_{AS} - C_{AL}} = 4\frac{L}{D}\frac{k_L}{v}$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{9.12\times10^{-3}}{2.10\times10^{-5}} = \underline{\underline{434}} \quad (a)$$

$$Re = \frac{1(50)}{9.12\times10^{-3}} = 5482$$

EQN (30-18) $Sh = 0.023\,Re^{0.83}Sc^{1/3}$

SUBSTITUTING VALUES – $\underline{\underline{Sh = 220}}$ (a)

$$k_L = \frac{D_{AB}}{D}Sh = \frac{2.10\times10^{-5}}{1}(220)$$

$$= 0.00463 \text{ cm/s}$$

$$4\frac{L}{D}\frac{k_L}{v} = 4\frac{(500)}{1}\frac{(0.00463)}{50} = 0.185$$

$$\ln\left[\frac{1.28 - 0}{1.28 - C_{AL}}\right] = 0.185$$

$$C_{AL} = \underline{\underline{0.213 \text{ mol/m}^3}}$$

30.16 CONTINUED -

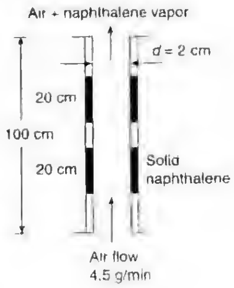
FOR $C_{AL} = 0.6\,C_{AS}$

$$\ln \frac{C_{AS}}{C_{AS}-0.6C_{AS}} = \ln 2.5 = 0.916$$

$$0.916 = \frac{4L}{1}\,\frac{(0.00463)}{50}$$

$$L = 2470 \text{ cm} = \underline{\underline{24.7 \text{ m}}}$$

30.17



NAPHTHALENE - AIR

$T = 373\text{ K}$ $\quad P = 1\text{ ATM}$

$p_0 = 1\text{ mm Hg}$

$D_{AB} = 0.086\ \text{cm}^2/\text{s}$

$\nu_B = 0.25\ \text{cm}^2/\text{s}$

$\rho_B = 9.5\times10^{-4}\ \text{g}/\text{cm}^3$

MASS BALANCE FOR A IN X (UP) DIRECTION

$$C_A v \frac{\pi D^2}{4}\Big|_x + k_c(C_{AS}-C_A)\pi D\Delta x = C_A v \frac{\pi D^2}{4}\Big|_{x+\Delta x}$$

DO ALGEBRA & EVALUATE IN LIMIT AS $\Delta x \to 0$

$$\frac{dC_A}{C_{AS}-C_A} = \frac{4}{D}\frac{k_c}{v}\,dx$$

LEFT-HAND-SIDE:

$$\int \frac{dC_A}{C_{AS}-C_A} = \int_0^{C_{A1}} \frac{dC_A}{C_{AS}-C_A} + \int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_{AS}-C_{A}}$$

$$= \ln\frac{C_{AS}-0}{C_{AS}-C_{A1}} + \ln\frac{C_{AS}-C_{A1}}{C_{AS}-C_{A2}}$$

$$= \ln\frac{C_{AS}}{C_{AS}-C_{A2}}$$

30.17 CONTINUED

RIGHT-HAND-SIDE -

$$\frac{4}{D}\frac{k_c}{v}\int dx = \frac{4}{D}\frac{k_c}{v}\left[\int_0^{20}dx + \int_x^{20+x}dx\right]$$

$$= \frac{4}{D}\frac{k_c}{v}(40)$$

FINAL EXPRESSION IS:

$$\underline{\underline{\ln\frac{C_{AS}}{C_{AS}-C_{AL}} = \frac{4}{D}\frac{k_c}{v}(40)}} \qquad (a)$$

$$c = \frac{P}{RT} = \frac{1}{(82.06)(373)} = 3.267\times10^{-5}\ \text{gmol}/\text{cm}^3$$

$$C_{AL} = y_{AL}c = 0.0066(c) = 2.15\times10^{-7} \quad ''$$

$$C_{AS} = \frac{p_{A0}}{RT} = \frac{0.0316}{(82.06)(373)} = 4.29\times10^{-7} \quad ''$$

$$v = 4.5\ \text{g/m}\left(\frac{\text{m}}{60\text{s}}\right)\left(\frac{\text{cm}^3}{9.5\times10^{-4}\text{g}}\right)\left(\frac{4}{\pi}\right)(2\text{cm})^{-2}$$

$$= 25.13\ \text{cm/s}$$

SUBSTITUTING VALUES & SOLVING:

$$\underline{\underline{k_c = 0.220\ \text{cm/s}}} \qquad (b)$$

$$Re = \frac{Dv}{\nu} = \frac{(2)(25.13)}{0.25} = 201$$

{LAMINAR}

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.25}{0.086} = 2.91$$

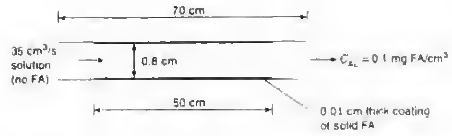
EQN (30-19) APPLIES:

$$k_c = \frac{D}{D_{AB}}(1.86)\left[\frac{D}{L}Re\,Sc\right]^{1/3}$$

WITH VALUES SUBSTITUTED $\underline{\underline{k_c = 0.246\ \text{cm/s}}}$

(c)

30.18



A INTO SOLVENT –

$C_A^* = 20 \text{ mg/cm}^3 \qquad \rho_A = 1.10 \text{ g/cm}^3$

$\nu = 0.02 \text{ cm}^2/\text{s} \qquad C_{AL} = 0.1 \text{ mg/cm}^3$

$\rho_{SOLV} = 1.04 \text{ g/cm}^3$

USUAL MASS BALANCE FOR A TRANSFERRING FROM TUBE WALL – SEE PROB 30.17

$$\int_0^{C_A} \frac{dC_A}{C_{AS}-C_A} = \frac{4}{D}\frac{k_c}{v}\int_0^L dx$$

$$\ln \frac{C_A}{C_{AS}-C_A} = \frac{4L}{D}\frac{k_c}{v} \qquad (a)$$

$$\ln \frac{20}{20-0.1} = 0.00501 = \frac{4(50)}{0.8}\frac{k_c}{v}$$

$$k_c = 2.005 \times 10^{-5}\, v$$

$$v = 35\left(\frac{4}{\pi}\right)(0.8)^{-2} = 69.63 \text{ cm/s}$$

$$k_c = 1.389 \times 10^{-3} \text{ cm/s}$$

$$Re = \frac{Dv}{\nu} = \frac{(0.8)(69.63)}{0.02} = 2793$$

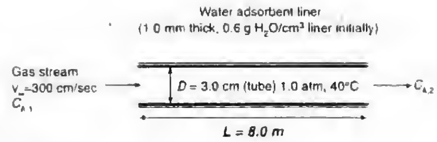
USE EQ. 30-18

$$k_c = \frac{D_{AB}}{D}(0.023)Re^{0.83}\left(\frac{\nu}{D_{AB}}\right)^{1/3}$$

SUBSTITUTING VALUES:

$$D_{AB} = 5.36 \times 10^{-5} \text{ cm}^2/\text{s} \qquad (b)$$

30.19



H_2O INTO AIR

$T = 313 \text{ K} \qquad P = 1 \text{ ATM}$

$p_A^0 = 55.4 \text{ mm Hg} = 0.0729 \text{ ATM}$

$\mu_{AIR} = 1.91 \times 10^{-4} \text{ g/cm.s} \qquad \nu = 0.169 \text{ cm}^2/\text{s}$

$\rho = 1.13 \times 10^{-3} \text{ g/cm}^3$

$$D_{AB} = 0.260\left(\frac{313}{298}\right)^{3/2} = 0.280 \text{ cm}^2/\text{s}$$

$$Sc = \frac{0.169}{0.280} = 0.60 \qquad (a)$$

USUAL MASS BALANCE – SEE PROB 30.17

$$\int_{C_{A1}}^{C_{A2}} \frac{dC_A}{C_A^*-C_A} = \frac{4}{D}\frac{k_c}{v}\int_0^L dx$$

$$\ln \frac{C_A^*-C_{A1}}{C_A^*-C_{A2}} = \frac{4L}{D}\frac{k_c}{v} \qquad (b)$$

$$Re = \frac{(3)(300)}{0.169} = 5325$$

EQN (30-18): $k_c = \frac{D_{AB}}{D}(0.023)Re^{0.83}Sc^{1/3}$

SUBSTITUTING VALUES – $k_c = 2.24 \text{ cm/s}$

$$\ln \frac{C_A^*-C_{A1}}{C_A^*-C_{A2}} = \frac{4(800)}{3}\frac{2.24}{300}$$

$$\frac{C_A^*-C_{A1}}{C_A^*-C_{A2}} = 2877$$

$$C_A^* = \frac{p_A^0}{RT} = \frac{0.0729}{(82.06)(313)} = 2.8 \times 10^{-6} \text{ mol/cm}^3$$

$$C_{A1} = yC = 0.01\left[\frac{1}{(82.06)(313)}\right] = 3.89 \times 10^{-7} \text{ mol/cm}^3$$

30.19 CONTINUED –

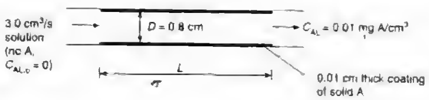
SUBSTITUTING VALUES: $C_{A2} = 28 \times 10^{-6}\ \text{mol}/\text{cm}^3$

MASS OF H_2O ABSORBED –

$$m = 0.6(0.10)(\pi)(800)\,3/18$$
$$= 25.13 \text{ gmol}$$
$$= (C_{A2} - C_{A1})\,v\left(\frac{\pi D^2}{4}\right)t$$
$$= (28 - 3.89)(10^{-7})(300)\left(\frac{\pi}{4}\right)(3)^2\, t$$
$$= 5.112 \times 10^{-3} \text{ gmol/s}$$

$$t = \frac{25.13}{5.112 \times 10^{-3}} = \underline{\underline{4916 \text{ s}}}$$
$$\underline{\underline{1.366 \text{ h}}}$$

30.20



$C_{AS} = 20\ \text{mg}/\text{cm}^3 \qquad \nu = 0.02\ \text{cm}^2/\text{s}$

$C_{AL} = 0.01$ " $\qquad D_{AB} = 4 \times 10^{-5}$ "

$\rho_{SOLID} = 1.10\ \text{g}/\text{cm}^3$

$$v = \frac{\dot{V}}{A} = \frac{(3)(4)}{\pi(0.8)^2} = 5.968 \text{ cm/s}$$

$$Re = \frac{(0.8)(5.968)}{0.02} = 238.7$$
{LAMINAR}

$$Sc = \frac{\nu}{D_{AB}} = \frac{0.02}{4 \times 10^{-5}} = 500$$

EQN (30-19) $k_c = \frac{D_{AB}}{D}(1.86)\left[\frac{D}{L} Re\, Sc\right]^{1/3}$

$$\Rightarrow k_c = 4.234 \times 10^{-4} L^{-1/3}$$

30.20 CONTINUED –

~ USUAL MASS BALANCE –

$$\ln \frac{C_A^* - C_A^0}{C_A^* - C_{AL}} = \frac{4 L k_c}{D\, v}$$

$$\ln \frac{20 - 0}{20 - 0.01} = 5.00 \times 10^{-4}$$
$$= \frac{4(4.234 \times 10^{-4})L^{2/3}}{0.8(5.968)}$$

$$\underline{\underline{L = 1.67 \text{ cm}}}$$

30.21 WETTED WALL COLUMN

$P = 1$ ATM $\qquad D_i = 5$ cm

$T = 300$ K $\qquad L = 600$ cm

$\nu = 0.157\ \text{cm}^2/\text{s} \qquad p_A^0 = 0.035$ ATM

FOR H_2O IN AIR @ 298 K

$$D_{AB} = \frac{0.260}{1}\left(\frac{300}{298}\right)^{3/2} = \underline{\underline{0.131 \text{ cm}^2/\text{s}}} \quad (a)$$

$$v = \dot{V}/A = \frac{4000}{(\pi/4)(5)^2} = 203 \text{ cm/s}$$

$$Re = \frac{Dv}{\nu} = \frac{(5)(203)}{0.157} = 6487$$

EQN (30-18) $k_c = \frac{D_{AB}}{D}(0.023) Re^{0.83} Sc^{1/3}$

$Sc = \frac{0.157}{0.131} = 1.198 \qquad k_c = 0.934$ cm/s

$$Sh = \frac{(0.934)(5)}{0.131} = \underline{\underline{35.65}} \quad (b)$$

$$k_G = \frac{k_c}{RT} = \frac{0.934}{(82.06)(300)} = \underline{\underline{3.79 \times 10^{-5}\ \text{gmol}/\text{cm}^2 \cdot \text{s} \cdot \text{atm}}}$$
(c)

30.21 CONTINUED -

(d)
$$C_A^* = \frac{P_A}{RT} = \frac{0.035}{(82.06)(300)} = 1.42\times10^{-6}\ \text{gmol/cm}^3$$

USUAL MASS BALANCE FOR CYL.

$$\int_{C_{Ai}}^{C_{AL}} \frac{dC_A}{C_A^*-C_A} = \frac{4}{D}\frac{k_c}{v}\int_0^L dx$$

$$\ln\frac{C_A^*-C_{Ai}}{C_A^*-C_{AL}} = \frac{4L}{D}\frac{k_c}{v}$$

$$\ln\frac{1.42\times10^{-6}}{1.42\times10^{-6}-C_{AL}} = \frac{4(600)(0.934)}{(5)(203)}$$

$$C_{AL} = 1.263\times10^{-6}\ \text{gmol/cm}^3$$

30.22 WETTED WALL COLUMN CO_2-H_2O

$T = 293$ K $\quad \rho_L = 998.2\ \text{kg/m}^3$

$P = 2.54$ ATM $\quad \mu_L = 993\times10^{-6}\ \text{kg/m.s}$

$H = 25.5$ ATM/(kg mol/m^3)

$L = 2$ m $\quad \dot{m}_{H_2O} = 2$ gmol/s

$D = 6$ cm $\quad \dot{m}_{CO_2} = 0.5$ "

(a)
$$p_{CO_2} = H C_{CO_2}^* \qquad C_{CO_2}^* = \frac{2.54}{25.4} = 0.1\ \text{kg mol/m}^3$$

$$Re_w = \frac{4\dot{m}_w}{\pi D \mu_w}$$

$$= \frac{4(2)(18)}{\pi(6)(993\times10^{-5})} = 769$$

EQN (30-20) APPLIES

$$k_L = \frac{D_{AB}}{z}(0.433)Sc^{1/2}\left(\frac{gz^3}{\nu_L^2}\right)^{1/6} Re_L^{0.4}$$

30.22 CONTINUED -

FOR CO_2 IN H_2O @ 293K

$D_{AB} = 1.77\times10^{-9}$ m/s

$$Sc = \frac{993\times10^{-6}}{(998.2)(1.77\times10^{-9})} = 562$$

$$\frac{gz^3}{\nu_L^2} = \frac{(9.81)(2)^3}{\left[993\times10^{-6}/998.2\right]^2} = 79.2\times10^{12}$$

(b)
SUBSTITUTING VALUES: $k_L = 2.686\times10^{-3}$ cm/s

USUAL MASS BALANCE:

$$\int_{C_{A0}}^{C_{AL}} \frac{dC_A}{C_A^*-C_A} = \frac{4}{D}\frac{k_c}{v}\int_0^L dz$$

$$Re = 769 = \frac{Dv}{\nu} \sim v = 0.0127\ \text{m/s}$$

$$\ln\frac{C_A^*}{C_A^*-C_A} = \frac{4(2)}{0.06}\left(\frac{2.686\times10^{-3}}{0.0127}\right)$$

$$\frac{C_A^*}{C_A^*-C_A} = 1.3258 = \frac{0.1}{0.1-C_A}$$

(c)
$$C_A = 0.0246\ \text{kg mol/m}^3$$

30.23 FALLING FILM TX TEOS (A) INTO H_2

$\dot{V}_L = 2000$ cm^3/s $\quad T = 333$ K

$D_i = 5$ cm $\quad \nu_G = 1.47$ cm^2/s

$L = 2$ m $\quad D_{AB} = 1.315$ cm^2/s

$p_A^\circ = 2133$ Pa

$$v = \frac{\dot{V}}{A} = \frac{2000}{\frac{\pi}{4}(5)^2} = 101.9\ \text{cm/s}$$

30.23 CONTINUED -

$$Re = \frac{DU}{\nu} = \frac{(5)(101.9)}{1.47} = 346.5$$

{LAMINAR}

EQN (30-19) $k_c = \frac{D_{AB}}{D}(1.86)\left[\frac{D}{L} Re\, Sc\right]^{1/3}$

$$Sc = \frac{1.47}{1.315} = 1.118$$

SUBSTITUTING VALUES: $k_c = 1.042$ cm/s

$$k_G = \frac{k_c}{RT} = \frac{1.042}{(82.06)(333)} = 3.813\times10^{-5}\ \text{gmol/cm}^2\text{s atm}$$

(a)

USUAL MASS BALANCE:

$$\int_{c_{A0}}^{c_{AL}} \frac{dc_A}{c_{AS}-c_A} = \frac{4}{D}\frac{k_c}{U}\int_0^L dx$$

$$\ln \frac{c_{AS}-c_{A0}}{c_{AS}-c_{AL}} = \frac{4L}{D}\frac{k_c}{U}$$

$$= \frac{4(200)}{5}\frac{1.042}{101.9} = 1.637$$

$$\frac{c_{AS}-0}{c_{AS}-c_{AL}} = 5.1387$$

$$c_{AL} = 0.805\, c_{AS}$$

$$c_{AS} = \frac{P_A^0}{RT} = \frac{2133}{(8.314)(333)} = 0.770\ \text{mol/m}^3$$

$$c_{AL} = 0.805(0.770) = 0.620\ \text{mol/m}^3$$

(b)

AT BOTTOM OF COLUMN: $c_{AS} - c_{AL} = 0.770 - 0$ mol/m³

AT TOP: $c_{AS} - c_{AL} = 0.770 - 0.620$ "

30.23 CONTINUED -

$$(c_{AS}-c_{AL})_{LM} = \frac{0.770 - 0.150}{\ln \frac{0.770}{0.150}} = 0.380\ \text{mol/m}^3$$

$$N_A = k_c (c_{AS}-c_{AL})_{LM}$$

$$= \frac{1.042(0.380)}{(100)^3} = 3.96\times10^{-7}\ \text{mol/cm}^2\text{s}$$

$$W_A = N_A A = (3.96\times10^{-7})(\pi)(5)(200)$$

$$= 1.24\times10^{-3}\ \text{mol/s}$$

$$\dot{m}_{H_2} = \frac{P\dot{V}}{RT} = \frac{(1.013\times10^5)(2\times10^{-3})}{(8.314)(333)}$$

$$= 0.0732\ \text{mol/s}$$

$$y_A = \frac{1.24\times10^{-3}}{1.24\times10^{-3} + 0.0732} = 0.0167 \quad \text{(b)}$$

$$\dot{m}_A = (1.24\times10^{-3})(208.33)$$

$$= 0.258\ \text{g/s} \quad \text{(c)}$$

30.24 WETTED-WALL COLUMN
- ETHYL ACETATE (A) INTO AIR

$U = 0.2$ m/s $\quad T = 300$ K $\quad P = 1$ ATM

$D = 0.05$ m $\quad P_A^0 = 0.080$ ATM

$L = 10$ m

$\nu_{AIR} = 1.569\times10^{-5}$ m²/s

$$D_{AB} = 0.0709\left(\frac{300}{273}\right)^{3/2} = 0.0817\ \text{cm}^2/\text{s}$$

$$Sc = \frac{(1.569\times10^{-5})(100)^2}{0.0817} = 1.92$$

$$Re = \frac{DU}{\nu} = \frac{(0.05)(0.2)}{1.569\times10^{-5}} = 637$$

{LAMINAR}

30,24 CONTINUED -

EQN (30-19): $k_c = \frac{D_{AB}}{D}(1.86)\left[\frac{D}{L} Re Sc\right]^{1/3}$

SUBSTITUTING VALUES: $\underline{\underline{k_c = 5.55 \times 10^{-4}\ m/s}}$ (a)

THE USUAL MASS BALANCE YIELDS:

$$\int_{c_{A0}}^{c_{AL}} \frac{dc_A}{c_{As} - c_A} = \frac{4}{D}\frac{k_c}{v}\int_0^x dy$$

$$\ln \frac{c_{As} - 0}{c_{As} - c_{AL}} = \frac{4L}{D}\frac{k_c}{v}$$

$$c_{As} = \frac{p_A^0}{RT} = \frac{0.08}{(82.06)(300)} = 3.25 \times 10^{-6}\ gmol/cm^3$$

$$= 3.25\ gmol/m^3$$

$$\ln \frac{3.25}{3.25 - c_{AL}} = \frac{4(10)}{0.05}\ \frac{5.55 \times 10^{-4}}{0.2}$$

GIVING $\underline{\underline{c_{AL} = 2.90\ gmol/m^3}}$

$$\dot{m} = c_{AL} v A$$

$$= 2.90(0.2)\left(\frac{\pi}{4}\right)(0.05)^2(3600)$$

$$\underline{\underline{= 4.1\ gmol/h}}$$

30,25 OZONE BUBBLED INTO H_2O

$T = 293\ K$ $V_{TANK} = 2\ m^3$

$P = 1\ ATM$

$c_A = 4\ gmol/m^3$ AFTER 10 m

$H = 6.67 \times 10^{-2}\ ATM/(gmol/m^3)$

30,25 CONTINUED -

FOR A WELL-MIXED PROCESS:
ALL OZONE IS DISSOLVED -

MASS BALANCE ON OZONE (A)

$$k_L a (c_A^* - c_A) = \frac{dc_A}{dt}$$

$$\int_0^{c_A} \frac{dc_A}{c_A^* - c_A} = k_L a \int_0^t dt$$

$$\ln \frac{c_A^*}{c_A^* - c_A} = k_L a t$$

$$p_A^* = H c_A^* = 6.67 \times 10^{-2}\ c_A^* = 1$$

$$c_A^* = 14.99\ gmol/m^3$$

$$\ln \frac{14.99}{14.99 - 4} = k_L a (10)(60)$$

$$\underline{\underline{k_L a = 5.14 \times 10^{-5}\ s^{-1}}}$$

30,26 USING EQN (30-21) $j_D = 1.17\, Re^{-0.415}$

$$j_D = \frac{k_c}{v_p} Sc^{2/3} = 1.17\, Re^{-0.415}$$

$$k_c = 1.17\, v_p\, Re^{-0.415}\, Sc^{-2/3}$$

AT $T = 311\ K$ $\nu = 1.673 \times 10^{-5}\ m^2/s$

$$D_{AB} = \frac{2.634}{1.013 \times 10^5}\left(\frac{311}{298}\right)^{3/2} = 2.772 \times 10^{-5}\ m^2/s$$

$$Sc = \frac{1.673 \times 10^{-5}}{2.772 \times 10^{-5}} = 0.60$$

30.26 CONTINUED –

$$Re = \frac{Dv}{\nu} = \frac{DG}{\rho \nu} = \frac{DG}{\mu}$$

AT 311 K – $\mu_{AIR} = 1.897 \times 10^{-5}$ Pa.S

$$Re = \frac{(0.00571)(0.816)}{1.897 \times 10^{-5}} = 246$$

SUBSTITUTING VALUES: $k_c = 0.120$ m/s

$$k_G = \frac{k_c}{RT} = \frac{0.120}{(8.314)(311)} = 4.64 \times 10^{-5}\ \text{gmol}/\text{m}^2\text{.s.Pa}$$

$$= 4.64 \times 10^{-8}\ \text{kgmol}/\text{m}^2\text{.s.Pa}$$

$$\underline{\underline{= 4.70 \times 10^{-3}\ \text{kgmol}/\text{m}^2\text{.S.ATM}}}$$

COMPARED WITH EXPERIMENTAL VALUE

$\Delta = 0.28 \times 10^{-3}$ kgmol/m^2.S.ATM ~ 6.33 %

METHOD 2:

$$\epsilon j_D = \frac{2.06}{Re^{0.575}} = \epsilon \frac{k_c}{v} Sc^{2/3}$$

$Re = 246 \quad Sc = 0.6 \quad \epsilon = 0.75$

$$k_c = \frac{2.06 (v_\infty) Re^{-0.575} Sc^{-2/3}}{0.75}$$

$$v = G/\rho = \frac{0.816}{1.136} = 0.718 \text{ m/s}$$

SUBSTITUTING VALUES: $k_c = 0.117$ m/s

$$k_G = \frac{k_c}{RT} = 4.52 \times 10^{-8}\ \text{kgmol}/\text{m}^2\text{.S.Pa}$$

$$\underline{\underline{= 4.58 \times 10^{-3}\ \text{kgmol}/\text{m}^2\text{.S.ATM}}}$$

~ 6.32% DIFFERENT FROM EXPERIMENT

30.27 FOR O_2 TRANSFER –

$$k_L a = 300\ h^{-1}$$

FOR $D_{O_2-H_2O}$ – EQN: (24-52)

$$\frac{D_{AB}\mu_B}{T} = \frac{7.4 \times 10^{-8} (\phi_B M_B)^{1/2}}{V_A^{0.6}}$$

VALUES: $\phi_B = 2.26 \quad M_B = 18 \quad T = 283K$

$V_A = 7.4 \quad \mu_B = 1.45$ cp

$D_{O_2-H_2O} = 2.77 \times 10^{-5}$ cm^2/s

TABLE J.2 $D_{CO_2-H_2O} = 1.46 \times 10^{-5}$ cm^2/s

FILM THEORY – $k_L a \sim D_{AB}^1$

$$k_L a_{CO_2} = k_L a_{O_2} \left[\frac{D_{CO_2-H_2O}}{D_{O_2-H_2O}}\right]^1$$

$$= 300 \left[\frac{1.46 \times 10^{-5}}{2.77 \times 10^{-5}}\right] = \underline{\underline{158\ h^{-1}}}$$

BOUNDARY-LAYER THEORY: $k_L a \sim D_{AB}^{2/3}$

$$k_L a_{CO_2} = 300 [\ \ ]^{2/3} = \underline{\underline{195.8\ h^{-1}}}$$

PENETRATION THEORY: $k_L a \sim D_{AB}^{1/2}$

$$k_L a_{CO_2} = 300 [\ \ ]^{1/2} = \underline{\underline{217.8\ h^{-1}}}$$

30.28 CO_2 INTO H_2O IN PACKED BED

$\dot{m}_w = 5 \text{ kgmol/m}$ $\quad P = 2 \text{ ATM}$

$\dot{m}_{CO_2} = 1 \text{ "}$ $\quad T = 293 K$

$D = 0.25 m$

$\rho_{H_2O} = 55.5 \text{ kgmol/m}^3$

$= 998.2 \text{ kg/m}^3$

$\mu_w = 993 \times 10^{-6} \text{ kg/m.s}$

$H = 25.4 \text{ ATM/(kgmol/m}^3)$

EQN (30-33) $\dfrac{k_L a}{D_{AB}} = \alpha \left(\dfrac{L}{\mu}\right)^{1-n} Sc^{1/2}$

FOR 1-IN. RASCHIG RINGS' $\alpha = 100$

$n = 0.22$

D_{AB} ~ CO_2 IN H_2O @ 293 K

$= 1.77 \times 10^{-9} \text{ m}^2/\text{s}$

$$Sc = \frac{993 \times 10^{-6}}{(998.2)(1.77 \times 10^{-9})} = 562$$

$A_x = \dfrac{\pi}{4}(0.25)^2 = 0.0491 \text{ m}^2$

$= 0.529 \text{ FT}^2$

$L = 5(18)(60)\dfrac{2.2}{0.529}$

$= 22500 \text{ Lbm/h.FT}^2$

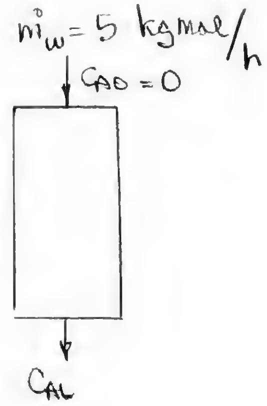
$D_{AB} = 1.77 \times 10^{-9} (0.3048)^2$

$= 6.86 \times 10^{-5} \text{ FT}^2/\text{h}$

SUBSTITUTING VALUES:

$\underline{\underline{k_L a = 0.0566 s^{-1}}}$ (a)

MASS BALANCE FOR A



$C_A^* = \dfrac{p_{A0}}{H} = \dfrac{2}{25.4}$

$= 0.075 \text{ kgmol/m}^3$

$C_{AL} = 0.95 C_A^*$

$= 0.95(0.075)$

$= 0.07125 \text{ kgmol/m}^3$

MASS BALANCE –

$k_L a (C_A^* - C_A) = v \dfrac{dC_A}{dz}$

$$\int_0^{C_{AL}} \frac{dC_A}{C_A^* - C_A} = \frac{k_L a}{v} \int_0^L dz$$

$$\ln \frac{C_A^*}{C_A^* - 0.95 C_A^*} = \frac{k_L a}{v} L$$

$$L = \frac{v}{k_L a} \ln 20$$

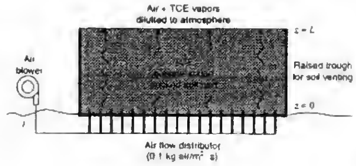
v (ASSUMING EMPTY X-SECTION)

$$= \frac{\dot{m}}{\rho A} = \frac{5}{(60)(55.5)(\pi/4)(0.25)^2}$$

$= 0.0306 \text{ m/s}$

$$L = \frac{(0.0306)(\ln 20)}{0.0566} = \underline{\underline{1.62 m}} \quad (b)$$

30.29



TCE (A) IN AIR

$D_p = 3\text{ mm}$ $\qquad \epsilon = 0.5$

$T = 293\text{ K}$ $\qquad G_B = 0.1\ \text{kg/m}^2\text{s}$

$p_A^0 = 58\text{ mm}$ $\qquad \rho = 1.206\ \text{kg/m}^3$

$D_{AB} = 8.08\times10^{-6}\ \text{m}^2/\text{s}$

$\nu_B = 1.505\times10^{-5}\ \text{m}^2/\text{s}$

$$Sc = \frac{1.505\times10^{-5}}{8.08\times10^{-6}} = 1.863$$

$$Re = \frac{D_p G}{\mu} = \frac{(0.003)(0.1)}{1.815\times10^{-5}} = 16.53$$

EQ IN (30.23) APPLIES:

$$\epsilon \frac{k_c}{v} Sc^{2/3} = 0.25\, Re^{-0.31}$$

$$k_c = \frac{0.25\, v\, Re^{-0.31}}{\epsilon\, Sc^{2/3}}$$

$$v = \frac{G}{\rho} = \frac{0.1}{1.206} = 0.083\ \text{m/s}$$

SUBSTITUTING VALUES: $\underline{\underline{k_c = 0.011\ \text{m/s}}}$ (a)

MASS BALANCE FOR A:

$$k_c \frac{A}{V}(c_A^* - c_A) = v \frac{dc_A}{dz}$$

$$\int_0^{0.9c_A^*} \frac{dc_A}{c_A^* - c_A} = \frac{k_c}{v}\frac{A}{V}\int_0^L dz$$

30.29 CONTINUED –

$$\ln \frac{c_A^*}{c_A^* - 0.9c_A^*} = \frac{k_c}{v}\frac{A}{V} L$$

$$L = \frac{vV}{k_c A} \ln 10$$

$\frac{V}{A}$ IS VOLUME TO SURFACE AREA RATIO OF PARTICLES IN TOWER

FOR A SPHERICAL PARTICLE –

$$A = \pi D^2$$

OCCUPYING A SPACE WITH $V = D^3$ IN TOWER –

$$\sim \frac{V}{A} = \frac{D^3}{\pi D^2} = \frac{D}{\pi}$$

$$L = \frac{v}{k_c}\frac{D}{\pi}\ln 10$$

$$= \frac{0.083\,(0.003)}{0.011\,(\pi)} \ln 10$$

$$= \underline{\underline{0.0166\ \text{m} = 1.66\ \text{cm}}} \quad \text{(b)}$$

CHAPTER 31

31.1 AERATION TANK w/ SPARGERS –

THIS PROBLEM IS SIMILAR TO EXAMPLE 2 IN SECTION 31.2.

FOR A WELL MIXED TANK – EQN (31-1) APPLIES & THE FINAL O_2 LEVEL IS DESCRIBED BY

$$C_A = C_A^* - (C_A^* - C_{A0})\exp(-K_L a t)$$

$$\dot{V}_{AIR} = 0.0078\ m^3/s$$

$$= \frac{(0.0078)(60)}{(0.3048)^3} = 16\ cfm$$

FOR 6 SPARGERS – & FIG 31.7 @ 15 cfm & 15 FT DEPTH

$$K_L a = K_L \frac{A}{V} = \frac{1200(6)}{V}$$

$$= \frac{(1200)(6)}{10{,}000} = 0.72\ h^{-1}$$

$$P_{AVG} = \frac{P_{TOP} + P_{BOTTOM}}{2}$$

$$\frac{1 + [1 + 14.93(0.0295)]}{2} = 1.22\ ATM$$

$$x^*_{O_2} = \frac{P}{H} = \frac{0.21(1.22)}{3.27 \times 10^4} = 7.83 \times 10^{-6}$$

$$C = \frac{1000}{18} = 55.56\ gmol/l$$

$$C_A^* = (7.83 \times 10^{-6})(55.56)$$

$$= 4.35 \times 10^{-4}\ gmol/l$$

SUBSTITUTING VALUES:

FOR $t = 9000\ s = 2.5\ h$

$$C_{O_2 t} = 3.64 \times 10^{-4}\ gmol/l$$

31.2 OZONE/H_2O TREATMENT USING 8 SPARGERS

SYSTEM IS ANALOGOUS TO EXAMPLE 2 –

$$t = \ln\left(\frac{C_A^* - C_{A0}}{C_A^* - C_{At}}\right)\left[\frac{1}{K_L a}\right]$$

FOR $\dot{V}_G = 17.8\ m^3/h = 4.9 \times 10^{-3}\ m^3/s = 10.4\ cfm$

& DEPTH = 3.2 m = 10.5 FT

FIG 31.7 GIVES $K_L \frac{A}{V} V \cong 400\ cfh$

$$K_L a = \frac{(400)(8)}{(80)/(0.3048)^3} = 1.132\ h^{-1}$$

BY PENETRATION THEORY:

$$\frac{K_L a|_{O_3}}{K_L a|_{O_2}} = \left[\frac{D_{O_3-H_2O}}{D_{O_2-H_2O}}\right]^{1/2} = \left[\frac{1.7 \times 10^{-5}}{2.14 \times 10^{-5}}\right]^{1/2} = 0.891$$

$$K_L a|_{O_3} = (1.132)(0.891) = 1.01\ h^{-1} \quad (a)$$

$$P_{AVG} = \frac{1 + (3.2)(0.0295)/0.3048 + 1}{2}$$

$$= 1.155\ ATM$$

$$p_{O_3} = 0.04(1.155) = 0.0462\ ATM$$

$$C^*_{O_3} = \frac{0.0462}{0.0667} = 0.682\ gmol/m^3$$

$$= 0.682\left(\frac{48}{1000}\right) = 32.7\ mg/l$$

$$C_{At} = 0.15\ gmol/m^3 = 7.2\ mg/l$$

SUBSTITUTING VALUES:

$$t = 0.246\ h = 886\ s \quad (b)$$

31.3 WASTEWATER TREATMENT USING 10 SPARGERS –

$V = 425\ m^3 = 15000\ ft^3$

$\dot{V} = 7.08 \times 10^{-3}\ m^3/s = 15\ cfm$

DEPTH = 3.2 m = 10.5 FT.

ANALYSIS PARALLELS EXAMPLE 2.

$$t = \ln\left(\frac{C_{O_2}^* - C_{O_2 0}}{C_{O_2}^* - C_{O_2 t}}\right)\left[\frac{1}{K_L a}\right]$$

FIG (31.7) $K_L a V = 800\ ft^3/h$

$$K_L a = \frac{(800)(10)}{15000} = 0.533\ h^{-1}$$

$P_{TOP} = 1$ ATM

$P_{BOTTOM} = 1 + (10.5)(0.0295) = 1.31$ ATM

$P_{AVG} = 1.155$ ATM

$p_{O_2} = 0.21\ (1.155) = 0.2425$ ATM

$$X_{O_2}^* = p_{O_2}/H = \frac{0.2425}{3.27 \times 10^4} = 7.42 \times 10^{-6}$$

$$C_L = \frac{1000}{18} = 55.56\ mol/l$$

$$C_{O_2}^* = (7.42 \times 10^{-6})(55.56)$$

$$= 4.12 \times 10^{-4}\ gmol/l$$

$$t = \ln\left[\frac{4.12 \times 10^{-4} - 8 \times 10^{-5}}{4.12 \times 10^{-4} - 2 \times 10^{-4}}\right]\left(\frac{1}{0.533}\right)$$

$$= 0.841\ h = 3028\ s$$

31.4 O_2 ABSORPTION USING 1 SPARGER

$V = 28.3\ m^3 = 1000\ ft^3$

$\dot{V} = 7.08 \times 10^{-3}\ m^3/s = 15\ cfm$

DEPTH = 3.2 m = 10.5 FT

FROM ANALYSIS ACCOMPANYING EXAMPLE 2

$$t = \frac{1}{K_L a} \ln \frac{C_{O_2}^* - C_{O_2 i}}{C_{O_2}^* - C_{O_2 t}}$$

$C_{O_2 i} = 0.04$ mmol/l

$C_{O_2 t} = 0.25$ "

FOR 10.5 FT DEPTH:

$P_{TOP} = 1$ ATM $\quad P_{BOTTOM} = 1 + (10.5)(0.0295) = 1.31$ ATM

$P_{AVG} = 1.155$ ATM

$p_{O_2} = 0.21(1.155) = 0.2426$ ATM

$$X_{O_2}^* = \frac{p_{O_2}}{H} = \frac{0.2426}{3.27 \times 10^4} = 7.42 \times 10^{-6}$$

$$C = \frac{1000}{18} = 55.56\ gmol/l$$

$$C_{O_2}^* = (7.42 \times 10^{-6})(55.56) = 4.12 \times 10^{-4}\ gmol/l$$

$$= 0.412\ mmol/l$$

FOR $t = 4\ h$ – SUBSTITUTION YIELDS

$$K_L a\big|_{O_2} = 0.208\ h^{-1} \quad \text{FOR 1 SPARGER} \quad (a)$$

$$\frac{K_L a|_{H_2S}}{K_L a|_{O_2}} = \left[\frac{D_{H_2S-H_2O}}{D_{O_2-H_2O}}\right]^{1/2} = \left[\frac{1.4 \times 10^{-5}}{2.14 \times 10^{-5}}\right] = 0.809$$

$$K_L a\big|_{H_2S} = 0.809(0.208) = 0.168\ h^{-1} \quad (b)$$

31.4 CONTINUED -

FOR H_2S — 10 SPARGERS, $K_La = 1.68\ h^{-1}$

$V = 425\ m^3 = 15000\ ft^3$

DEPTH = 10.5 FT $t = 4\ h$

$y_{H_2S} = p_{H_2S} = C^*_{H_2S} = 0$

$$t = 4 = \frac{1}{1.68}\ln\left[\frac{0 - C_{H_2Si}}{0 - C_{H_2St}}\right]$$

$C_{H_2Si} = 0.03\ mmol/l$

(C)

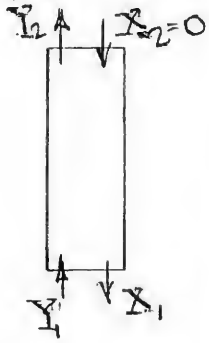
SOLVING : $\underline{\underline{C_{H_2St} = 3.61\times10^{-4}\ gmol/l}}$

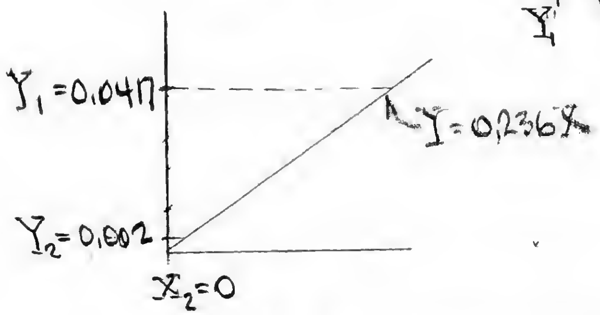
31.5 COUNTERCURRENT ABSORPTION TOWER

$X_1 = ?,\ X_2 = 0$

$Y_1 = \frac{0.04}{0.96} = 0.0417$

$Y_2 = \frac{0.002}{0.998} = 0.002$





$$\left.\frac{L_S}{G_S}\right|_{MIN} = \frac{Y_1 - Y_2}{X_1^* - X_2} = \frac{0.0417 - 0.002}{0.177 - 0} = 0.224$$

$$\left.\frac{L_S}{G_S}\right|_{ACTUAL} = (0.224)(1.5) = 0.336\ \frac{\text{MOL SOLV}}{\text{MOL C.G.}}$$

31.5 CONTINUED -

$$0.336 = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0417 - 0.002}{X_1 - 0}$$

$$X_1 = \frac{0.0397}{0.336} = 0.118$$

$$x_1 = \frac{X_1}{1+X_1} = \frac{0.118}{1.118} = \underline{\underline{0.106}}$$

31.6 TCE STRIPPED FROM H_2O IN A COUNTERCURRENT TOWER

FOR DILUTE STREAMS

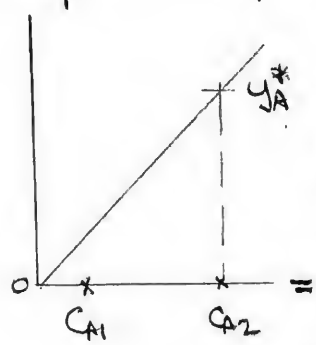
$X \cong x \quad Y \cong y$

$$C_{A2} = \frac{40\ (1000)}{131.5} = 3.04\times10^{-4}\ mol/m^3$$

$$C_{A1} = \frac{5\ (1000)(10^{-6})}{131.5} = 3.80\times10^{-5}\ ''$$

$y_{A1} = 0$

$$y^*_{A2} = \frac{H}{P}C_{A2} = \frac{11.7\times10^{-3}(3.04\times10^{-4})}{1} = 3.557\times10^{-6}$$



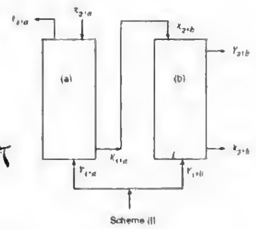
$$\left.\frac{L'}{G'}\right|_{MIN} = \frac{\Delta y}{\Delta C_A} = \frac{3.557\times10^{-6}}{3.04\times10^{-4} - 3.8\times10^{-5}} = 0.013$$

$$\left.\frac{L'}{G'}\right|_{ACTUAL} = \frac{0.013}{3} = 4.33\times10^{-3} = \frac{y_{A2} - 0}{2.66\times10^{-4}}$$

$y_{A2} = 1.15\times10^{-6}$

$p_{A2} = y_{A2}P = \underline{\underline{1.15\times10^{-6}\ ATM}}$

31.7

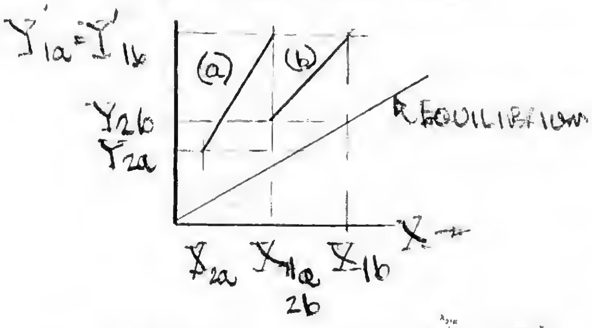


TOWER (a)
- COUNTERCURRENT

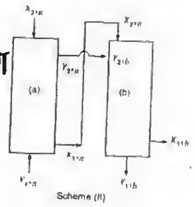
TOWER (b)
- COUNTERCURRENT

$Y_{1a}' = Y_{1b}' > Y_{2b} > Y_{2a}$

$X_{1b} > X_{1a} = X_{2b} > X_{2a}$

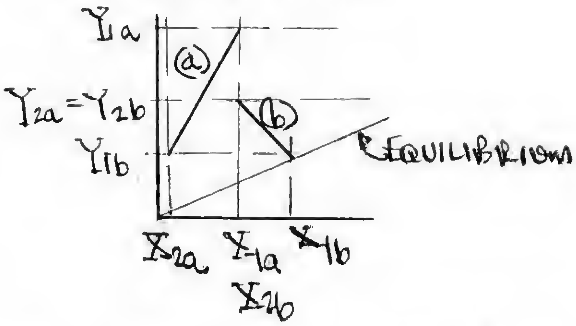


TOWER (a) - COUNTERCURRENT
" (b) - COCURRENT

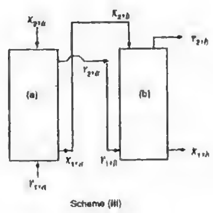


$X_{1b} > X_{1a} = X_{2b} > X_{2a}$

$Y_{1a} > Y_{2a} = Y_{2b} > Y_{1b}$



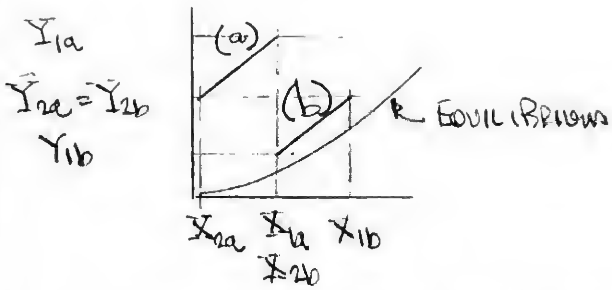
BOTH TOWERS ARE COUNTERCURRENT



$X_{1b} > X_{1a} = X_{2b} > X_{2a}$

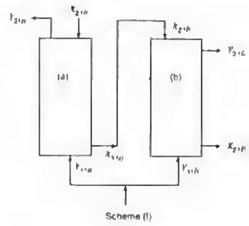
$Y_{1a} > Y_{2a} = Y_{1b} > Y_{2b}$

31.7 CONTINUED



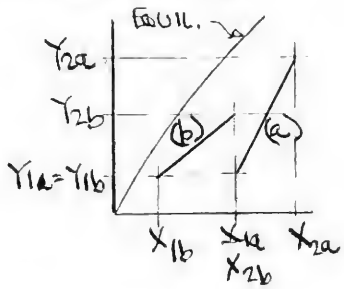
31.8 SAME FLOW SCHEMES AS IN PROB 31.7 EXCEPT PROCESSES ARE NOW DESORPTION/STRIPPING

BOTH ARE COUNTERCURRENT

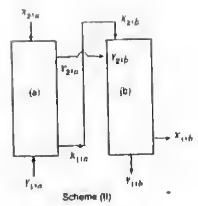


$X_{2a} > X_{1a} = X_{2b} = X_{1b}$

$Y_{2a} > Y_{2b} > Y_{1a} = Y_{1b}$

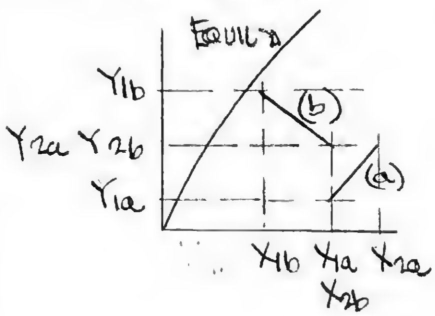


(a) COUNTERCURRENT
(b) COCURRENT



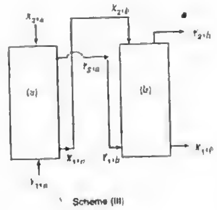
$X_{2a} > X_{1a} = X_{2b} > X_{1b}$

$Y_{1b} > Y_{2a} = Y_{2b} > Y_{1a}$



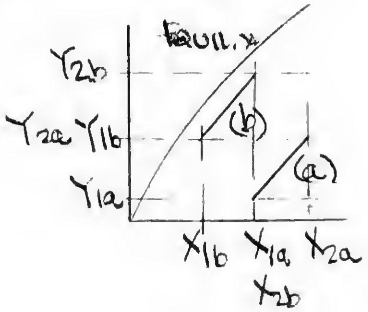
31.8 CONTINUED -

BOTH TOWERS ARE COUNTERCURRENT



$X_{2a} > X_{1a} = X_{2b} > X_{1b}$

$Y_{2b} > Y_{2a} = Y_{1b} > Y_{1a}$



31.9

$X_2 = 0$ $\quad Y_2 = 0.002$

X_1 $\quad Y_1 = \dfrac{0.05}{0.95} = 0.526$

$G_1 = 136 \text{ mol}/m^2 \cdot s$

$G_S = G_1(1-y_1)$

$= 136(0.95)$

$= 129.2 \text{ mol air}/m^2 \cdot s$

$L_S = L_2 = \dfrac{3400}{18}$

$= 188.9 \text{ mol } H_2O/m^2 \cdot s$

$L_S(X_1 - X_2) = G_S(Y_1 - Y_2)$

$188.9(X_1 - 0) = 129.2(0.0526 - 0.002)$

$X_1 = 0.0346$

$x_1 = \dfrac{X_1}{1+X_1} = \dfrac{0.0346}{1.0346} = \underline{\underline{0.033}}$ (a)

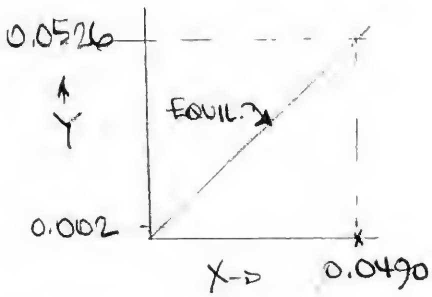
$\left.\dfrac{L_S}{G_S}\right|_{ACTUAL} = \dfrac{188.9}{129.2} = 1.462$

31.9 CONTINUED -

EQUILIBRIUM DATA: $y = 1.075x$

$Y = \dfrac{y}{1-y} \quad X = \dfrac{x}{1-x}$

Y	y	x	X
0	0	0	0
0.0054	0.0054	0.005	0.005
0.0109	0.0108	0.01	0.0101
0.0220	0.0215	0.02	0.0204
0.0333	0.0323	0.03	0.0309
0.0449	0.0430	0.04	0.0417
0.0568	0.0538	0.05	0.0526



$\left.\dfrac{L_S}{G_S}\right|_{MIN} = \dfrac{0.0526 - 0.002}{0.0490 - 0} = 1.033$

$\dfrac{L_S/G_S|_{ACT}}{L_S/G_S|_{MIN}} = \dfrac{1.462}{1.033} = 1.415$ (b)

MASS BALANCE - REFERENCE IS TOP ~ (2)

$G_S(Y - Y_2) = L_S(X - \cancel{X_2})$ 0

$Y = \dfrac{y_z}{1-y_z} = \dfrac{0.02}{0.98} = 0.0204$

$129.2(0.0204 - 0.002) = 188.9 X_z$

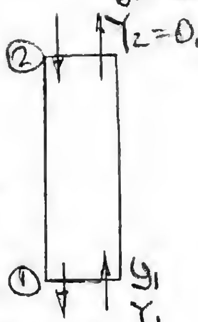
$X_z = 0.01258$

$x_z = \dfrac{0.01258}{1.01258} = \underline{\underline{0.0124}}$ (c)

31.10

$x_2 = 0$
$X_2 = 0$

$y_2 = 0.002$
$Y_2 = 0.002$



$G_1 = 136 \text{ mol/m}^2\cdot s$

x_1
X_1

y_1
Y_1

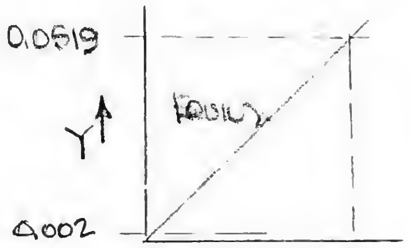
$y_1 = 0.0493$

$Y_1 = \frac{0.0493}{0.9507} = 0.0519$

$G_S = G_1(1-y_1) = 136(0.9507)$

$= 129.3 \text{ mol/m}^2\cdot s$

EQUILIBRIUM DATA — SEE TABLE FOR PROB 31.9



$X_1^* = 0.0480$

$\left.\frac{L_S}{G_S}\right|_{MIN} = \frac{Y_1 - Y_2}{X_1^* - X_2} = \frac{0.0519 - 0.002}{0.0480 - 0}$

$= 1.0396$

$\left.\frac{L_S}{G_S}\right|_{ACTUAL} = 1.4(1.0396) = 1.455$

$= \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0519 - 0.002}{X_1 - 0}$

$X_1 = 0.0345$

MOLAR FLUX OF NH_3 $= G_S(Y_{A1} - Y_{A2})$

31.10 CONTINUED

$= G_S(Y_{A1} - Y_{A2})$

$= 129.3(0.0519 - 0.002)$

$= 6.45 \text{ gmol/m}^2\cdot s \times \frac{17}{1000}$

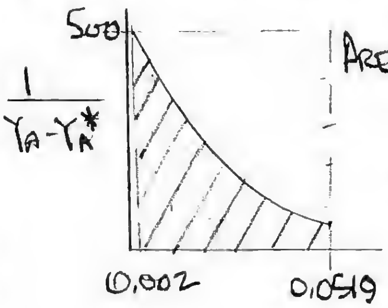
$= 0.1096 \text{ kg/m}^2\cdot s$ (a)

HEIGHT OF PACKING: $Z = \frac{G_S}{K_y a}\int_{0.002}^{0.0519} \frac{dY_A}{Y_A - Y_A^*}$

SINCE INTEGRAND INFORMATION IS NOT IN ANALYTIC FORM – EVALUATION OF Z MUST BE DONE GRAPHICALLY OR NUMERICALLY –

Y_A	Y_A^*	$Y_A - Y_A^*$	$(Y_A - Y_A^*)^{-1}$
0.002	0	0.002	500
0.010	0.0057	0.0043	232.6
0.015	0.0095	0.0055	181.8
0.020	0.0132	0.0068	147.6
0.025	0.0170	0.0080	125.0
0.030	0.0208	0.0092	108.7
0.035	0.0247	0.0103	97.1
0.040	0.0284	0.0116	86.2
0.045	0.0321	0.0129	77.5
0.050	0.0358	0.0142	70.4
0.0519	0.0372	0.0147	68.0

A PLOT WILL YIELD



$\text{AREA} = \int_{0.002}^{0.0519} \frac{dY_A}{Y_A - Y_A^*}$

$= 8.27$

31.10 CONTINUED –

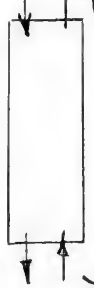
$$Z = \frac{G_S}{K_y a}\int_{0.002}^{0.0ϵ8} \frac{dY_A}{Y_A - Y_A^*}$$

$$= \frac{129.3}{107}(8.27) \cong 10\ m$$

31.11

$x_2 = 0$
$X_2 = 0$

$y_2 = 0.002$
$Y_2 = 0.002$



$y_1 = 0.037$

$$Y_1 = \frac{0.037}{0.963} = 0.0384$$

$$X_1^* = \frac{Y_1}{48.3} = \frac{0.0384}{48.3}$$
$$= 7.95 \times 10^{-4}$$

$$\left.\frac{L_S}{G_S}\right|_{MIN} = \frac{Y_1 - Y_2}{X_1^* - X_2^*}$$
$$= \frac{0.0384 - 0.002}{7.95 \times 10^{-4}} = 45.8$$

(a)

$$\left.\frac{L_S}{G_S}\right|_{ACT} = 1.5(45.8) = \underline{\underline{68.7}}\ \text{Mol } H_2O/\text{Mol C.G.}$$

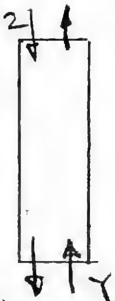
$$= \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.0384 - 0.002}{X_1 - 0}$$

$$X_1 = 5.30 \times 10^{-4}$$

$$x_1 = \frac{X_1}{1 + X_1} = 5.30 \times 10^{-4} \quad (b)$$

31.12

X_2 Y_2



X_1 $Y_1 = 0$

$$X_2 = \frac{0.07}{0.93} = 0.075$$

31.12 CONTINUED –

EQUILIBRIUM DATA →

x mole benzene / mole wash oil	0.00	0.02	0.04	0.06	0.08	0.10	0.12	0.14
y mole benzene / mole steam	0.00	0.07	0.14	0.22	0.31	0.405	0.515	0.65

$L_S' = 6.94$ mol wash oil/s

Benzene in $L_2' = 0.075(6.94)$
$= 0.52$ mol Bz/s

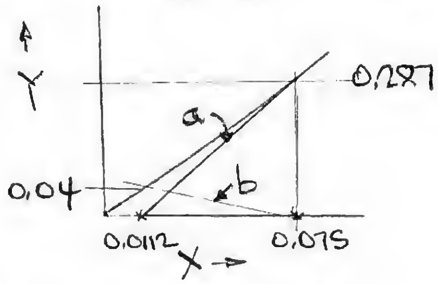
Bz to be removed $= (0.52)(0.85)$
$= 0.442$ mol/s

Bz remaining in liquid $= 0.078$ "

$$X_1 = \frac{0.078}{6.94} = 0.0112\ \frac{\text{mol Bz}}{\text{mol W.O.}}$$

For countercurrent flow streams:

$$\left.\frac{G_S}{L_S}\right|_{MIN} = \frac{X_2 - X_1}{Y_2^* - Y_1} = \frac{0.075 - 0.0112}{0.287 - 0}$$



$G_{S\,MIN} = 0.222(6.94) = 1.54$ mol/s

$G_{S\,ACTUAL} = 1.4(1.54) = \underline{\underline{2.16\ \text{mol/s}}}$ (a)

Cocurrent flow: –

$$\left.\frac{G_S}{L_S}\right|_{MIN} = \frac{X_2 - X_1}{Y_1^* - Y_2} = \frac{0.075 - 0.0112}{0.040 - 0} = 1.595$$

$G_{S\,MIN} = 1.595(6.94) = 11.07$ mol/s

$G_{S\,ACT} = 1.4(11.07) = \underline{\underline{15.5\ \text{mol/s}}}$ (b)

31.13

$x_{A2} = 0$ $\quad y_{A2} = 0.003$

$x_{A1} = 0.0365$ $\quad y_{A1} = 0.05$

$$X_{A1} = \frac{0.0365}{0.9635} = 0.0379$$

$$Y_{A1} = \frac{0.05}{0.95} = 0.0526$$

$$Y_{A2} = \frac{0.003}{0.997} \doteq 0.003$$

$$G_1' = \dot{V}\frac{P}{RT} = \frac{(15/60)(1.013\times10^5)}{(8.314)(289)}$$

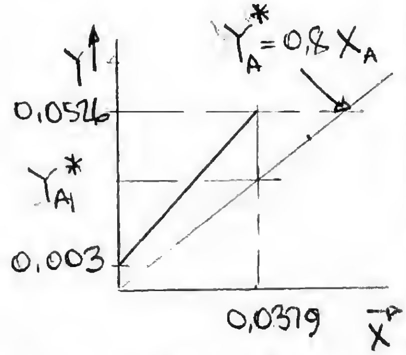
$$= 10.54 \text{ gmol/s}$$

$$G_S' = G_1'(1-y_{A1}) = (10.54)(0.95)$$

$$= 10.01 \text{ gmol/s}$$

$$\frac{G_S'}{L_S'} = \frac{X_{A1}-X_{A2}}{Y_{A1}-Y_{A2}} = \frac{0.0379-0}{0.0526-0.003}$$

$$L_S' = 13.1 \text{ gmol/s}$$



$$Y_{A1}^* = 0.8\,X_{A1} = 0.8(0.0379)$$

$$= 0.0303$$

$$Y_{A1} - Y_{A1}^* = 0.0526 - 0.0303$$

$$= 0.0223$$

$$Y_{A2} - Y_{A2}^* = 0.003 - 0 = 0.003$$

31.13 CONTINUED -

$$Z = \frac{G_S}{K_y a}\,\frac{Y_{A1}-Y_{A2}}{(Y_{A1}-Y_A^*)_{L.M.}}$$

$$(Y_{A1}-Y^*)_{L.M.} = \frac{0.0223-0.003}{\ln\frac{0.0223}{0.003}}$$

$$= 0.0096$$

$$G_S = G_S'/A = \frac{10.01}{0.2} = 50.05 \text{ g mol/m}^2\cdot\text{s}$$

$$Z = \frac{(50.05)(0.0223-0.003)}{(52)(0.0096)}$$

$$\underline{\underline{= 4.97 \text{ m}}}$$

31.14 $\quad x_{A2} = 0$ $\quad X_{A2} = 0$

$y_{A2} = 0.003$

$$Y_{A2} = \frac{0.003}{0.997} \doteq 0.003$$

X_{A1}

$y_{A1} = 0.036$

$$Y_{A1} = \frac{0.036}{0.964} = 0.0373$$

$$A = \frac{\pi}{4}(0.15) = 0.0177 \text{ m}^2$$

$$L_1 = L_S = \frac{14.5}{0.0177} = 819.2 \text{ mol/m}^2\cdot\text{s}$$

$$G_1 = \frac{8}{0.0177} = 452 \text{ mol/m}^2\cdot\text{s}$$

$$G_S = G_1(1-y_1) = 452(0.964)$$

$$= 435.7 \text{ mol/m}^2\cdot\text{s}$$

$$\left.\frac{L_S}{G_S}\right|_{ACT} = \frac{819.2}{435.7} = 1.88$$

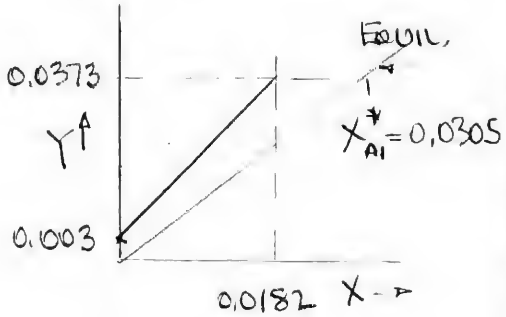
31.14 CONTINUED -

$$\frac{L_S}{G_S} = 1.88 = \frac{Y_{A1}-Y_{A2}}{X_{A1}-X_{A2}} = \frac{0.0373-0.003}{X_{A1}-0}$$

$$X_{A1} = 0.0182$$

EQUILIBRIUM DATA →

x mole NH_3 / mole NH_3-free water	0.00	0.0164	0.0252	0.0349	0.0445	0.0722
y mole NH_3 / mole NH_3-free air	0.00	0.021	0.032	0.042	0.053	0.080



$$\left.\frac{L_S}{G_S}\right|_{MIN} = \frac{Y_{A1}-Y_{A2}}{X^*_{A1}-X_{A2}} = \frac{0.0375-0.003}{0.0305-0}$$

$$= 1.125$$

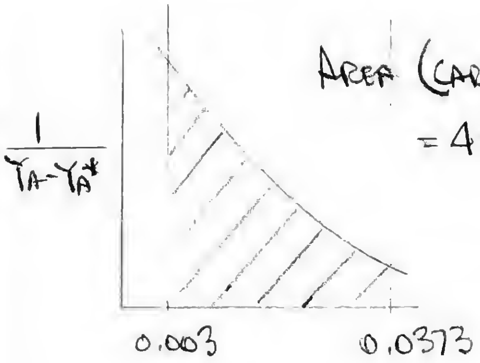
$$\frac{L_S/G_S|_{ACT}}{L_S/G_S|_{MIN}} = \frac{1.88}{1.125} = \underline{\underline{1.67}} \quad (a)$$

USING GRAPHICAL INTEGRATION:

$$Z = \frac{G_S}{K_y a}\int_{0.003}^{0.0373} \frac{dY_A}{Y_A - Y_A^*}$$

Y_{AG}	Y_A^*	$Y_{AG}-Y_A^*$	$(Y_{AG}-Y_A^*)^{-1}$
0.003	0	0.003	333.3
0.010	0.0048	0.0052	192.3
0.015	0.0083	0.0067	149.2
0.020	0.0116	0.0084	119.1
0.025	0.0150	0.010	100.0
0.030	0.0183	0.0117	85.5
0.0373	0.023	0.0143	69.9

31.14 CONTINUED -



AREA (CAREFULLY DONE) = 4.81

$$Z = \frac{435.7}{71}(4.81) = \underline{\underline{29.52\ m}} \quad (b)$$

31.15 SAME SPECS & SYSTEM AS IN PROB 31.14

FROM PROB 31.14 SOLUTION:

$G_S = 435.7$ mol/m²·s

$K_y a = 71$ mol/m²·s·ΔY_A

$$Y_{A1} - Y_{A1}^* = 0.0373 - 0.023 = 0.0143$$

$$Y_{A2} - Y_{A2}^* = 0.003$$

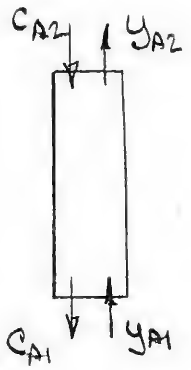
$$(Y_A - Y_A^*)_{L.M.} = \frac{0.0143-0.003}{\ln\frac{0.0143}{0.003}} = 0.0072$$

$$Z = \frac{G_S}{K_y a}\,\frac{Y_{A1}-Y_{A2}}{(Y_A - Y_A^*)_{L.M.}}$$

$$= \frac{435.7}{71}\,\frac{(0.0373-0.003)}{0.0072}$$

$$= \underline{\underline{29.2\ m}}$$

31.16



$$C_{A2} = 0.0394 \times 10^{-3}\ \text{gmol}/\ell$$

$$= 0.0394\ \text{mol}/\text{m}^3$$

$$C_{A1} = 0.0131 \times 10^{-3}\ \text{gmol}/\ell = 0.0131\ \text{mol}/\text{m}^3$$

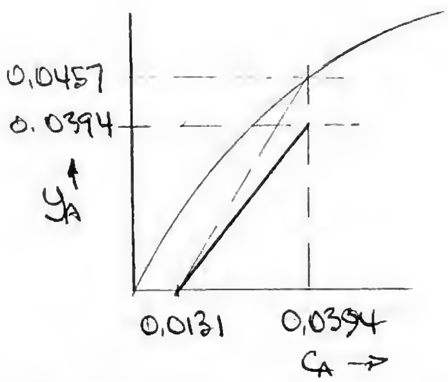
$$y_{A1} = 0$$

$$Q_L' = \frac{5000(3.785)}{1000} = 18.92\ \text{m}^3/\text{h}$$

$$G' = \frac{\mathring{V}_L'}{1.5} = \frac{18.92}{1.5} = 12.62\ \text{mol}/\text{h}$$

EQUILIBRIUM DATA →

c_A, moles VOC/m^3	0.014	0.0240	0.0349	0.0498
y_A, VOC	0.018	0.030	0.042	0.053



SINCE STREAMS ARE RELATIVELY DILUTE Q_L' & G' ARE CONSTANT

$$Q_L'(C_{A1} - C_{A2}) = G'(y_{A1} - y_{A2})$$

31.16 CONTINUED -

$$\left.\frac{Q_L'}{G'}\right|_{\text{MIN}} = \frac{y_{A2}^* - y_{A1}}{C_{A2} - C_{A1}} = \frac{0.0457 - 0}{0.0394 - 0.0131}$$

$$= 1.738$$

$$G'_{\text{MIN}} = \frac{18.92}{1.738} = \underline{\underline{10.89\ \text{mol}/\text{h}}} \quad \text{(a)}$$

$$Q_L'(C_{A1} - C_{A2}) = G_L'(y_{A1} - y_{A2})$$

$$18.92(0.0131 - 0.0394) = 12.62(0 - y_{A2})$$

$$y_{A2} = 0.0394$$

TOWER HEIGHT: $Z = \dfrac{\mathring{V}_L}{K_L a} \dfrac{C_{A2} - C_{A1}}{(C_A - C_A^*)_{L.M.}}$

$$(C_A - C_A^*)_2 = 0.0394 - 0.0305 = 9.0 \times 10^{-3}$$

$$(C_A - C_A^*)_1 = 1.31 \times 10^{-2} - 0 = 0.0131$$

$$(C_A - C_A^*)_{L.M.} = \frac{0.0131 - 0.009}{\ln \frac{0.0131}{0.009}} = 0.0109$$

$$\frac{Q'}{A} = \frac{18.92}{\frac{\pi}{4}(0.6)^2(3600)} = 0.0186\ \text{m/s}$$

$$Z = \frac{0.0186(0.0394 - 0.0131)}{0.01\ (0.0109)}$$

$$\underline{\underline{= 4.49\ \text{m}}} \quad \text{(b)}$$

31.17 THE EXOTHERMIC REACTION WILL CAUSE THE TEMPERATURE IN THE TOWER TO INCREASE, WHICH, IN TURN, WILL CAUSE THE EQUILIBRIUM LINE TO SHIFT UPWARD. THE RESULT WILL BE A SMALLER DRIVING FORCE, $Y_A - Y_A^*$ & A TALLER TOWER WILL BE REQUIRED RELATIVE TO ONE OPERATING ISO-THERMALLY.

31.18

$x_2 = 0$ $\quad y_2 = 0.003$

$X_2 = 0$ $\quad Y_2 = 0.003$

$x_1 = 0.0305$ $\quad y_1 = 0.05$

$Y_1 = \frac{0.05}{0.95} = 0.0526$

$X_1 = \frac{0.0305}{0.9695} = 0.0315$

$G_1' = \frac{\dot{V}P}{RT} = \frac{(0.236)(1.013\times10^5)}{(8.314)(293)}$

$= 9.814 \text{ gmol/s}$

$G_S' = G'(1-y_1) = (9.814)(0.95)$

$= 9.323 \text{ gmol/s}$

$L_S' = \frac{G_S'(Y_1 - Y_2)}{X_1 - X_2}$

$= \frac{9.323(0.0526 - 0.003)}{0.0315 - 0}$

$= 14.68 \text{ gmol/s}$

31.18 CONTINUED.

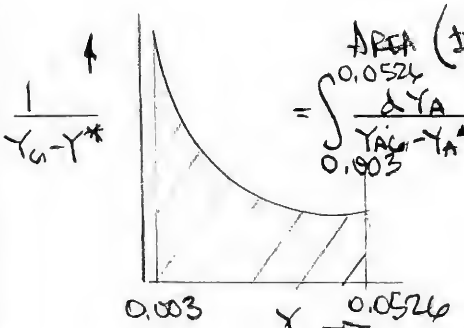
EQUILIBRIUM DATA →

X moles mercaptan / mole mercaptan-free solvent	0.00	0.01	0.02	0.03	0.04
Y moles mercaptan / mole mercaptan free air	0.00	0.0045	0.0145	0.0310	0.0545

TOWER HEIGHT: $Z = \frac{G_S}{K_y a}\int_{Y_2}^{Y_1} \frac{dY_A}{Y_A - Y_A^*}$

USING EQUILIBRIUM DATA PROVIDED:

Y_G	Y^*	$Y_G - Y^*$	$(Y_G - Y^*)^{-1}$
0.003	0	0.003	333.3
0.010	0.0016	0.0084	119.05
0.016	0.0038	0.0122	81.97
0.020	0.0057	0.0143	69.93
0.028	0.0100	0.0180	55.56
0.034	0.0140	0.0200	50.0
0.040	0.0193	0.0207	48.31
0.048	0.0281	0.0199	50.25
0.0526	0.0340	0.0186	53.76



AREA (DONE CAREFULLY)

$= \int_{0.003}^{0.0526} \frac{dY_A}{Y_{AG} - Y_A^*} = 3.67$

$Z = \frac{9.323(3.67)}{(0.2)(40)} = \underline{\underline{4.28 \text{ m}}}$

31.19 SAME SYSTEM & FEED STREAMS AS IN PROB 31.18

FROM PROB 31.18 SOLUTION –

$G_1' = 9.814$ gmol/s

$L_S' = 14.68$ "

IN THIS CASE –

$Z = 4.5$ m $\quad C_f = 155$

1-IN. RASCHIG RINGS

FOR GAS: $\Delta P/Z = 300\ N/m^3$

PARAMETERS FOR FIG 31.25:

$$\frac{L'}{G'}\left[\frac{\rho_G}{\rho_L-\rho_G}\right]^{1/2}$$

$$G_1' = \frac{(9.814)(30.1)}{1000} = 0.295\ kg/s$$

$$L_1 = \frac{L_S}{1-X_1} = \frac{14.68}{1-0.0315}$$

$$= 15.16\ gmol/s$$

$$L_1' = \frac{(15.16)(180)}{1000} = 2.729\ kg/s$$

$$\rho_g = \frac{P}{RT}M = \frac{(1.013\times10^5)(30.1)}{(8.314)(293)(1000)}$$

$$= 1.252\ kg/m^3$$

$$\rho_L = 0.81(1000) = 810\ kg/m^3$$

SUBSTITUTING VALUES –

$$\frac{L'}{G'}\left[\frac{\rho_g}{\rho_L-\rho_G}\right]^{1/2} = 0.364$$

31.19 CONTINUED –

FROM FIG 31.25 –

$$\frac{G'^2 C_f \mu_L^{0.1} J}{\rho_G(\rho_L-\rho_G)g_c} = 0.03$$

SUBSTITUTING VALUES

$\mu_L = 0.0039$ Pa.s

$g_c = 1$

OTHERS ALREADY CALCULATED

$$G' = 0.592\ kg/m^2.s$$

$$\text{TOWER AREA} = \frac{G_1'}{G'} = \frac{0.295}{0.592}$$

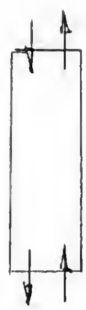
$$= 0.498\ m^2$$

$$D = \left(\frac{0.498}{\pi/4}\right)^{1/2} = \underline{\underline{0.796\ m}}$$

OR $\underline{\underline{\sim 0.8\ m}}$

31.20 $x_{A2} = 0$

$X_{A2} = 0$



$y_{A2} = 0.004$

$$Y_{A2} = \frac{0.004}{0.996} \doteq 0.004$$

x_{A1}

$y_{A1} = 0.125$

$$Y_{A1} = \frac{0.125}{0.875} = 0.143$$

FRACTION OF HCl REMOVED –

$$= \frac{G_S(Y_{A1}-Y_{A2})}{G_S(Y_{A1})} = \frac{0.143-0.004}{0.143}$$

$$= \underline{\underline{0.972 \sim 97.2\%}}\ (a)$$

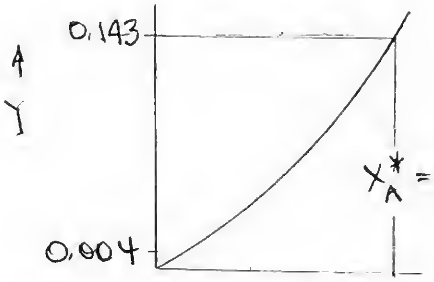
31,20 CONTINUED -

EQUILIB. DATA →

x_{HCl}	0.210	0.243	0.287	0.330	0.353	0.375	0.400	0.425
y_{HCl}	0.0023	0.00956	0.0215	0.0523	0.0852	0.135	0.203	0.322

DETERMINE EQUILIB. VALUES IN TERMS OF X_A, Y_A:

x_A	X_A	y_A	Y_A
0,210	0,266	0,0023	0,0023
0,243	0,321	0,0095	0,0096
0,287	0,403	0,0215	0,0220
0,330	0,493	0,0523	0,0552
0,353	0,546	0,0852	0,0931
0,375	0,600	0,135	0,156
0,400	0,666	0,203	0,255
0,425	0,739	0,322	0,475



$$\left.\frac{L_S}{G_S}\right|_{MIN} = \frac{Y_{A1} - Y_{A2}}{X_{A1}^* - X_{A2}} = \frac{0,143 - 0,004}{0,588 - 0}$$

$$= 0,236$$

$$\left.\frac{L_S}{G_S}\right|_{ACT} = 0,236(1,64) = 0,387$$

$$= \frac{Y_{A1} - Y_{A2}}{X_{A1} - X_{A2}} = \frac{0,143 - 0,004}{X_{A1} - 0}$$

$$X_{A1} = 0,359$$

$$x_{A1} = \frac{X_{A1}}{1 + X_{A1}} = \frac{0,359}{1,359} = \underline{\underline{0,264}} \quad \text{(b)}$$

31,20 CONTINUED -

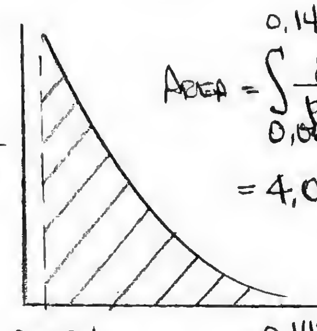
$K_G a = 8.8$ kg mol/m³·s·Pa

WITH $K_G a$ EXPRESSED IN THIS MANNER DRIVING FORCE MUST BE IN $p_A - p_A^*$

$$Z = \frac{G_S}{K_G a} \int_{0,004}^{0,143} \frac{dY_A}{p_{AG} - p_A^*}$$

Y_{AG}	Y_A^*	y_{AG}	y_A^*	p_{AG}
0,004	0	0,004	0	405
0,02	0,0005	0,0196	0,0005	1985
0,04	0,0010	0,0385	0,0010	3900
0,06	0,0015	0,0566	0,0015	5734
0,08	0,0019	0,0741	0,0019	7506
0,10	0,0021	0,0909	0,0021	9208
0,12	0,0060	0,1071	0,0060	10850
0,143	0,015	0,1250	0,0148	12662

p_A^*	$p_{AG} - p_A^*$	$(p_{AG} - p_A^*)^{-1} \times 10^4$
0	405	24,7
50,6	1935	5,17
101	3799	2,63
152	5582	1,79
192	7314	1,37
213	8996	1,11
608	10240	0,98
1499	11160	0,90



$$\text{AREA} = \int_{0,004}^{0,143} \frac{dY_A}{p_{AG} - p_A^*}$$

$$= 4,06 \times 10^{-5}$$

{DONE WITH CARE}

31.20 CONTINUED –

$$\dot{V} = 5\ m^3/m$$

$$G_1' = \frac{\dot{V}P}{RT} = \frac{(5)(1.013\times10^5)}{(8.314)(293)(60)}$$

$$= 3.465\ mol/s$$

$$G_s' = G_1'(1-y_{A1}) = (3.465)(1-0.125)$$

$$= 3.03\ mol/s$$

$$G_s = \frac{3.03}{(\pi/4)(0.6)^2} = 10.72\ mol/m^2\cdot s$$

$$K_g a = 8.8\times10^{-8}\ kg\,mol/m^3\cdot s\cdot Pa$$

$$= 8.8\times10^{-5}\ mol/m^3\cdot s\cdot Pa$$

SUBSTITUTING:

$$z = \frac{10.72\ (4.06\times10^{-5})}{8.8\times10^{-5}}$$

$$\underline{\underline{= 4.94\ m}}$$